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MODELING IMPACTS OF SOCIOECONOMIC STATUS AND VACCINATION PROGRAMS ON TYPHOID FEVER EPIDEMICS

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Abstract. Typhoid fever is one of the most common endemic diseases in tropical and developing countries. Socioeconomic gaps among the populations in these countries may play a major role in the transmission and control of Typhoid fever as well as in the effectiveness of vaccination programs. In this study, we develop a mathematical model that describes impacts of socioeconomic status and vaccination programs on the dynamics of Typhoid epidemics. We establish that the global dynamics of Typhoid is determined by the basic reproduction number, $\mathfrak{R}_0$, which helps identify the socioeconomic condition and vaccination program for successful mitigation of the disease. Using numerical simulations of our model, we show that socioeconomic status plays a significant role in Typhoid dynamics. We find that a low socioeconomic status results in increased Typhoid cases and a higher $\mathfrak{R}_0$ value. Furthermore, increasing vaccination of the low socioeconomic population results in a lower $\mathfrak{R}_0$ value, lower Typhoid infections, and a lower disease prevalence. However, both low and high socioeconomic class populations need to be targeted by vaccination programs to achieve successful disease eradication.

1. Introduction

Typhoid fever is a well documented disease that affects mainly Southern Asia [3, 13], Sub-Saharan Africa [1, 5, 6, 12], and most of other developing countries. It is estimated that over 21 million Typhoid cases occur worldwide each year, with more than half million annual Typhoid deaths, most of which occur in Africa [1, 6, 11, 12]. Several previous studies have assessed Typhoid epidemics and its treatment and control strategies [4, 8, 9, 10, 13, 14]. Edward [4] explored the effects of education as a potential means for eradication, while other studies [8, 9, 10] have used mathematical modeling to evaluate the impact of vaccines on controlling Typhoid fever. Pitzer et al. [13, 14] studied the periodic impact of vaccination and water sanitation methods, and concluded that vaccination alone does not fully clear the Typhoid fever in a given population. While these studies significantly improve knowledge of Typhoid epidemics and its control, much about its quantitative understanding still remains unknown.

Corner et al. [3] observed that socioeconomic status plays an important role in determining the burden of Typhoid fever. In particular, they show that people in

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lower socioeconomic class usually live near lakes, rivers and in environment with poor sanitation, whereas people with higher socioeconomic status class usually live further away from the water sources and in clean environment. Therefore, the living styles governed by the socioeconomic status clearly put people with different socioeconomic categories into different levels of Typhoid burden. As indicated by Watson and Edmunds [19], funding for vaccination programs, such as that from the World Health Organization (WHO), are often limited, causing difficulty for such programs to reach the entire susceptible population. In general, lower class individuals are less educated, and thus less likely to have access to the vaccination programs. Combined all these socioeconomic factors imply that high-class individuals have more access to care, and thus are less likely to become infected and more likely to recover quickly upon infection, compared to low-class individuals. Therefore, including socioeconomic factors into the modeling of Typhoid epidemic dynamics is critical to accurately evaluate prevention strategies, including vaccination programs.

In this study we develop a novel mathematical model to evaluate the effects of socioeconomic status and vaccination programs on the spread of Typhoid. We derive a formula for the basic reproduction number, and analyze how vaccination of high-class and low-class populations affects the basic reproduction number. We establish the local and global stability criteria of our model and compute the important epidemiological quantities, such as new infection and prevalence, over a typhoid epidemic season.

2. Model formulation

We develop a model for transmission dynamics of Typhoid fever by incorporating socioeconomic status into the models based on previous studies [11, 13]. For this, we divide the susceptible population into two classes: susceptible high- and susceptible low-class, denoted by S_h and S_l, respectively. Both susceptible classes may lead to infection (I) - either by person to person infection or by infection from the bacteria in the environment (B). However, the rate at which the low-class susceptible individuals become infected is increased by a factor of $k > 1$ compared to the infection rates β_p (direct person to person) and β_B (indirect through environmental bacteria) of the high-class susceptible individuals. The infected individuals either recover (R) at rate η or become asymptomatic carrier (C) at rate γ. Carriers fully recover from typhoid bacteria at a rate of τ.

We assume that within one season (100 days) of Typhoid epidemic, individuals moving from either high class to low class, or from low class to high class are negligible. σ_h and σ_l denote the per capita rates, at which individuals from high-class and low-class are vaccinated (V). The vaccinated individuals lose effectiveness at a rate of ω. Infected individuals and asymptomatic carriers produce bacteria into the environment at per capita rates p_i and p_c, respectively. Bacteria in the environment also grows logistically with a per capita rate r and carrying capacity κ, and becomes non-infectious at a rate ξ.

We denote μ to be the natural mortality rate and let δ represent the mortality rate due to Typhoid infection. The birth rate of the susceptible population for high class and low class individuals is given by Λ_h and Λ_l, respectively. Table 2 provides the description of all model parameters along with the source of their numerical values and Figure 1 provides a schematic diagram of the model. The full

mathematical model can be given in a system of differential equations as follows:

$$\begin{aligned}
\frac{\mathrm{d}S_h}{\mathrm{d}t} &= \Lambda_h + \omega V - (\beta_p I + \beta_B B + \sigma_h + \mu)S_h,\\
\frac{\mathrm{d}S_l}{\mathrm{d}t} &= \Lambda_l + \omega V - (k(\beta_p I + \beta_B B) + \sigma_l + \mu)S_l,\\
\frac{\mathrm{d}V}{\mathrm{d}t} &= \sigma_h S_h + \sigma_l S_l - (2\omega + \mu)V,\\
\frac{\mathrm{d}I}{\mathrm{d}t} &= (\beta_p I + \beta_B B)S_h + k(\beta_p I + \beta_B B)S_l - (\delta + \mu + \gamma + \eta)I,\\
\frac{\mathrm{d}C}{\mathrm{d}t} &= \gamma I - (\mu + \tau)C,\\
\frac{\mathrm{d}R}{\mathrm{d}t} &= \tau C + \eta I - \mu R,\\
\frac{\mathrm{d}B}{\mathrm{d}t} &= p_i I + p_c C + rB\big(1 - \frac{B}{\kappa}\big) - \xi B.
\end{aligned} \tag{2.1}$$

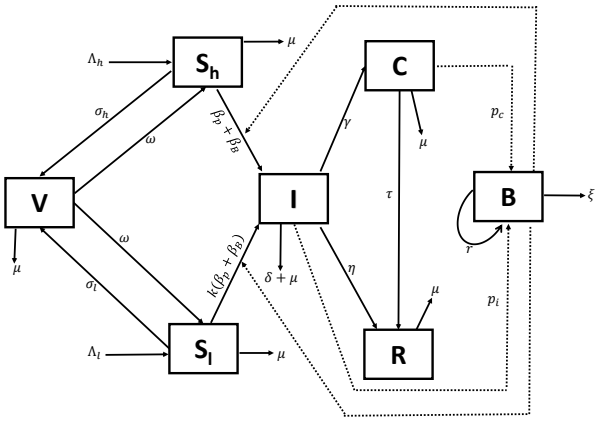

FIGURE 1. The model scheme.

3. Model Analysis

3.1. **Feasibility.** Note that from the system (2.1), the total human population, N, is given by

$$N = S_h + S_l + I + C + R + V.$$

Also, since all parameters are positive, it can be shown that $S_h(t) \geq 0$, and similarly all other state variables are also non-negative.

Adding up all states yields that $\frac{\mathrm{d}N}{\mathrm{d}t} \leq \Lambda - \mu N$, where $\Lambda = \Lambda_h + \Lambda_l$. This implies that as $t \to \infty$, $N \leq \Lambda/\mu$. This shows that $N(t)$ is ultimately bounded. Note also that $\frac{\mathrm{d}B}{\mathrm{d}t} \leq p_i I + p_c C + rB(1 - \frac{B}{\kappa})$. Since $I, C \leq N \leq \frac{\Lambda}{\mu}$, $\frac{\mathrm{d}B}{\mathrm{d}t} \leq (p_i + p_c)\frac{\Lambda}{\mu} + rB(1 - \frac{B}{\kappa})$. As discussed in Mutua et al. [11], we can easily show that B is ultimately bounded. Thus, the solutions of system (2.1) exist globally on the interval $[0, \infty)$ and the model is mathematically well-posed.

TABLE 1. Model parameter values

Parameter Definition	**Parameter Symbol**	**Parameter Value**	**Source**
Natural Birthrate (high class)	Λ_h	168.12 day^{-1}	Estimated
Natural Birthrate (low class)	Λ_l	298.88 day^{-1}	Estimated
Natural Mortality Rate	μ	0.00004 day^{-1}	[11]
Disease-induced Mortality	δ	0.002 day^{-1}	[11]
Rate of progression to carriers	γ	0.04 day^{-1}	[11]
Bacterial growth rate	r	0.014 day^{-1}	Estimated
Bacterial decay rate	ξ	0.0645 day^{-1}	Estimated
Rate of shedding into water supply from infected class	p_i	10 bacteria per individual day^{-1}	[11]
Rate of shedding into water supply from carrier class	p_c	1 bacteria per individual day^{-1}	[11]
Recovery rate from infection	η	0.0657 day^{-1}	[11]
Recovery rate from carriers	τ	0.000315 day^{-1}	[11]
Infection rate (person to person)	β_p	2.1397E-11 day^{-1}	Estimated
Infection rate (bacteria to person)	β_B	1.37E-09 day^{-1}	[11]
Vaccination rate (high class)	σ_h	0 day^{-1}	Varied over [0,1]
Vaccination rate (low class)	σ_l	0 day^{-1}	Varied over [0,1]
Waning rate of vaccination effect	ω	9.0411E-04 day^{-1}	[10]
Modifier for infection rate for low class	k	1.25 day^{-1}	Varied over [1,10]
Basic Reproduction Number	$\mathfrak{R}_0$	18.2	Computed

3.2. **Basic Reproduction Number.** The basic reproduction number, $\mathfrak{R}_0$, is defined as the average number of secondary infections caused by a single infectious individual, introduced into the entire susceptible populations, during his or her

infectious period. We calculated $\mathfrak{R}_0$ for the system (2.1) using the next generation matrix method [18]. For the sake of simplicity in carrying out our analysis, we assume that, in a short period of one Typhoid epidemic season, the vaccine effectiveness does not wane ($\omega = 0$). We, however, note that our computation with $\omega \neq 0$ did not make any noticeable change on the numerical values of the reproduction number. System (2.1) has the following disease free equilibrium

$$\mathcal{E}_0 = \left(\frac{\Lambda_h}{\sigma_h+\mu}, \frac{\Lambda_l}{\sigma_l+\mu}, \frac{\sigma_h\Lambda_h}{\mu(\sigma_h+\mu)} + \frac{\sigma_l\Lambda_l}{\mu(\sigma_l+\mu)}, 0, 0, 0, 0\right)^T.$$

We now introduce the matrices

$$F = \begin{pmatrix} \beta_p(h_1+kh_2) & 0 & \beta_B(h_1+kh_2) \\ 0 & 0 & 0 \\ p_i & p_c & r \end{pmatrix}, \quad V = \begin{pmatrix} h_3 & 0 & 0 \\ -\gamma & h_4 & 0 \\ 0 & 0 & \xi \end{pmatrix}$$

where

$$h_1 = \frac{\Lambda_h}{\sigma_h+\mu}, \quad h_2 = \frac{\Lambda_l}{\sigma_l+\mu},$$
$$h_3 = \delta+\mu+\gamma+\eta, \text{ and } \quad h_4 = \mu+\tau.$$

Then, the basic reproduction number, $\mathfrak{R}_0$, which is the spectral radius of the matrix FV^{-1}, is

$$\mathfrak{R}_0 := \frac{1}{2h_3h_4\xi}\big[h_2h_4k\xi\beta_p + \Psi_4 + \sqrt{\Psi_1 - 2\Psi_2 + 4\Psi_3(h_2k+h_1) + \Psi_4^2 - 2\Psi_5}\big],$$

where

$$\Psi_1 = (h_2h_4k\xi\beta_p)^2 + 2h_2h_1k(h_4\xi\beta_p)^2, \quad \Psi_2 = rh_2h_3h_4^2k\xi\beta_p + rh_1h_3h_4^2\xi\beta_p,$$
$$\Psi_3 = \gamma h_3h_4p_c\xi\beta_B + h_3h_4^2p_i\xi\beta_B, \quad \Psi_4 = h_1h_4\xi\beta_p + rh_3h_4, \quad \Psi_5 = h_1h_3h_4^2r\xi\beta_p.$$

3.3. **Stability analysis.** The following theorem follows from [18, Theorem 2].

Theorem 3.1. *The disease-free equilibrium $\mathcal{E}_0$ of system* (2.1) *is locally asymptotically stable if $\mathfrak{R}_0 < 1$, and unstable if $\mathfrak{R}_0 > 1$.*

Furthermore, we are able to show that $\mathfrak{R}_0$ can also provide the condition for the global stability of $\mathcal{E}_0$. The global asymptotic stability of $\mathfrak{R}_0$ is investigated using the procedure previously implemented for typhoid model by Mutua et al. [11]. We prove the global stability result in the following theorem.

Theorem 3.2. *If $\mathfrak{R}_0 < 1$, the disease-free equilibrium $\mathcal{E}_0$ of the system* (2.1) *is globally asymptotically stable.*

Proof. We define the spectral bound or the stability modulus of an $n \times n$ matrix M, denoted by $s(M)$, by $s(M) := \max\{Re(\lambda) : \lambda \text{ is an eigenvalue of } M\}$.

Using the equations for infectious compartments of the linearized system of (2.1) at $\mathcal{E}_0$, we define the following matrix:

$$J = \begin{bmatrix} \beta_p\left(\frac{\Lambda_h}{\sigma_h+\mu}\right) + k\beta_p\left(\frac{\Lambda_l}{\sigma_l+\mu}\right) - (\delta+\mu+\gamma+\eta) & 0 & \beta_B\left(\frac{\Lambda_h}{\sigma_h+\mu}\right) + k\beta_B\left(\frac{\Lambda_l}{\sigma_l+\mu}\right) \\ \gamma & -(\mu+\tau) & 0 \\ p_i & p_c & (r-\xi) \end{bmatrix}.$$

Clearly, J is irreducible and has non-negative off-diagonal elements. Then $s(J)$ is a simple eigenvalue of J with a positive eigenvector (see,e.g., [15, Theorem A.5]).

Assume that $\mathcal{R}_0 < 1$. Then we have $s(J) < 0$ from the local stability result. Thus, we can find a sufficiently small positive number ρ_0 such that $s(J_{\rho_0}) < 0$ (see, e.g., [7, Section II.5.8]), where

$$J_{\rho_0} = \begin{bmatrix} a_{11} & 0 & a_{13} \\ \gamma & -(\mu+\tau) & 0 \\ p_i & p_c & (r-\xi) \end{bmatrix},$$

$$a_{11} = \beta_p\Big(\frac{\Lambda_h}{\sigma_h+\mu} + \rho_0\Big) + k\beta_p\Big(\frac{\Lambda_l}{\sigma_l+\mu} + \rho_0\Big) - (\delta+\mu+\gamma+\eta),$$
$$a_{13} = \beta_B\Big(\frac{\Lambda_h}{\sigma_h+\mu} + \rho_0\Big) + k\beta_B\Big(\frac{\Lambda_l}{\sigma_l+\mu} + \rho_0\Big)$$

is irreducible and has non-negative off-diagonal elements. From the first and the second equations of the system (2.1), we obtain $\frac{dS_h}{dt} \le \Lambda_h - (\sigma_h+\mu)S_h$ and $\frac{dS_l}{dt} \le \Lambda_l - (\sigma_l+\mu)S_l$. This implies that $S_h(t) \le \hat{S}_h(t) \to \frac{\Lambda_h}{\sigma_h+\mu}$ as $t \to \infty$ and $S_l(t) \le \hat{S}_l(t) \to \frac{\Lambda_l}{\sigma_l+\mu}$ as $t \to \infty$. Then, it follows that there is a $t_1 > 0$ such that

$$S_h(t) \le \frac{\Lambda_h}{\sigma_h+\mu} + \rho_0 \quad \text{and} \quad S_l(t) \le \frac{\Lambda_l}{\sigma_l+\mu} + \rho_0, \ \forall\, t \ge t_1.$$

Now, from system (2.1), we obtain for $t \ge t_1$ that

$$\begin{aligned} \frac{dI}{dt} &\le (\beta_p I + \beta_B B)\Big(\frac{\Lambda_h}{\sigma_h+\mu} + \rho_0\Big) + k(\beta_p I + \beta_B B)\Big(\frac{\Lambda_l}{\sigma_l+\mu} + \rho_0\Big) \\ &\quad - (\delta+\mu+\gamma+\eta)I, \\ \frac{dC}{dt} &= \gamma I - (\mu+\tau)C, \\ \frac{dB}{dt} &\le p_i I + p_c C + (r-\xi)B. \end{aligned}$$

Consider the auxiliary system

$$\begin{aligned} \frac{d\hat{I}}{dt} &= (\beta_p \hat{I} + \beta_B \hat{B})\Big(\frac{\Lambda_h}{\sigma_h+\mu} + \rho_0\Big) + k(\beta_p \hat{I} + \beta_B \hat{B})\Big(\frac{\Lambda_l}{\sigma_l+\mu} + \rho_0\Big) \\ &\quad - (\delta+\mu+\gamma+\eta)\hat{I}, \quad t \ge t_1, \\ \frac{d\hat{C}}{dt} &= \gamma \hat{I} - (\mu+\tau)\hat{C}, \quad t \ge t_1, \\ \frac{d\hat{B}}{dt} &= p_i \hat{I} + p_c \hat{C} + (r-\xi)\hat{B}, \quad t \ge t_1. \end{aligned} \tag{3.1}$$

Since J_{ρ_0} is irreducible and has non-negative off-diagonal elements, it follows that $s(J_{\rho_0})$ is simple and associates a strongly positive eigenvector $\tilde{v} \in \mathbb{R}^3$ (see,e.g., [16, Theorem A.5]). For any solution $(S_h(t), S_l(t), V(t), I(t), C(t), R(t), B(t))$ of (2.1) with nonnegative initial value $(S_h(0), S_l(0), V(0), I(0), C(0), R(0), B(0))$, there is a sufficiently large $b > 0$ such that $(I(t_1), C(t_1), B(t_1)) \le b\tilde{v}$ holds. It is easy to see that $G(t) := be^{s(J_{\rho_0})(t-t_1)}\tilde{v}$ is a solution of (3.1) with $G(t_1) := b\tilde{v}$. By the comparison principle [16, Theorem B.1], it follows that

$$(I(t), C(t), B(t)) \le be^{s(J_{\rho_0})(t-t_1)}\tilde{v}, \quad \forall t \ge t_1.$$

Since $s(J_{\rho_0}) < 0$, it follows that

$$\lim_{t\to\infty} (I(t), C(t), B(t)) = (0, 0, 0).$$

It then follows that the equations for S_h and S_l are asymptotic to the following systems

$$\frac{dS_h(t)}{dt} = \Lambda_h - (\sigma_h + \mu)S_h(t),$$

and

$$\frac{dS_l(t)}{dt} = \Lambda_h - (\sigma_l + \mu)S_l(t)$$

and hence,

$$\lim_{t\to\infty} S_h(t) = \frac{\Lambda_h}{\sigma_h + \mu} \quad \text{and} \quad \lim_{t\to\infty} S_l(t) = \frac{\Lambda_l}{\sigma_l + \mu},$$

by the theory for asymptotically autonomous semiflows (see, e.g., [17, Corollary 4.3]). These results, along with 3rd and 6th equations of the system (2.1), also imply $\lim_{t\to\infty} V(t) = \frac{\sigma_h\Lambda_h}{\mu(\sigma_h+\mu)} + \frac{\sigma_l\Lambda_l}{\mu(\sigma_l+\mu)}$ and $\lim_{t\to\infty} R(t) = 0$. Thus, $\mathcal{E}_0$ is globally asymptotically stable. □

4. Numerical Results

4.1. **Base Case Scenario.** We consider a base case without any vaccination programs, i.e. $\sigma_h = \sigma_l = 0$. Using the mathematical formula derived in section 3.2 and the parameter values in Table 1, we computed the basic reproduction number for the base case to be $\mathfrak{R}_0 = 18.20$. The computed reproduction number is consistent with the previous estimate in Mutua et al. [11]. Based on our model, we also calculated the total number of new Typhoid cases generated in a single epidemic season (≈ 100 days) using the formula

$$\int_0^{100} [(\beta_p I(t) + \beta_B B(t))S_h(t) + (k\beta_p I(t) + k\beta_B B(t))S_l(t)]dt.$$

In this formula, the integrand $(\beta_p I(t) + \beta_B B(t))S_h(t) + (k\beta_p I(t) + k\beta_B B(t))S_l(t)$ is the rate of new infection per unit time, and thus the integral of this rate over the entire epidemic season gives the total new infections. Also for $t = 0$ to $t = 100$, we calculated the Typhoid peak prevalence as percentage given by $\max\{100(I+C)/N\}$. Based on our simulations, we estimated approximately 2.6 million of new Typhoid cases. During this epidemic, the peak prevalence reached is 22%.

4.2. **Sensitivity to base case $\mathfrak{R}_0$.** To identify important parameters that affect $\mathfrak{R}_0$, we performed the sensitivity analysis by calculating the sensitivity index S_X given by

$$S_X = \frac{X}{\mathfrak{R}_0} \cdot \frac{\partial \mathfrak{R}_0}{\partial X},$$

where X is a parameter whose sensitivity is sought. The larger the magnitude of the number, the greater impact that parameter has on $\mathfrak{R}_0$ and correspondingly, the smaller the magnitude, the weaker the impact on $\mathfrak{R}_0$. Also, the negative (or positive) sensitivity value indicates whether the reproduction number decreases (or increases) when the parameter is increased. The sensitivity result is shown in Figure 2. Figure 2 suggests that while the rate of infection from bacteria and the natural death rate have the largest impact on the basic reproduction number, the parameter k, related to socioeconomic factor, also has significant impact on $\mathfrak{R}_0$. This shows that the socioeconomic factor can not be ignored while developing prevention strategies.

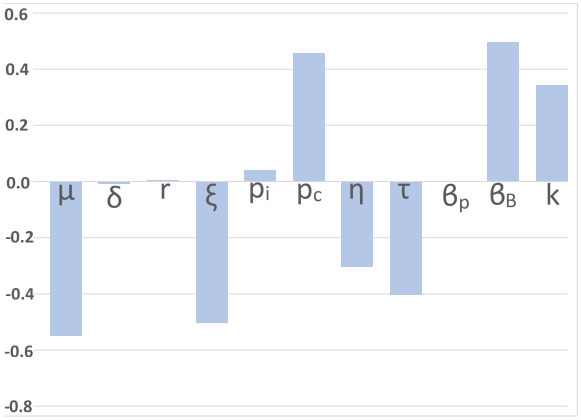

FIGURE 2. Sensitivity of Parameter Estimations to $\mathfrak{R}_0$. The bar corresponding to a parameter X represents the value of the sensitivity index S_X.

4.3. **Effect of vaccination.** We studied the effects of vaccination of high and low class populations by varying the corresponding vaccination rates σ_h and σ_l, respectively. Our results (Figure 3, left) show that increasing the vaccination rate of only low class population, i.e. increasing σ_l with $\sigma_h = 0$, decreases $\mathfrak{R}_0$ from 18 to 10, whereas increasing the vaccination rate of only high class populations, i.e. increasing σ_h with $\sigma_l = 0$, decreases $\mathfrak{R}_0$ from 18 to 15. While vaccinating low-class population seems more effective on reducing $\mathfrak{R}_0$, this result shows that vaccination programs targeted at only one class of the population might not be enough to avoid typhoid epidemics. However, increasing the vaccination rates of both population classes simultaneously can bring the value of $\mathfrak{R}_0$ below 1, thereby avoiding the epidemics. Therefore both classes need to be taken into consideration while designing proper vaccination programs.

Also, increasing the rate of vaccination of only high-class populations from $\sigma_h = 0$ to $\sigma_h = 1$ with $\sigma_l = 0$ fixed leads to a decrease in the total new infections by nearly 1 million (from 2.6 million to about 1.6 million) (Figure 3, middle), while a similar vaccination program targeted to low-class population only (i.e. increasing σ_l from 0 to 1 with $\sigma_h = 0$ fixed) can decrease the new infection by 2 million (from 2.6 million to about 0.5 million) (Figure 3, middle). These results again suggest that vaccination programs which target the low-class population are more effective towards reducing new Typhoid cases. As expected, vaccinating both classes simultaneously can reduce the new infection to a negligible level. We also analyzed the effects of vaccination on the peak prevalence reached during an epidemic season (Figure 3, right) and found the similar results in the sense that vaccination programs targeting low-class population produce lower peak of the disease prevalence. We found that the peak prevalence dropped from 22% to 14% (an 8% drop) with vaccination for only high-class, compared to 16% drop (from 22% to 6%) with vaccination for low-class only. Again, vaccination of both classes brought peak prevalence further down to below 4%.

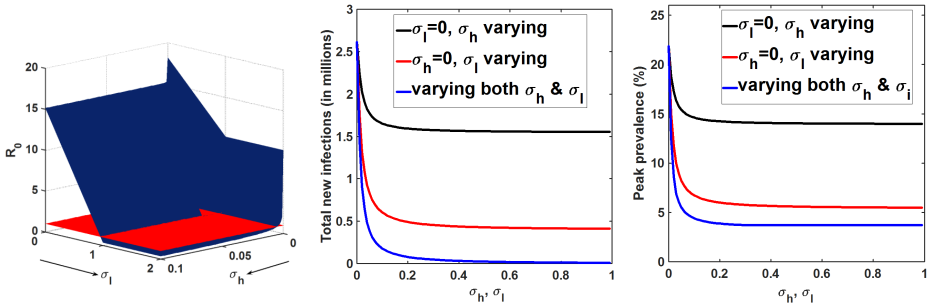

FIGURE 3. Effect of vaccination on $\mathfrak{R}_0$ (left), the total new infections (middle) and the peak prevalence (right).

4.4. **Effects of socioeconomic factor (k).** Socioeconomic status impacts the living standards, including access to important resources such as clean water among others. To study the effect of variation of the socioeconomic status in the dynamics of Typhoid, we can consider the parameter k in our model, which represents the Typhoid infection rate exacerbated by the deteriorated situation in the low class population. We let k vary from $k = 1$ (no effect of socioeconomic status) to $k = 10$. We note that $k = 10$ is an arbitrary maximum and is chosen for the purpose of demonstration. However, results for any value greater than $k = 10$ can similarly be obtained using our model simulations. We also study how this effect of k is altered when vaccination program is introduced. Specifically, we vary k at four levels of vaccination: no vaccination ($\sigma_h = \sigma_l = 0$), vaccination of high class only ($\sigma_h = 0.05$ and $\sigma_l = 0$), vaccination of low class only ($\sigma_h = 0$ and $\sigma_l = 0.05$), and vaccination of both high and low classes ($\sigma_h = 0.05$ and $\sigma_l = 0.05$).

4.4.1. *Effect of k with no vaccination.* In Figure 4 (left) we show the effect of k on the reproduction number $\mathfrak{R}_0$. In the absence of vaccination ($\sigma_h = \sigma_l = 0$), the reproduction number grows from $\mathfrak{R}_0 = 18.2$ to $\mathfrak{R}_0 = 44.0$ when k is increased from 1 to 10. The effect of k on new Typhoid infections and the peak prevalence of the disease is presented in Figure 4 (middle and right). We observe that with no vaccination the total new infections grow rapidly from 2.6 millon to 4.5 million. As k increases from $k = 1$ to $k = 10$ the peak prevalence grows to 41% from 20%.

4.4.2. *Effect of k under high-class targeted vaccination.* Simulating the model with vaccination for the high class only (i.e. $\sigma_h = 0.05$, and $\sigma_l = 0$), we observe that $\mathfrak{R}_0$ increases from 13.5 to 43.0 (Figure 4, left). This change is almost the same as the case with no vaccination discussed above, indicating that the effect of k remains almost unaltered due to high-class targeted vaccines. However, the effect of high-class targeted vaccination is greater on the total new infections and the peak prevalence (Figure 4, middle and right). Our simulation results show that on increasing k from 1 to 10, the total new infections over one Typhoid epidemic season increase from 1.4 million to 3.8 million, and the peak prevalence rises from 12.7% to 38.9% (Figure 4).

4.4.3. *Effect of k under low-class targeted vaccination.* Simulating the model with vaccination for the low class only ($\sigma_h = 0$, and $\sigma_l = 0.05$) we observe that $\mathfrak{R}_0$ begins at nearly half of the base case (Figure 4, left) for $k = 1$. In this case, the

reproduction number is hardly affected by the increase in the value of k (Figure 4, left). On increasing k from 1 to 10, the total new infections grow from 0.7 million to 2.9 million (Figure 4, middle), and the peak prevalence increases from 7.6% to 26.4% (Figure 4, right). Compared to high-class targeted vaccines, in the presence of low-class targeted vaccines, increase in the total new infections and the prevalence due to the socioeconomic factor k is smaller. Therefore, the effect of k is smaller in the presence of low-class targeted vaccination program than the high-class targeted vaccines.

4.4.4. *Effect of k under both-class targeted vaccination.* The effect of k on all of $\mathfrak{R}_0$, the total new infection and the peak prevalence becomes pronounced under both-class targeted vaccination ($\sigma_h = 0.05$ and $\sigma_l = 0.05$). In this case (Figure 4, left) we see that the basic reproduction number, $\mathfrak{R}_0$, changes from 0.6 at $k = 1$ to 1.3 at $k = 10$. Since an increase in k can cause $\mathfrak{R}_0$ greater than 1, the socioeconomic factor can be a determinant factor for the success of vaccination programs. Under both-class targeted vaccination, on increasing k from 1 to 10, the total new infections over one Typhoid epidemic season increases from 0.3 million to 2.3 million (Figure 4, middle). Similarly, the peak prevalence of the disease increases from 5% to 25.2% when k increases from 1 to 10 (Figure 4, right).

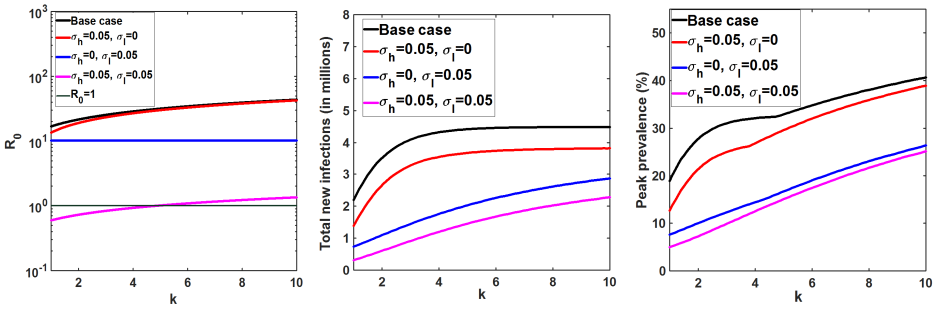

FIGURE 4. Effect of socioeconomic factor (k) $\mathfrak{R}_0$ (left), the total new infections (middle), and the peak prevalence (right).

5. Conclusion

Typhoid fever continues to be a significant burden on populations in developing countries, most of which are in Southern Asia and Sub-Saharan Africa. Here, we present a novel deterministic mathematical model to study the impact of varying socioeconomic status on Typhoid fever epidemics. Using mathematical analysis and simulations of our model, we show how socioeconomic status and vaccination program in combination impact the key features of Typhoid epidemics, including the basic reproduction number, the new Typhoid cases, and the peak prevalence of the disease. Given the significant effects of socioeconomic status on disease epidemic outcomes revealed by our results, we recommend targeting both-class population rather than the single-class population for developing Typhoid intervention strategies including effective vaccination programs, even though the targeting low-class population provides better outcomes than the high-class.

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