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INFINITELY MANY SOLUTIONS FOR FRACTIONAL SCHRÖDINGER-POISSON SYSTEMS WITH SIGN-CHANGING POTENTIAL

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Abstract. In this article, we prove the existence of multiple solutions for following fractional Schrödinger-Poisson system with sign-changing potential

$$\begin{gathered}(-\Delta)^s u + V(x)u + \lambda\phi u = f(x,u), \quad x \in \mathbb{R}^3,\\ (-\Delta)^t \phi = u^2, \quad x \in \mathbb{R}^3,\end{gathered}$$

where $(-\Delta)^\alpha$ denotes the fractional Laplacian of order $\alpha \in (0,1)$, and the potential V is allowed to be sign-changing. Under certain assumptions on f, we obtain infinitely many solutions for this system.

1. Introduction and preliminaries

This article concerns the fractional Schrödinger-Poisson system

$$\begin{gathered}(-\Delta)^s u + V(x)u + \lambda\phi u = f(x,u), \quad x \in \mathbb{R}^3,\\ (-\Delta)^t \phi = u^2, \quad x \in \mathbb{R}^3,\end{gathered} \tag{1.1}$$

where $(-\Delta)^\alpha$ denotes the fractional Laplacian operator, λ is a positive parameter and V is allowed to be sign-changing. In (1.1), the first equation is a nonlinear fractional Schrödinger equation in which the potential ϕ satisfies a nonlinear fractional Poisson equation. For this reason, system (1.1) is called a fractional Schrödinger-Poisson system, also known as the fractional Schrödinger-Maxwell system, which is not only a physically relevant generalization of the classical NLS but also an important model in the study of fractional quantum mechanics. For more details about the physical background, we refer the reader to [14, 15] and the references therein.

If $\lambda = 1$, then system (1.1) reduces to the fractional Schrödinger-Poisson system

$$\begin{gathered}(-\Delta)^s u + V(x)u + \phi u = f(x,u), \quad x \in \mathbb{R}^3,\\ (-\Delta)^t \phi = u^2, \quad x \in \mathbb{R}^3,\end{gathered} \tag{1.2}$$

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which has been studied by Zhang [29] by using the fountain theorem. The author proved the existence of multiple solutions under the condition (A4) and (A5) below. Meanwhile, the author proved that (A4) and (A5) are more weaker than (A8).

Let $s = 1$, $t = 1$ and $\lambda = 1$, then system (1.1) can be simplified to the classical fractional Schrödinger-Poisson system

$$\begin{gathered} -\Delta u + V(x)u + \phi u = f(x,u), \quad x \in \mathbb{R}^3, \\ -\Delta\phi = u^2, \quad x \in \mathbb{R}^3, \end{gathered} \tag{1.3}$$

which has been considered to prove the existence of infinitely many solutions for (1.3) via the fountain theorem. For more details, see the references [11, 13, 17, 20, 24] and the references therein, for more results about applying the critical point theory to second-order elliptic equations, we refer the reader to [2, 3, 4, 5, 26, 27, 28, 32] and the references therein.

However, it is well known that the fractional Schrödinger-Poisson system was first introduced by Giammetta [10] and the diffusion is fractional only in the Poisson equation. Afterwards, in [23], the authors proved the existence of radial ground state solutions of (1.1) when $V(x) \equiv 0$ and nonlinearity $f(x,u)$ is of subcritical or critical growth. Recently, in [29], the author proved infinitely many solutions via fountain theorem in (1.1) when $\lambda = 1$ and $V(x)$ is positive. However, to the best of our knowledge, for the sign-changing potential case, there are not many results for problem (1.1).

In 2013, Tang [21] gave some more weaker conditions and studied the existence of infinitely many solutions for Schrödinger equation via the symmetric mountain pass theorem with sign-changing potential. Using Tang's conditions, some authors studied the existence of infinitely many solutions for different equations with sign-changing potential. See, e.g., [6, 7, 9, 24, 25, 31] and the references quoted in them. These results generalized and extended some known results.

In [29], the author proved the existence of multiple solutions for the fractional Schrödinger-Possion equation with the following super-quadratic conditions:

(A1) $\inf_{x\in\mathbb{R}^3} V(x) \geq V_0 > 0$, where V_0 is a constant. Moreover, for every $M > 0$, $\operatorname{meas}(\{x \in \mathbb{R}^3 : V(x) \leq M\}) < \infty$, where $\operatorname{meas}(\cdot)$ denote the Lebesgue measure in $\mathbb{R}^3$.

(A2) There exists $a_1 > 0$ and $q \in (2, 2_s^*)$ such that
$$|f(x,u)| \leq a_1(1 + |u|^{p-1}), \quad \forall (x,u) \in \mathbb{R}^3 \times \mathbb{R},$$
where $2_s^* = \frac{6}{3-2s}$ is the critical exponent in fractional Sobolev inequalities. Moreover, $f(x,u) = o(u)$ as $u \to 0$.

(A3) $\lim_{|u|\to\infty} \frac{F(x,u)}{|u|^4} = \infty$, uniformly for $x \in \mathbb{R}^3$.

(A4) there exists a constant $\theta \geq 1$ such that
$$\theta\mathcal{F}(x,u) \geq \mathcal{F}(x,\tau u), \quad \forall (x,u) \in \mathbb{R}^3 \times \mathbb{R},\ \forall \tau \in [0,1],$$
where $\mathcal{F}(x,u) := \frac{1}{4}uf(x,u) - F(x,u)$.

(A5) there exists $r_1 > 0$ such that
$$4F(x,u) \leq uf(x,u), \quad \forall (x,u) \in \mathbb{R}^3 \times \mathbb{R},\ |u| \geq r_1\,.$$

(A6) $f(x,-u) = -f(x,u)$, $\forall\ (x,u) \in \mathbb{R}^3 \times \mathbb{R}$.

Under the conditions (A1)–(A4), (A6) and (A1)–(A3), (A5) and (A6), respectively, the author obtained multiple solutions for (1.1) in [29, 30]. However, in [12], Jeanjean gave the following condition and application to Landesman-Lazer type problems in $\mathbb{R}^N$.

(A7) $\frac{f(x,u)}{u^3}$ is increasing in $u>0$ and decreasing in $u<0$.

In [8], the following Ambrosetti and Rabinowitz condition was assumed to prove the existence of high energy solutions.

(A8) (Also known as (AR) condition) There exist $\mu>4$ and $r_1>0$ such that
$$0<\mu F(x,u)\le uf(x,u),\quad \forall x\in\mathbb{R}^3,\ |u|>r_1,$$
where $F(x,u)=\int_0^u f(x,\eta)d\eta$.

Inspired by the above results, we consider problem (1.1) with sign-changing potential and without the (AR) type superlinear condition, and establish the existence of infinitely many solutions by the symmetric mountain pass theorem in [21]. To state our results, we use the following conditions on V:

(A9) $V\in C(\mathbb{R}^3,\mathbb{R})$ and $\inf_{x\in\mathbb{R}^3}V(x)>-\infty$;

(A10) there exists a constant $d_0>0$ such that
$$\lim_{|y|\to\infty}\operatorname{meas}\big(\{x\in\mathbb{R}^3:|x-y|\le d_0, V(x)\le M\}\big)=0,\quad \forall M>0,$$
where meas denotes the Lebesgue measure on $\mathbb{R}^3$.

Condition (A10) was first introduced by Bartsch and Wang [4]. From (A9), we give the following equivalent equations for problem (1.1):

Remark 1.1. By (A9), we known that $V(x)$ is bounded from below. Hence there exists $V_0>0$ such that $\inf_{x\in\mathbb{R}^3}\widetilde{V}(x)>0$ for all $x\in\mathbb{R}^3$, where $\widetilde{V}(x):=V(x)+V_0$. Let $\widetilde{f}(x,u):=f(x,u)+V_0u$. Then problem (1.1) is equivalent to the problem
$$\begin{gathered}(-\Delta)^s u+\widetilde{V}(x)u+\lambda\phi u=\widetilde{f}(x,u),\quad x\in\mathbb{R}^3,\\ (-\Delta)^t\phi=u^2,\quad x\in\mathbb{R}^3.\end{gathered}$$

To achieve our results, we need to make the following assumptions on F and f.

(A11) $f\in C(\mathbb{R}^3,\mathbb{R})$, and there exist $c_1>0$, $c_2>0$ and $q\in(4,2_s^*)$ such that
$$|f(x,u)|\le c_1|u|^3+c_2|u|^{q-1},\quad \forall (x,u)\in\mathbb{R}^3\times\mathbb{R},$$
where $2_s^*=\frac{6}{3-2s}$ is the critical exponent in fractional Sobolev inequalities.

(A12) $\lim_{|u|\to\infty}\frac{|F(x,u)|}{|u|^4}=\infty$, a.e. $x\in\mathbb{R}^3$ and there exists $r_0\ge 0$ such that
$$F(x,u)\ge 0,\quad \forall (x,u)\in\mathbb{R}^3\times\mathbb{R},\ |u|\ge r_0;$$

(A13) there exists $\theta_0>0$ such that
$$4F(x,u)-uf(x,u)\le\theta_0u^2,\quad \forall (x,u)\in\mathbb{R}^3\times\mathbb{R}.$$

Next, we illustrate that F and f satisfying (A11)-(A13) and (A6) is not equivalent to $\widetilde{F}$ and $\widetilde{f}$ satisfying (A11)–(A13) and (A6).

Remark 1.2. First, we prove that $\widetilde{f}$ satisfying (A11) is not equivalent to f satisfying (A11). In fact, if $\widetilde{f}$ satisfies (A11), then we have
$$|f(x,u)|\le|\widetilde{f}(x,u)-V_0u|\le c_1|u|^3+c_2|u|^{q-1}+V_0|u|.$$
Thus f does not satisfy (A11). Now, if f satisfy (A11), similar to the discussion of f, we can obtain
$$|\widetilde{f}(x,u)|\le c_1|u|^3+c_2|u|^{q-1}+V_0|u|.$$

Thus $\widetilde{f}$ does not satisfy (A11).

Second, if $\lim_{|u|\to\infty}\frac{|F(x,u)|}{|u|^4}=\infty$, a.e. $x\in\mathbb{R}^3$ then $\lim_{|u|\to\infty}\frac{|\widetilde{F}(x,u)|}{|u|^4}=\infty$, a.e. $x\in\mathbb{R}^3$. the converse also holds. Moreover, if $F(x,u)\geq 0$ for any $(x,u)\in\mathbb{R}^3\times\mathbb{R}$, $|u|\geq r_0$, then $\widetilde{F}(x,u)\geq 0$ for any $(x,u)\in\mathbb{R}^3\times\mathbb{R}$, $|u|\geq r_0$. Conversely, it does not hold.

Finally, $\widetilde{f}$ satisfying (A13) is not equivalent to f satisfying (A13). In fact, if $4F(x,u)-uf(x,u)\leq\theta_0u^2$, then using $\widetilde{f}(x,u):=f(x,u)+V_0u$, we have

$$4\big[\widetilde{F}(x,u)-\frac{1}{2}V_0u^2\big]-u\big[\widetilde{f}(x,u)+V_0u\big]=4\widetilde{F}(x,u)-u\widetilde{f}(x,u)-V_0u^2\leq\theta_0u^2$$

which implies

$$4\widetilde{F}(x,u)-u\widetilde{f}(x,u)\leq(\theta_0+V_0)u^2.$$

This shows that $\widetilde{F}$ and $\widetilde{f}$ satisfy (A13). On the contrary, if $4\widetilde{F}(x,u)-u\widetilde{f}(x,u)\leq\theta_0u^2$, then similar to the proof the above inequalities, we obtain

$$4F(x,u)-uf(x,u)\leq(\theta_0-V_0)u^2.$$

But we do not know whether $\theta_0>V_0$ or not. Thus F and f do not satisfy (A13).

As for (A6), it is easy to check that f satisfying (A6) is equivalent to $\widetilde{f}$ satisfying (A6).

Now, we are ready to state the main results of this paper. Note that the space E is defined in (2.2).

Theorem 1.3. *Suppose that* (A6), (A9)–(A13) *are satisfied. Then when* $s\in(3/4,1)$, $t\in(0,1)$ *satisfying* $4s+2t\geq 3$, *problem* (1.1) *has infinitely many nontrivial solutions.* $\{(u_k,\phi_{u_k}^t)\}$ *in* $E\times D^{t,2}(\mathbb{R}^3)$ *satisfying* $J(u_k)\to+\infty$ *as* $k\to\infty$, *where the functional* J *is defined in* (2.8).

In [21], the author used the following conditions to prove the existence of infinitely many solutions for Schrödinger equation.

(A15) there exist $\mu>4$ and $\varrho>0$ such that

$$\mu F(x,u)\leq uf(x,u)+\varrho u^2,\quad\forall(x,u)\in\mathbb{R}^3\times\mathbb{R};$$

(A16) there exist $\mu>4$ and $r_1>0$ such that

$$\mu F(x,u)\leq uf(x,u),\quad\forall(x,u)\in\mathbb{R}^3\times\mathbb{R},\ |u|\geq r_0;$$

It is easy to check that (A15) imply (A13). Thus, we have the following corollary.

Corollary 1.4. *Suppose that* (A6), (A9)–(A12), (A15) *are satisfied. Then when* $s\in(3/4,1)$, $t\in(0,1)$ *satisfy* $4s+2t\geq 3$, *problem* (1.1) *has infinitely many nontrivial solutions.* $\{(u_k,\phi_{u_k}^t)\}$ *in* $E\times D^{t,2}(\mathbb{R}^3)$ *satisfying* $J(u_k)\to+\infty$ *as* $k\to\infty$, *where the functional* J *is defined in* (2.8).

It is easy to check that (A11) and (A15) imply (A16). Thus, we have the following corollary.

Corollary 1.5. *Suppose that* (A6), (A9)–(A12), (A16) *are satisfied. Then when* $s\in(3/4,1)$, $t\in(0,1)$ *satisfy* $4s+2t\geq 3$, *problem* (1.1) *has infinitely many nontrivial solutions.* $\{(u_k,\phi_{u_k}^t)\}$ *in* $E\times D^{t,2}(\mathbb{R}^3)$ *satisfying* $J(u_k)\to+\infty$ *as* $k\to\infty$, *where the functional* J *is defined in* (2.8).

Remark 1.6. In our results, $F(x,u)$ is allowed to be sign-changing. Thus (A12) is much weaker than (A3). In addition, it is obvious that (A15) is somewhat weaker than (A16). Moreover, (A16) implies (A5) and that (A15) implies (A13). Hence, our condition (A13) is somewhat weaker than (A5), (A15), (A16).

Remark 1.7. If $s \in (3/4, 1)$ then we can infer that $2_s^* > 4$. Hence (A11) is feasible.

2. Variational framework and main results

In this section, we need assumptions (A17) and (A18) instead of (A9) and (A10).

- (A17) $\widetilde{V} \in C(\mathbb{R}^3, \mathbb{R})$ and $\inf_{x\in\mathbb{R}^3} \widetilde{V}(x) > 0$;
- (A18) there exists a constant $d_0 > 0$ such that
$$\lim_{|y|\to\infty} \operatorname{meas}\big(\{x \in \mathbb{R}^3 : |x-y| \le d_0, \widetilde{V}(x) \le M\}\big) = 0, \quad \forall M > 0,$$
where meas denotes the Lebesgue measure on $\mathbb{R}^3$.

We define the Gagliardo seminorm by
$$[u]_{\alpha,p} = \Big(\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \frac{|u(x)-u(y)|^p}{|x-y|^{N+\alpha p}} dxdy\Big)^{1/p},$$
where $u : \mathbb{R}^3 \to \mathbb{R}$ is a measurable function.

On the one hand, we define fractional Sobolev space by
$$W^{\alpha,p}(\mathbb{R}^3) = \{u \in L^p(\mathbb{R}^3) : u \text{ is measurable and } [u]_{\alpha,p} < \infty\}$$
endowed with the norm
$$\|u\|_{\alpha,p} = \Big([u]_{\alpha,p}^p + \|u\|_p^p\Big)^{1/p}, \tag{2.1}$$
where
$$\|u\|_p = \Big(\int_{\mathbb{R}^3} |u(x)|^p dx\Big)^{1/p}.$$

If $p = 2$, the space $W^{\alpha,2}(\mathbb{R}^3)$ is an equivalent definition of the fractional Sobolev spaces is based on the Fourier analysis; that is,
$$H^\alpha(\mathbb{R}^3) := W^{\alpha,2}(\mathbb{R}^3) = \big\{u \in L^2(\mathbb{R}^3) : \int_{\mathbb{R}^3} (1+|\xi|^{2\alpha})|\widetilde{u}|^2 d\xi < \infty\big\},$$
endowed with the norm
$$\|u\|_{H^\alpha} = \Big(\int_{\mathbb{R}^3} |\xi|^{2\alpha}|\widetilde{u}|^2 d\xi + \int_{\mathbb{R}^3} |u|^2 d\xi\Big)^{1/2},$$
where $\widetilde{u}$ denotes the usual Fourier transform of u. Furthermore, we know that $\|\cdot\|_{H^\alpha}$ is equivalent to the norm
$$\|u\|_{H^\alpha} = \Big(\int_{\mathbb{R}^3} |(-\Delta)^{\alpha/2}u|^2 dx + \int_{\mathbb{R}^3} u^2 dx\Big)^{1/2}.$$

On the other hand, in view of the potential $\widetilde{V}(x)$, we consider the subspace
$$E = \big\{u \in H^\alpha(\mathbb{R}^3) : \int_{\mathbb{R}^3} \widetilde{V}(x)u^2 dx < \infty\big\}. \tag{2.2}$$
Thus, E is a Hilbert space with the inner product
$$(u,v)_{E_V} = \int_{\mathbb{R}^3} \big(|\xi|^{2\alpha}\widetilde{u}(\xi)\widetilde{v}(\xi) + \widetilde{u}(\xi)\widetilde{v}(\xi)\big)\, d\xi + \int_{\mathbb{R}^3} \widetilde{V}(x)u(x)v(x)dx$$

and the norm

$$\|u\|_{E_V} = \Big(\int_{\mathbb{R}^3} \big(|\xi|^{2\alpha}|\widetilde{u}(\xi)|^2 + |\widetilde{u}(\xi)|^2\big)\,d\xi + \int_{\mathbb{R}^3} \widetilde{V}(x)u^2(x)dx\Big)^{1/2}.$$

Moreover, $\|\cdot\|_{E_V}$ is equivalent to the norm

$$\|u\| := \|u\|_E = \Big(\int_{\mathbb{R}^3} |(-\Delta)^{\alpha/2}u|^2 dx + \int_{\mathbb{R}^3} \widetilde{V}(x)u^2 dx\Big)^{1/2},$$

where the corresponding inner product is

$$(u,v)_E = \int_{\mathbb{R}^3} \Big((-\Delta)^{\alpha/2}u(-\Delta)^{\alpha/2}v + \widetilde{V}(x)uv\Big)\,dx.$$

The homogeneous Sobolev space $D^{\alpha,2}(\mathbb{R}^3)$ is defined by

$$D^{\alpha,2}(\mathbb{R}^3) = \big\{u \in L^{2^*_\alpha}(\mathbb{R}^3) : |\xi|^\alpha \widetilde{u}(\xi) \in L^2(\mathbb{R}^3)\big\},$$

which is the completion of $C_0^\infty(\mathbb{R}^3)$ under the norm

$$\|u\|_{D^{\alpha,2}} = \Big(\int_{\mathbb{R}^3} |(-\Delta)^{\alpha/2}u|^2 dx\Big)^{1/2} = \Big(\int_{\mathbb{R}^3} |\xi|^{2\alpha}|\widetilde{u}(\xi)|^2 d\xi\Big)^{1/2},$$

endowed with the inner product

$$(u,v)_{D^{\alpha,2}} = \int_{\mathbb{R}^3} (-\Delta)^{\alpha/2}u(-\Delta)^{\alpha/2}v\,dx.$$

Then $D^{\alpha,2}(\mathbb{R}^3) \hookrightarrow L^{2^*_\alpha}(\mathbb{R}^3)$; that is, there exists a constant $C_0 > 0$ such that

$$\|u\|_{2^*_\alpha} \le C_0\|u\|_{D^{\alpha,2}}. \tag{2.3}$$

Next, we give the following lemmas which discuss the continuous and compact embedding for $E \hookrightarrow L^p(\mathbb{R}^3)$ for all $p \in [2, 2^*_\alpha]$. In the rest of this article, we use the norm $\|\cdot\|$ in E. Motivated by [33, Lemma 3.4], we can prove the following lemma. Here we omit its proof.

Lemma 2.1. *Space E is continuously embedded in $L^p(\mathbb{R}^3)$ for $2 \le p \le 2^*_\alpha := \frac{6}{3-2\alpha}$ and compactly embedded in $L^p(\mathbb{R}^3)$ for all $s \in [2, 2^*_\alpha)$.*

By Lemma 2.1, we can conclude that there exists a constant $\gamma_p > 0$ such that

$$\|u\|_p \le \gamma_p\|u\|, \tag{2.4}$$

where $\|u\|_p$ denotes the usual norm in $L^p(\mathbb{R}^3)$ for all $2 \le p \le 2^*_\alpha$.

Lemma 2.2 ([18, Theorem 6.5])**.** *For any $\alpha \in (0,1)$, $D^{\alpha,2}(\mathbb{R}^3)$ is continuously embedded int $L^{2^*_\alpha}(\mathbb{R}^3)$, that is, there exists $S_\alpha > 0$ such that*

$$\Big(\int_{\mathbb{R}^3} |u|^{2^*_\alpha} dx\Big)^{2/2^*_\alpha} \le S_\alpha \int_{\mathbb{R}^3} |(-\Delta)^{\alpha/2}u|^2 dx \quad \forall u \in D^{\alpha,2}(\mathbb{R}^3).$$

Next, let $\alpha = s \in (0,1)$. Using Hölder's inequality, for every $u \in E$ and $s,t \in (0,1)$, we have

$$\begin{aligned}\int_{\mathbb{R}^3} u^2 v dx &\le \Big(\int_{\mathbb{R}^3} |u|^{\frac{12}{3+2t}} dx\Big)^{\frac{3+2t}{6}} \Big(\int_{\mathbb{R}^3} |v|^{2^*_t} dx\Big)^{1/2^*_t} \\ &\le \gamma_{\frac{12}{3+2t}} S_t^{1/2}\|u\|^2\|v\|_{D^{t,2}},\end{aligned} \tag{2.5}$$

where we use the embedding

$$E \hookrightarrow L^{\frac{12}{3+2t}}(\mathbb{R}^3) \quad \text{when } 2t + 4s \ge 3.$$

By the Lax-Milgram theorem, there exists a unique $\phi_u^t \in D^{t,2}(\mathbb{R}^3)$ such that

$$\int_{\mathbb{R}^3} v(-\Delta)^t \phi_u^t dx = \int_{\mathbb{R}^3} (-\Delta)^{t/2}\phi_u^t (-\Delta)^{t/2} v dx = \int_{\mathbb{R}^3} u^2 v dx, \quad v \in D^{t,2}(\mathbb{R}^3). \tag{2.6}$$

Hence, ϕ_u^t satisfies the Poisson equation

$$(-\Delta)^t \phi_u^t = u^2, \quad x \in \mathbb{R}^3.$$

Moreover, ϕ_u^t has the integral expression

$$\phi_u^t(x) = c_t \int_{\mathbb{R}^3} \frac{u^2(y)}{|x-y|^{3-2t}} dy, \quad x \in \mathbb{R}^3,$$

which is called t-Riesz potential, where

$$c_t = \pi^{-3/2} 2^{-2t} \frac{\Gamma(\frac{3}{2} - 2t)}{\Gamma(t)}.$$

Thus $\phi_u^t(x) \geq 0$ for all $x \in \mathbb{R}^3$, from (2.1) and (2.6), we have

$$\|\phi_u^t\|_{D^{t,2}} \leq S_t^{1/2} \|u\|^2_{L^{\frac{12}{3+2t}}} \leq C_1 \|u\|^2 \quad \text{when } 2t + 4s \geq 3. \tag{2.7}$$

Therefore, by Hölder's inequality and Lemma 2.1, there exist $\widetilde{C}_1 > 0$, $\widetilde{C}_2 > 0$ such that

$$\begin{aligned} \int_{\mathbb{R}^3} \phi_u^t u^2 dx &\leq \Big(\int_{\mathbb{R}^3} |\phi_u^t|^{2_t^*} dx \Big)^{1/2_t^*} \Big(\int_{\mathbb{R}^3} |u|^{\frac{12}{3+2t}} dx \Big)^{\frac{3+2t}{6}} \\ &\leq \widetilde{C}_1 \|\phi_u^t\|_{D^{t,2}} \|u\|^2 \leq \widetilde{C}_2 \|u\|^4. \end{aligned}$$

Next, we define the energy functional J on E by

$$J(u) = \frac{1}{2}\|u\|^2 + \frac{\lambda}{4} \int_{\mathbb{R}^3} \phi_u^t u^2 dx - \int_{\mathbb{R}^3} \widetilde{F}(x,u) dx, \quad \forall u \in E. \tag{2.8}$$

By [19], the energy functional $J : E \to \mathbb{R}$ is well defined and of class $C^1(E, \mathbb{R})$. Moreover, the derivative of J is

$$\langle J'(u), v \rangle = \int_{\mathbb{R}^3} \Big((-\Delta)^{s/2} u (-\Delta)^{s/2} v + \widetilde{V}(x) uv + \lambda \phi_u^t uv - \widetilde{f}(x,u) v \Big) dx, \tag{2.9}$$

for all $u, v \in E$. Obviously, it can be proved that if u is a critical point of J, then the pair (u, ϕ_u^t) is a solution of system (1.1).

A sequence $\{u_n\} \subset E$ is said to be a $(C)_c$-sequence if $J(u) \to c$ and $\|J'(u)\|(1 + \|u_n\|) \to 0$. J is said to satisfy the $(C)_c$-condition if any $(C)_c$-sequence has a convergent subsequence. To prove our results, we state the following symmetric mountain pass theorem, see [1, Lemma 2.4] and [19, Lemma 912]

Lemma 2.3. *Let X be an infinite dimensional Banach space, $X = Y \oplus Z$, where Y is finite dimensional. If $J \in C^1(X, \mathbb{R})$ satisfies the $(C)_c$ condition for all $c > 0$, and*

1. *$J(0) = 0$, $J(-u) = J(u)$ for all $u \in X$;*
2. *there exist constants $\rho, \alpha > 0$ such that $J|_{\partial B_\rho \cap Z} \geq \alpha$;*
3. *for any finite dimensional subspace $\widetilde{X} \subset X$, there is $R = R(\widetilde{X}) > 0$ such that $J(u) \leq 0$ on $\widetilde{X} \backslash B_R$;*

then J possesses an unbounded sequence of critical values.

Lemma 2.4. *Assume that a sequence $\{u_n\} \subset E$, $u_n \rightharpoonup u$ in E as $n \to \infty$ and $\{\|u_n\|\}$ is a bounded sequence. Then, as $n \to \infty$,*

$$\int_{\mathbb{R}^3} (\phi_{u_n}^t u_n - \phi_u^t u)(u_n - u)dx \to 0. \tag{2.10}$$

Proof. Take a sequence $\{u_n\} \subset E$ such that $u_n \rightharpoonup u$ in E as $n \to \infty$ and $\{\|u_n\|\}$ is a bounded sequence. By Lemma 2.1, we have $u_n \to u$ in $L^p(\mathbb{R}^3)$ where $2 \le p < 2_s^*$, and $u_n \to u$ a.e. on $\mathbb{R}^3$. Hence $\sup_{n\in\mathbb{N}} \|u_n\| < \infty$ and $\|u\|$ is finite. Since $s \in (3/4, 1)$, then we know that $E \hookrightarrow L^{\frac{6}{2s}}$. Hence by (2.3) and (2.7), we have

$$\begin{aligned}
&|\int_{\mathbb{R}^3} (\phi_{u_n}^t u_n - \phi_u^t u)(u_n - u)dx| \\
&\le \Big(\int_{\mathbb{R}^3} (\phi_{u_n}^t u_n - \phi_u^t u)^2 dx\Big)^{1/2} \Big(\int_{\mathbb{R}^3} (u_n - u)^2 dx\Big)^{1/2} \\
&\le \sqrt{2}\Big[\int_{\mathbb{R}^3} (|\phi_{u_n}^t u_n|^2 + |\phi_u^t u|^2)\Big]^{1/2} \|u_n - u\|_2 \\
&\le C_3\Big(\|\phi_{u_n}^t\|_{2_s^*}^2 \|u_n\|_{\frac{6}{2s}}^2 + \|\phi_u^t\|_{2_s^*}^2 \|u\|_{\frac{6}{2s}}^2\Big)^{1/2} \|u_n - u\|_2 \\
&\le C_3\Big(\|u_n\|^4 + \|u\|^4\Big)^{1/2} \|u_n - u\|_2 \to 0, \quad \text{as } n \to \infty.
\end{aligned}$$

This completes the proof. ◻

The next lemmas are needed for our proofs.

Lemma 2.5 ([21])**.** *Assume that $p_1, p_2 > 1$, $r, q \ge 1$ and $\Omega \subseteq \mathbb{R}^N$. Let $g(x,t)$ be a Carathéodory function on $\Omega \times \mathbb{R}$ and satisfies*

$$|g(x,t) \le a_1|t|^{\frac{p_1-1}{r}} + a_2|t|^{\frac{p_2-1}{r}}, \quad \forall (x,t) \in \Omega \times \mathbb{R}, \tag{2.11}$$

where $a_1, a_2 \ge 0$. t If $u_n \to u$ in $L^{p_1}(\Omega) \cap L^{p_2}(\Omega)$, and $u_n \to u$ a.e. $x \in \Omega$, then for any $v \in L^{p_1 q}(\Omega) \cap L^{p_2 q}(\Omega)$,

$$\lim_{n\to\infty} \int_\Omega |g(x,u_n) - g(x,u)|^r |v|^q dx = 0. \tag{2.12}$$

Lemma 2.6. *Suppose that* (A9)–(A13) *are satisfied. Then any $\{u_n\} \subset E$ satisfying*

$$J(u_n) \to c > 0, \quad \|J'(u_n)\|(1 + \|u_n\|) \to 0 \tag{2.13}$$

is bounded in E.

Proof. To prove the boundedness of $\{u_n\}$ we argue by contradiction. Assume that $\|u_n\| \to \infty$ and let $v_n = u_n/\|u_n\|$. Then $\|v_n\| = 1$ and $\|v_n\|_p \le \gamma_p \|v_n\| = \gamma_p$ for $2 \le p < 2_s^*$. By (A13) and (2.13), for n large enough, we have

$$\begin{aligned}
c + 1 &\ge J(u_n) - \frac{1}{4}\langle J'(u_n), u_n\rangle \\
&= \frac{1}{4}\|u_n\|^2 + \int_{\mathbb{R}^3} \Big(\frac{1}{4}\widetilde{f}(x,u_n)u_n - \widetilde{F}(x,u_n)\Big)dx \\
&= \frac{1}{4}\|u_n\|^2 + \int_{\mathbb{R}^3} \Big[\frac{1}{4}f(x,u_n) - F(x,u_n) - \frac{1}{4}V_0 u^2\Big]dx \qquad (2.14)\\
&\ge \frac{1}{4}\|u_n\|^2 - \frac{1}{4}(\theta_0 + V_0)\|u_n\|_2^2 \\
&\ge \frac{1}{4}\|u_n\|^2 - \frac{1}{4}(\theta_0 + V_0)\|v_n\|_2^2 \|u_n\|^2,
\end{aligned}$$

which implies

$$\frac{c+1}{\|u_n\|^2} \geq \frac{1}{4} - \frac{1}{4}(\theta_0 + V_0)\|v_n\|_2^2. \tag{2.15}$$

For $0 < a < b$, let

$$\Omega_n(a,b) = \{x \in \mathbb{R}^3 : a \leq |u_n| < b\}. \tag{2.16}$$

Passing to a subsequence, we may assume that $v_n \rightharpoonup v$ in E, then by Lemma 2.1, $v_n \to v$ in $L^p(\mathbb{R}^3)$, $2 \leq p < 2_s^*$, and $v_n \to v$ a.e. on $\mathbb{R}^3$. Hence, if $\|u_n\| \to \infty$ in (2.15), then

$$\|v_n\|_2^2 \geq \frac{4}{\theta_0 + V_0} + o_n(1),$$

which shows that $v_n \rightharpoonup v \neq 0$.

We only need to consider the case $v \neq 0$. Set $A := \{x \in \mathbb{R}^3 : v(x) \neq 0\}$. Thus $\operatorname{meas}(A) > 0$. For a.e. $x \in A$, we have $\lim_{n\to\infty} |u_n(x)| = \infty$. Hence $A \subset \Omega_n(r_0, \infty)$ for large $n \in \mathbb{N}$. which implies that $\chi_{\Omega_n(r_0,\infty)} = 1$ for large n, where χ_{Ω_n} denotes the characteristic function on Ω_n and $\Omega_n(0, r_0)$ is the same as in (2.16). By (A13), then we have

$$|\widetilde{F}(x, u_n)| \leq c_1 u_n^4 + c_2 |u_n|^q + \frac{1}{2} V_0 u_n^2, \tag{2.17}$$

which implies that there exists a constant $C_3 > 0$ such that $|\widetilde{F}(x, u_n)| \leq C_3 u_n^2$, for any $|u_n| \leq r_0$. It follows from (2.8), (2.16), (2.17) and Fatou's Lemma that

$$\begin{aligned}
0 &= \lim_{n\to\infty} \frac{c+o(1)}{\|u_n\|^4} = \lim_{n\to\infty} \frac{J(u_n)}{\|u_n\|^4} \\
&= \lim_{n\to\infty} \Big[\frac{1}{2\|u_n\|^2} - \frac{1}{\|u_n\|^4}\int_{\mathbb{R}^3} \widetilde{F}(x,u_n)dx + \frac{\lambda}{4}\frac{1}{\|u_n\|^4}\int_{\mathbb{R}^3} \phi_{u_n}^t u_n^2 dx\Big] \\
&= \lim_{n\to\infty} \Big[-\frac{1}{\|u_n\|^4}\int_{\Omega_n(0,r_0)} \widetilde{F}(x,u_n)dx \\
&\quad - \int_{\Omega_n(r_0,\infty)} \frac{\widetilde{F}(x,u_n)}{u_n^4} v_n^4 dx + \frac{\lambda}{4}\frac{1}{\|u_n\|^4}\int_{\mathbb{R}^3} \phi_{u_n}^t u_n^2 dx\Big] \\
&\leq \limsup_{n\to\infty} \Big[\frac{1}{\|u_n\|^4}\|u_n\|_2^2 - \int_{\Omega_n(r_0,\infty)} \frac{\widetilde{F}(x,u_n)}{u_n^4} v_n^4 dx\Big] + \frac{\lambda \widetilde{C}_2}{4} \\
&\leq \limsup_{n\to\infty} \Big[\frac{\|v_n\|_2^2}{\|u_n\|^2} - \int_{\Omega_n(r_0,\infty)} \frac{\widetilde{F}(x,u_n)}{u_n^4} v_n^4 dx\Big] + \frac{\lambda \widetilde{C}_2}{4} \\
&= \limsup_{n\to\infty} \Big[\frac{\gamma_2^2}{\|u_n\|^2} - \int_{\Omega_n(r_0,\infty)} \frac{\widetilde{F}(x,u_n)}{u_n^4} v_n^4 dx\Big] + \frac{\lambda \widetilde{C}_2}{4} \\
&\leq -\liminf_{n\to\infty} \int_{\Omega_n(r_0,\infty)} \frac{\widetilde{F}(x,u_n)}{u_n^4} v_n^4 dx + \frac{\lambda \widetilde{C}_2}{4} \\
&= -\liminf_{n\to\infty} \int_{\mathbb{R}^3} \frac{|\widetilde{F}(x,u_n)|}{u_n^4} [\chi_{\Omega_n(r_0,\infty)}(x)] v_n^4 dx + \frac{\lambda \widetilde{C}_2}{4} \\
&\leq -\int_{\mathbb{R}^3} \liminf_{n\to\infty} \frac{\widetilde{F}(x,u_n)}{u_n^4} [\chi_{\Omega_n(r_0,\infty)}(x)] v_n^4 dx + \frac{\lambda \widetilde{C}_2}{4} = -\infty,
\end{aligned} \tag{2.18}$$

which is a contradiction. Thus $\{u_n\}$ is bounded in E. □

Lemma 2.7. *Suppose that* (A9)–(A13) *are satisfied. Then each* $\{u_n\} \subset E$ *satisfying* (2.13) *has a convergent subsequence in* E.

Proof. By Lemma 2.6, $\{u_n\}$ is bounded in E. If necessary going to a subsequence, we can assume that $u_n \rightharpoonup u$ in E. From Lemma 2.1, we have $u_n \to u$ in $L^p(\Omega)$ for all $2 \le p < 2_s^*$. Hence, by Lemma 2.5, one has

$$\Big| \int_{\mathbb{R}^3} (\widetilde{f}(x, u_n) - \widetilde{f}(x, u))(u_n - u)dx\Big| \to 0, \quad \text{as } n \to \infty. \tag{2.19}$$

Observe that

$$\begin{aligned} \|u_n - u\|^2 = \langle J'(u_n) - J'(u), u_n - u\rangle + \int_{\mathbb{R}^3} (\phi^t_{u_n} - \phi^t_u)(u_n - u)dx \\ + \int_{\mathbb{R}^3} (\widetilde{f}(x, u_n) - \widetilde{f}(x, u))(u_n - u)dx. \end{aligned} \tag{2.20}$$

It is clear that

$$\langle J'(u_n) - J'(u), u_n - u\rangle \to 0, \quad \text{as } n \to \infty. \tag{2.21}$$

From (2.19), (2.20) and (2.21), we have $\|u_n - u\| \to 0$, as $n \to \infty$. $\square$

Lemma 2.8. *Suppose that* (A9)–(A13) *are satisfied. Then for each $\widetilde{E} \subset E$, it holds*

$$J(u) \to -\infty, \quad \|u\| \to \infty, \quad u \in \widetilde{E}. \tag{2.22}$$

Proof. Arguing indirectly, assume that for some sequence $\{u_n\} \subset \widetilde{E}$ with $\|u_n\| \to \infty$, there is $M > 0$ such that $J(u_n) \ge -M$ for all $n \in \mathbb{N}$. Set $v_n = \frac{u_n}{\|u_n\|}$, then $\|v_n\| = 1$. Passing to a subsequence, we may assume that $v_n \rightharpoonup v$ in E. Since $\widetilde{E}$ is finite dimensional, then $v_n \to v \in \widetilde{E}$ in E, $v_n \to v$ a.e. on $\mathbb{R}^N$, and so $\|v\| = 1$. Hence, we can conclude a contradiction by a similar methods as (2.18). $\square$

Corollary 2.9. *Suppose that* (A9)–(A13) *are satisfied. Then for any $\widetilde{E} \subset E$, there exists $R = R(\widetilde{E}) > 0$ such that*

$$J(u_n) \le 0, \quad \|u\| \ge R, \quad \forall u \in \widetilde{E}.$$

Let $\{e_j\}$ is a total orthonormal basis of E and define $X_j = \mathbb{R}e_j$,

$$Y_k = \oplus_{j=1}^k X_j, \quad Z_k = \oplus_{j=k+1}^\infty X_j, \quad \forall k \in \mathbb{Z}. \tag{2.23}$$

Lemma 2.10. *Suppose that* (A9) *and* (A10) *are satisfied. Then for $2 \le p < 2_s^*$, we have*

$$\beta_k(s) := \sup_{u \in Z_k, \|u\|=1} \|u\|_p \to 0, \quad k \to \infty.$$

Proof. It is clear that $0 < \beta_{k+1} \le \beta_k$, so that $\beta_k \to \beta \ge 0 (k \to \infty)$. For every $k \in \mathbb{N}$, there exists $u_k \in Z_k$ such that $|u_k|_2 > \frac{\beta_k}{2}$ and $\|u_k\| = 1$. For any $v \in E$, writing $v = \Sigma_{j=1}^\infty c_j e_j$, we have, by the Cauchy-Schwartz inequality,

$$\begin{aligned} |(u_k, v)| &= |(u_k, \Sigma_{j=1}^\infty c_j e_j)| = |(u_k, \Sigma_{j=k+1}^\infty c_j e_j)| \\ &\le \|u_k\| \|\Sigma_{j=k+1}^\infty c_j e_j\| = (\Sigma_{j=k+1}^\infty c_j^2)^{1/2} \to 0 \end{aligned}$$

as $k \to \infty$, which implies that $u_k \rightharpoonup 0$. By Lemma 2.1, the compact embedding of $E \hookrightarrow L^p(\mathbb{R}^3)$ $(2 \le p < 2_s^*)$ implies that $u_k \to 0$ in $L^p(\mathbb{R}^3)$. Hence, letting $k \to \infty$, we obtain $\beta = 0$, which completes the proof. $\square$

By Lemma 2.10, we can choose an integer $m \geq 1$ such that

$$\|u\|_2^2 \leq \frac{1}{2V_0}\|u\|^2, \quad \|u\|_4^2 \leq \sqrt{\frac{1}{c_1}}\|u\|^2, \quad \|u\|_q^q \leq \frac{q}{4c_2}\|u\|^q, \quad \forall u \in Z_m, \tag{2.24}$$

where $q \in (4, 2_s^*)$.

Lemma 2.11. *Suppose that* (A9)–(A11) *are satisfied. Then there exist constants* $\rho, \alpha > 0$ *such that*

$$J\big|_{\partial B_\rho \cap Z_m} \geq \alpha.$$

Proof. From (2.17) and (2.24), for $u \in Z_m$, choosing $\rho := \|u\| = \frac{1}{2}$, we obtain

$$\begin{aligned} J(u) &= \frac{1}{2}\|u\|^2 + \frac{\lambda}{4}\int_{\mathbb{R}^3} \phi_u^t u^2 dx - \int_{\mathbb{R}^3} \widetilde{F}(x,u)dx \\ &\geq \frac{1}{2}\|u\|^2 - \int_{\mathbb{R}^3} \widetilde{F}(x,u)dx \\ &\geq \frac{1}{2}\|u\|^2 - \frac{c_1}{4}\|u\|_4^4 - \frac{c_2}{q}\|u\|_q^q - \frac{1}{2}V_0\|u\|_2^2 \\ &\geq \frac{1}{4}\left(\|u\|^2 - \|u\|^4 - \|u\|^q\right) \\ &= \frac{1}{4}\Big[\frac{3}{16} - \frac{1}{2^q}\Big] := \alpha > 0, \end{aligned}$$

since $q \in (4, 2_s^*)$. This completes the proof. □

Proof of Theorem 1.3. Let $X = E$, $Y = Y_m$ and $Z = Z_m$. By Lemmas 2.6, 2.7, 2.11 and Corollary 2.9, all conditions of Lemma 2.3 are satisfied. Thus problem (1.1) possesses infinitely many nontrivial solutions. □

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References

[1] T. Bartolo, V. Benci, D. Fortunato; *Abstract critical point theorems and applications to some nonlinear problems with strong resonance at infinity*, Nonlinear Anal., 7 (1983), 981-1012.

[2] T. Bartsch; *Infinitely many solutions of a symmetric Dirichlet problem.* Nonlinear Anal., 20 (1993), 1205-1216.

[3] T. Bartsch, M. Willem; *On an elliptic equation with concave and convex nonlinearities.* Proc. Am. Math. Soc., 123 (1995), 3555-3561.

[4] T. Bartsch, Z. Q. Wang; *Existence and multiplicity results for some superlinear elliptic problems on* $\mathbb{R}^N$, Comm. Partial Differ. Equ., 20 (1995), 1725-1741.

[5] T. Bartsch, Z. Q. Wang, M. Willem; *The Dirichlet problem for superlinear elliptic equations*, In: Chipot, M., Quittner, P.: (eds.) Handbook of Differential Equations-Stationary Partial Differential Equations, vol. 2, pp. 1-5.

[6] J. Chen, B. Cheng, X. H. Tang; *New existence of multiple solutions for nonhomogeneous Schrödinger-Kirchhoff problems involving the fractional p-Laplacian with sign-changing potential*, Rev. Real Acad. Cienc. Exactas F., (2016) DOI: 10.1007/s13398-016-0372-5.

[7] J. Chen, X. H. Tang, Z. Gao; *Existence of multiple solutions for modified Schrödinger-Kirchhoff-Poisson type systems via perturbation method with sign-changing potential*, Comput. Math. Appl., 73 (2017), 505-519.
[8] S. Chen, C. Tang; *High energy solutions for the Schrödinger-Maxwell equations*, Nonlinear Anal. 71 (2009) 4927-4934.
[9] B. Cheng, X. H. Tang; *High energy solutions of modified quasilinear fourth-order elliptic equations with sign-changing potential*, Comput. Math. Appl., 73 (2017), 27-36.
[10] A. R. Giammetta; *Fractional Schrödinger-Poisson-Slater system in one dimension*, arXiv:1405.2796v1.
[11] E. Hebey, J. Wei; *Schrödinger-Poisson systems in the 3-sphere*, Calc. Var. Partial Differ. Equ., 47 (2013), 25-54.
[12] L. Jeanjean; *On the existence of bounded Palais-Smale sequences and application to a landesman-Lazer type problem on* $\mathbb{R}^N$, Proc. Roy. Soc. Edinburgh Sect. A, 129 (1999), 787-809.
[13] H. Kikuchi; *On the existence of solution for elliptic system related to the Maxwell-Schrödinger equations*, Nonlinear Anal., 27 (2007), 1445-1456.
[14] N. Laskin; *Fractional quantum mechanics and Lévy path integrals*, Phys. Lett. A., 268 (2000), 298-305.
[15] N. Laskin; *Fractional Schrödinger equation*, Phys. Rev. E., 66 056108 (2002).
[16] S. B. Liu, S. J. Li; *Infinitely many solutions for a superlinear elliptic equation*, Acta Math. Sinica (Chin. Ser.), 46 (4) (2003), 625-630 (in Chinese).
[17] D. Mugnai; *The Schrödinger-Poisson system with positive potential*, Comm. Partial Differ. Equ., 36 (2011), 1099-1117.
[18] E. D. Nezza, G. Palatucci, E. Valdinoci; *Hitchhiker's guide to the fractional Sobolev spaces*, Bull. Sci. Math., 136 (2012), 512-573.
[19] P. H. Rabinowitz; *Minimax methods in critical point theory with applications to differential equations*, in: CBMS Reg. Conf. Ser. in Math., vol. 65, Amer. Math. Soc., Providence, RI, 1986.
[20] J. Seok; *On nonlinear Schrödinger-Poisson equations with general potentials*, J. Math. Anal. Appl., 401 (2013), 672-681.
[21] X. H. Tang; *Infinitely many solutions for semilinear Schrödinger equations with sign-changing potential and nonlinearity*, J. Math. Anal. Appl., 401 (2013) 407-415.
[22] L. Zhao, H. Liu, F. Zhao; *Existence and concentration of solutions for the Schrödinger-Poisson equations with steep well potential*, J. Differential Equations, 255 (2013) 1-23.
[23] J. Zhang, J. M. do Ó, M. Squassina; *Fractional Schrödinger-Possion equations with a general subcritical or critical nonlinearity*, Adv. Nonlinear Stud., 16 (2016), 15-30.
[24] J. Zhang, X. H. Tang, W. Zhang; *Infinitely many solutions of quasilinear Schrödinger equation with sign-changing potential*, J. Math. Anal. Appl., 420 (2014), 1762-1775.
[25] J. Zhang, X. H. Tang, W. Zhang; *Existence of multiple solutions of Kirchhoff type equation with sign-changing potential*, Appl. Math. Comput., 242 (2014), 491-499.
[26] J. Zhang, X. H. Tang, W. Zhang; *Existence and multiplicity of stationary solutions for a class of Maxwell-Dirac system*, Nonlinear Anal., 127 (2015), 298-311.
[27] J. Zhang, X. H. Tang, W. Zhang; *Ground states for diffusion system with periodic and asymptotically periodic nonlinearity*, Comput. Math. Appl., 71 (2016), 633-641.
[28] J. Zhang, W. Zhang, X. L. Xie; *Existence and concentration of semiclassical solutions for Hamiltonian elliptic system*, Commu. Pure Appl. Anal., 15 (2016), 599-622.
[29] J. G. Zhang; *Existence and multiplicity results for the Fractional Schrödinger-Poisson systems*, arXiv:1507.01205v1.
[30] Q. Zhang, Q. Wang; *Multiple solutions for a class of sublinear Schrödinger equations*, J. Math. Anal. Appl., 389 (2012), 511-518.
[31] W. Zhang, X. H. Tang, J. Zhang; *Infinitely many solutions for fourth-order elliptic equations with sign-changing potential*, Taiwanese J. Math., 18 (2014), 645-659.
[32] W. Zhang, X. H. Tang, J. Zhang; *Infinitely many radial and non-radial solutions for a fractional Schrödinger equation*, Comput. Math. Appl., 71 (2016), 737-747.
[33] W. M. Zou, M. Schechter; *Critical Point Theory and Its Applications*, Springer, New York, 2006.

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