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HOMOCLINIC SOLUTIONS FOR A CLASS OF SECOND-ORDER HAMILTONIAN SYSTEMS WITH LOCALLY DEFINED POTENTIALS

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Abstract. In this article, we establish sufficient conditions for the existence of homoclinic solutions for a class of second-order Hamiltonian systems

$$\ddot{u}(t) - L(t)u(t) + \nabla W\big(t, u(t)\big) = f(t),$$

where $L(t)$ is a positive definite symmetric matrix for all $t \in \mathbb{R}$. It is worth pointing out that the potential function $W(t,u)$ is locally defined and can be superquadratic or subquadratic with respect to u.

1. Introduction and statement of main results

The purpose of this article is to investigate the second-order Hamiltonian systems

$$\ddot{u}(t) - L(t)u(t) + \nabla W\big(t, u(t)\big) = f(t) \tag{1.1}$$

where $t \in \mathbb{R}$, $u \in \mathbb{R}^n$, $L \in C(\mathbb{R}, \mathbb{R}^{n\times n})$ is a positive definite and symmetric matrix for all $t \in \mathbb{R}$, $W : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}^n$. Here, we say that a solution $u(t)$ of (1.1) is nontrivial homoclinic (to 0) if $u \not\equiv 0$ and $u(t) \to 0$ as $t \to \pm\infty$. Moreover, $\nabla W(t,x)$ denotes the gradient with respect to x, $(\cdot,\cdot) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denotes the standard inner product in $\mathbb{R}^n$ and $|\cdot|$ is the induced norm.

If $f = 0$, then (1.1) degenerates to the following second-order Hamiltonian system

$$\ddot{u}(t) - L(t)u(t) + \nabla W\big(t, u(t)\big) = 0 \tag{1.2}$$

In physics, Hamiltonian systems describe the evolution equations of a physical system, which can present important insight about the dynamics, even if the analytical solution of the initial value problem cannot be obtained. It is well known that a homoclinic orbit lies in the intersection of the stable manifold and the unstable manifold of a saddle point, which is a fundamental tool in the study of chaos. In the past decades, there have been a lot of results about the existence and multiplicity of homoclinic orbits for Hamiltonian systems via critical point theory, see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] and the references therein.

In the case that $L(t)$ and $W(t,x)$ are either independent of t or periodic in t, it has been studied by many authors, see [1, 4, 7, 8, 9, 11, 17, 16, 18, 22]. In particular, in [18], Rabinowitz has proved the existence of homoclinic orbits as a limit of $2kT$-periodic solutions of (1.2). Motivated by the work of Rabinowitz,

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applying the same procedure, the existence of homoclinic solutions of (1.1) or (1.2) was obtained as the limit of subharmonic solutions, see Izydorek and Janczewska [8, 9] and so on.

In the case that $L(t)$ and $W(t,x)$ are not periodic with respect to t, the problem of existence and multiplicity of homoclinic orbits for (1.1) will become much more difficult, due to the lack of compactness of the Sobolev embedding. In [20], Rabinowitz and Tanaka considered (1.2) without a periodicity assumption, both for L and W. To deal with the case that the nonlinearity W is superquadratic, they introduced the Ambrosetti-Rabinowitz growth condition, i.e., the following assumption (A1) and assumed that the smallest eigenvalue of $L(t)$ tends to $+\infty$ as $|t| \to \infty$. Using a variant of the Mountain Pass theorem without the Palais-Smale condition, they proved that (1.2) possesses a nontrivial homoclinic orbit.

For the next theorem we use the following assumptions:

- (A1) $L(t)$ is positive definite symmetric matrix for all $t \in \mathbb{R}$ and there exists an $l \in C(\mathbb{R}, (0,\infty))$ such that $l(t) \to +\infty$ as $|t| \to \infty$ and
$$\big(L(t)x, x\big) \geq l(t)|x|^2 \quad \text{for all } t \in \mathbb{R} \text{ and } x \in \mathbb{R}^n;$$
- (A2) $W \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$ and there is a constant $\mu > 2$ such that
$$0 < \mu W(t,x) \leq \big(x, \nabla W(t,x)\big) \quad \text{for all } t \in \mathbb{R} \text{ and } x \in \mathbb{R}^n \setminus \{0\};$$
- (A3) $|\nabla W(t,x)| = o(|x|)$ as $|x| \to 0$ uniformly with respect to $t \in \mathbb{R}$;
- (A4) There is a $\overline{W} \in C(\mathbb{R}^n, \mathbb{R})$ such that
$$|W(t,x)| + |\nabla W(t,x)| \leq |\overline{W}(x)| \quad \text{for all } t \in \mathbb{R} \text{ and } x \in \mathbb{R}^n.$$

Theorem 1.1 ([20])**.** *Assume that L and W satisfy* (A1)–(A4)*. Then* (1.2) *possesses a nontrivial homoclinic solution.*

Motivated by [11, 20], in this paper, we study the existence of Homoclinic solutions for (1.1), where we only give some local assumptions on $W(t,u)$ and $W(t,u)$ can be superquadratic or subquadratic with respect to u. Our main results are stated in the next theorem, under the following conditions:

- (A5) $W \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R})$, $W(t,0) \equiv 0$ and $\nabla W(t,0) \equiv 0$ for all $t \in \mathbb{R}$;
- (A6) there exist $\rho > 0$ and $a \in L^\alpha(\mathbb{R}, \mathbb{R}_+)$ such that
$$W(t,x) \leq a(t)|x|^\mu \quad \text{for all } t \in \mathbb{R} \text{ and } |x| \leq \rho, \tag{1.3}$$
where $\alpha > 1$, $\mu > 1$ if $\frac{2(\alpha-1)}{\alpha} \leq 1$ or $\mu \geq \frac{2(\alpha-1)}{\alpha}$ if $\frac{2(\alpha-1)}{\alpha} > 1$.
- (A7) $f \not\equiv 0$ is a continuous and bounded function such that $\int_{\mathbb{R}} |f(t)|^\beta dt < \infty$ and
$$\frac{1 \wedge l_*}{4}\rho - \frac{M_a}{\sqrt[\alpha^*]{2}}\rho^{\mu-1} - \frac{M_f}{\sqrt[\beta^*]{2}} > 0, \tag{1.4}$$
where $1 < \beta \leq 2$, $\frac{1}{\alpha^*} + \frac{1}{\alpha} = 1$, $\frac{1}{\beta^*} + \frac{1}{\beta} = 1$, $l_* = \inf_{t \in \mathbb{R}} l(t) > 0$,
$$M_a = \Big(\int_{\mathbb{R}} |a(t)|^\alpha dt\Big)^{1/\alpha} \quad \text{and} \quad M_f = \Big(\int_{\mathbb{R}} |f(t)|^\beta dt\Big)^{1/\beta}.$$

Theorem 1.2. *Assume that* (A1), (A5)–(A7)*. Then* (1.1) *possesses a nontrivial homoclinic solution.*

2. Proof of main results

Motivated by [10, 13], we first consider the existence of the homoclinic solutions for (1.1), which can be obtained as the limit of periodic solutions for the following boundary-value problem

$$\begin{gathered} \ddot{u}(t) - L(t)u(t) + \nabla W\big(t, u(t)\big) = f(t), \quad t \in [-T, T] \\ u(-T) - u(T) = \dot{u}(-T) - \dot{u}(T) = 0, \end{gathered} \tag{2.1}$$

for all $T \in \mathbb{R}^+$.

Given any $T \in \mathbb{R}^+$, let

$$\begin{aligned} E_T :=& W^{1,2}\big([-T, T], \mathbb{R}^n\big) \\ =& \big\{u : [-T, T] \to \mathbb{R}^n : u \text{ is absolutely continuous}, \\ & u(-T) = u(T) \text{ and } \dot{u} \in L^2([-T, T], \mathbb{R}^n)\big\} \end{aligned}$$

and for $u \in E_T$, define

$$\|u\|_{E_T} = \Big\{ \int_{-T}^{T} [|\dot{u}(t)|^2 + |u(t)|^2] dt \Big\}^{1/2},$$

then E_T is a Hilbert space endowed with the above norm.

Next, we define a functional $I_T : E_T \to \mathbb{R}$ by

$$I_T(u) = \int_{-T}^{T} \big[\frac{1}{2}|\dot{u}(t)|^2 + \frac{1}{2}\big(L(t)u(t), u(t)\big) - W\big(t, u(t)\big) + \big(f(t), u(t)\big)\big] dt. \tag{2.2}$$

We can easily see that $I_T \in C^1(E_T, \mathbb{R})$ is weakly lower semi-continuous because it is the sum of a convex continuous function and of a weakly continuous one. By the direct calculation, it follows that

$$\begin{aligned} \langle I_T'(u), v\rangle = \int_{-T}^{T} \big[&\big(\dot{u}(t), \dot{v}(t)\big) + \big(L(t)u(t), v(t)\big) \\ &- \big(\nabla W\big(t, u(t)\big), v(t)\big) + \big(f(t), v(t)\big)\big] dt \end{aligned} \tag{2.3}$$

for all $u, v \in E_T$. Moreover, it is well known that the critical points of I_T in E_T are classical solutions of (2.1) (see [15, 19]).

To prove our main result, we apply a critical point theorem, which is stated precisely as follows.

Lemma 2.1 (See [11]). *Let X be a real reflexive Banach space and $\Omega \subset X$ be a closed bounded convex subset of X. Suppose that $\varphi : X \to \mathbb{R}$ is weakly lower semi-continuous. If there exists a point $x_0 \in \Omega \setminus \partial\Omega$ such that*

$$\varphi(x) > \varphi(x_0) \quad \text{for all } x \in \partial\Omega. \tag{2.4}$$

Then there exists a $x^* \in \Omega \setminus \partial\Omega$ such that

$$\varphi(x^*) = \inf_{u\in\Omega} \varphi(u).$$

Lemma 2.2 (See [8]). *Let $u : \mathbb{R} \to \mathbb{R}^n$ be a continuous mapping such that $\dot{u} \in L^2_{loc}(\mathbb{R}, \mathbb{R}^n)$. Then for every $t \in \mathbb{R}$, we have*

$$|u(t)| \le \sqrt{2}\Big[\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \big(|\dot{u}(s)|^2 + |u(s)|^2\big) ds\Big]^{1/2}. \tag{2.5}$$

Lemma 2.3. *Let $u \in E_T$. It follows that*

$$\|u\|_{L^\infty_{[-T,T]}} \le \Big(\int_{-T}^{T} |u(t)|^2 dt\Big)^{1/2} + \Big(\int_{-T}^{T} |\dot u(t)|^2 dt\Big)^{1/2}. \tag{2.6}$$

Note that the above lemma is a special case of [22, Corollary 2.2].

Corollary 2.4. *Let $u \in E_T$. It follows that*

$$\|u\|_{L^\infty_{[-T,T]}} \le \sqrt{2}\|u\|_{E_T} = \sqrt{2}\Big\{\int_{-T}^{T} \big[|\dot u(t)|^2 + |u(t)|^2\big] dt\Big\}^{1/2}. \tag{2.7}$$

Proof. Combining (2.6) and the inequality $\sqrt{a} + \sqrt{b} \le \sqrt{2}(a+b)^{1/2}$, it is obvious that (2.7) holds. □

Lemma 2.5. *Under the conditions of Theorem 1.2, the boundary-value problem* (2.1) *admits a solution $u_T \in E_T$ such that*

$$\int_{-T}^{T} \big[|\dot u_T(t)|^2 + |u_T(t)|^2\big] dt < \frac{1}{2}\rho^2 \quad \text{for all } T \in \mathbb{R}_+. \tag{2.8}$$

Proof. Clearly, $I_T(0) = 0$ by (A5) for all $T \in \mathbb{R}_+$. For the purpose of using Lemma 2.1, we first need to construct a closed bounded convex subset of E_T for all $T \in \mathbb{R}_+$. Given any $T \in \mathbb{R}_+$, let $\Omega_T := \{u \in E_T : \int_{-T}^{T} \big[|\dot u(t)|^2 dt + |u(t)|^2\big] dt \le \frac{1}{2}\rho^2\}$, where ρ is the constant defined in (1.3). It is evident that Ω_T is a closed bounded convex subset of E_T for all $T \in \mathbb{R}_+$.

For any $T \in \mathbb{R}_+$, we will prove that (2.8) holds. If $u \in \partial\Omega_T$, it follows that $\int_{T}^{T} \big[|\dot u(t)|^2 dt + |u(t)|^2\big] dt = \frac{1}{2}\rho^2$. Applying Corollary 2.4, it is obvious that $\|u\|_{L^\infty_{[-T,T]}} \le \rho$ for all $u \in \partial\Omega_T$. That is $|u(t)| \le \rho$ for all $t \in [-T,T]$. Combining this inequality, (A1), (A6) and (A7), we get that $\mu \ge \frac{2}{\alpha^*}$ and

$$\begin{aligned}
&I_T(u)\\
&= \int_{-T}^{T} \Big[\frac{1}{2}|\dot u(t)|^2 + \frac{1}{2}\big(L(t)u(t), u(t)\big) - W\big(t, u(t)\big) + \big(f(t), u(t)\big)\Big] dt\\
&\ge \frac{1}{2}\int_{-T}^{T} |\dot u(t)|^2 dt + \frac{1}{2}\int_{-T}^{T} l(t)|u(t)|^2 dt - \int_{-T}^{T} a(t)|u(t)|^\mu dt + \int_{-T}^{T} \big(f(t), u(t)\big) dt\\
&\ge \frac{1}{2}\int_{-T}^{T} |\dot u(t)|^2 dt + \frac{l_*}{2}\int_{-T}^{T} |u(t)|^2 dt - \Big(\int_{-T}^{T} |a(t)|^\alpha dt\Big)^{1/\alpha}\Big(\int_{-T}^{T} |u(t)|^{\mu\alpha^*} dt\Big)^{1/\alpha^*}\\
&\quad - \Big(\int_{-T}^{T} |f(t)|^\beta dt\Big)^{1/\beta}\Big(\int_{-T}^{T} |u(t)|^{\beta^*} dt\Big)^{1/\beta^*}\\
&\ge \frac{1}{2}\int_{-T}^{T} |\dot u(t)|^2 dt + \frac{l_*}{2}\int_{-T}^{T} |u(t)|^2 dt - \|u\|_{L^\infty_{[-T,T]}}^{\mu - \frac{2}{\alpha^*}}\Big(\int_{\mathbb{R}} |a(t)|^\alpha dt\Big)^{1/\alpha}\\
&\quad \times \Big(\int_{-T}^{T} |u(t)|^2 dt\Big)^{1/\alpha^*} - \|u\|_{L^\infty_{[-T,T]}}^{1 - \frac{2}{\beta^*}}\Big(\int_{\mathbb{R}} |f(t)|^\beta dt\Big)^{1/\beta}\Big(\int_{-T}^{T} |u(t)|^2 dt\Big)^{1/\beta^*}\\
&\ge \frac{1 \wedge l_*}{4}\rho^2 - \frac{M_a}{\sqrt[\alpha^*]{2}}\rho^\mu - \frac{M_f}{\sqrt[\beta^*]{2}}\rho\\
&> 0 = I_T(0)
\end{aligned}$$

for all $u \in \partial\Omega_T$. Consequently, using Lemma 2.1, we can have that for all $T \in \mathbb{R}_+$, there exists $u_T \in \operatorname{int}\Omega_T$ such that

$$I_T(u_T) = \inf_{u\in\Omega_T} I_T(u),$$

where

$$\operatorname{int}\Omega_T = \Big\{u \in E_T : \int_{-T}^{T} [|\dot{u}(t)|^2 + |u(t)|^2]dt < \frac{1}{2}\rho^2\Big\}$$

Furthermore, we note that $\operatorname{int}\Omega_T$ is an open subset of E_T. This together with [15, Theorem 1.3] implies that

$$I'_T(u_T) = 0.$$

That is, u_T is the solution of the boundary-value problem (2.1) and

$$\int_{-T}^{T} [|\dot{u}_T(t)|^2 + |u_T(t)|^2]dt < \frac{1}{2}\rho^2.$$

The proof is complete. □

Proof of Theorem 1.2. First, we can choose a sequence $T_m \to \infty$ and study the boundary-value problem (2.1) on the bounded closed interval $[-T_m, T_m]$ for all $m \in \mathbb{N}$. Using the result of Lemma 2.5, it follows that there exists a sequence of solutions u_m such that $\|u_m\|_{E_{T_m}}$ is uniformly bounded with respect to $m \in \mathbb{N}$.

According to the inequality

$$|u_m(t_1) - u_m(t_2)| \le \int_{t_1}^{t_2} |\dot{u}_m(t)|dt \le \sqrt{t_2 - t_1}\Big(\int_{t_1}^{t_2} |\dot{u}_m(t)|^2 dt\Big)^{1/2}$$

we can assert that the sequence $\{u_m\}_{m\in\mathbb{N}}$ is equicontinuous and uniformly bounded on every bounded closed interval $[-T_m, T_m]$, $m \in \mathbb{N}$. Therefore, we can select a subsequence $\{u_{m_k}\}_{k\in\mathbb{N}}$ such that it converges uniformly on any bounded closed interval to a continuous function u. Furthermore, using (2.1), it is clear that the sequence $\{\ddot{u}_{m_k}\}_{k\in\mathbb{N}}$ and so $\{\dot{u}_{m_k}\}_{k\in\mathbb{N}}$ converges uniformly on any bounded closed intervals. Noting that

$$u_{m_k}(t) = \int_0^t (t - s)\ddot{u}_{m_k}(s)ds + t\dot{u}_{m_k}(0) + u_{m_k}(0),$$

it is obvious that $u \in C^2(\mathbb{R}, \mathbb{R}^n)$ and $\ddot{u}_{m_k} \to \ddot{u}$ uniformly on any bounded closed intervals as $k \to \infty$. Consequently, we can first study the boundary-value problem (2.1) on bounded closed interval $[-T_m, T_m]$, $m \in \mathbb{N}$. Next, using the diagonal process and let $m \to \infty$, we can easily see that u is a classical solution of (1.1).

Since $\|u_m\|_{E_{T_m}}$ is uniformly bounded with respect to $m \in \mathbb{N}$, under the above analysis, it is evident that

$$\int_{\mathbb{R}} [|\dot{u}(t)|^2 + |u(t)|^2]dt \le \frac{1}{2}\rho^2. \tag{2.9}$$

By Lemma 2.2, we have

$$|u(t)| \le \sqrt{2}\Big[\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \big(|\dot{u}(s)|^2 + |u(s)|^2\big)ds\Big]^{1/2} \quad \text{for all } t \in \mathbb{R}.$$

This together with (2.9) implies that the limit of $u(t)$ is zero as $|t| \to \infty$, i.e., $u(\pm\infty) = 0$. Moreover, since $f \not\equiv 0$, it follows that u is a nontrivial homoclinic orbit of (1.1).

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