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SOLVABILITY OF A NONLOCAL PROBLEM FOR A HYPERBOLIC EQUATION WITH INTEGRAL CONDITIONS

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Abstract. We study a nonlocal problem with integral conditions for a hyperbolic equation two independent variables. By introducing additional functional parameters, we investigated the solvability and construction of approximate solutions. The original problem is reduced to an equivalent problem consisting of the Goursat problems for a hyperbolic equation with parameters and the boundary value problem with integral condition for the ordinary differential equations with respect to the parameters. Based on the algorithms for finding solutions to the equivalent problem, we propose algorithms for finding the approximate solutions, and prove their convergence. Coefficient criteria for the unique solvability of nonlocal problem with integral conditions for hyperbolic equation with mixed derivative are also established.

1. Introduction

On the domain $\Omega = [0,T]\times[0,\omega]$, we consider the nonlocal problem for hyperbolic equation with integral conditions

$$\frac{\partial^2 u}{\partial t\partial x} = A(t,x)\frac{\partial u}{\partial x} + B(t,x)\frac{\partial u}{\partial t} + C(t,x)u + f(t,x), \tag{1.1}$$

$$\int_0^a K(t,\xi)u(t,\xi)d\xi = \psi(t), \quad t \in [0,T], \tag{1.2}$$

$$\int_0^b M(\tau,x)u(\tau,x)d\tau = \varphi(x), \quad x \in [0,\omega], \tag{1.3}$$

where $u(t,x)$ is unknown function, the functions $A(t,x)$, $B(t,x)$, $C(t,x)$, and $f(t,x)$ are continuous on Ω, the functions $K(t,x)$ and $\psi(t)$ are continuously differentiable by t on Ω and $[0,T]$, respectively, the functions $M(t,x)$ and $\varphi(x)$ are continuously differentiable by x on Ω and $[0,\omega]$, respectively, $0 < a \le \omega$, $0 < b \le T$. The compatibility condition is given below.

Let $C(\Omega,\mathbb{R})$ be a space of functions $u : \Omega \to R$ continuous on Ω with norm $\|u\|_0 = \max_{(t,x)\in\Omega} |u(t,x)|$.

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A function $u(t,x) \in C(\Omega,\mathbb{R})$, having the partial derivatives $\frac{\partial u(t,x)}{\partial x} \in C(\Omega,\mathbb{R})$, $\frac{\partial u(t,x)}{\partial t} \in C(\Omega,\mathbb{R})$, and $\frac{\partial^2 u(t,x)}{\partial t \partial x} \in C(\Omega,\mathbb{R})$ is called a classical solution to problem (1.1)–(1.3), if it satisfies equation (1.1) and integral conditions (1.2), (1.3).

Mathematical modelling of various physical processes often leads to the nonlocal problems for hyperbolic equations. Problems with integral conditions arise while researching the processes of heat distribution, plasma physics, clean technology of silicon ores, moisture transfer in capillary-porous media, etc. [8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 24, 25, 26, 27]. Some classes of nonlocal boundary value problems with integral conditions for hyperbolic equations are studied in [6, 8-28]. Solvability conditions for the considered problems are obtained in the different terms. Problem (1.1)–(1.3) for $K(t,x) = M(t,x)$ and $K(t,x) = M(t,x) = 1$ are studied in [10, 11, 19, 20, 21, 22, 23]. Under the assumptions of continuous differentiability of the equation coefficients, the conditions for the unique solvability of that problem have been obtained. For $K(t,x) = K(x)$, $M(t,x) = M(t)$, the problem (1.1)–(1.3) for the system of hyperbolic equations is studied in [28] by contractive mapping principle.

For $K(t,x) = M(t,x)$, problem (1.1)–(1.3) for the system of hyperbolic equations is studied in [7]. The unique solvability conditions for this problem are established in the terms of initial data. Nonlocal problems with general integral conditions for hyperbolic equations belong to the field of little studied problems of mathematical physics. This formulation of problem is considered for the first time.

The aim of this work is to construct algorithms for finding a solution to problem (1.1)–(1.3) and establish conditions for the existence and uniqueness of classical solution to problem (1.1)–(1.3).

In Section 2, a scheme of method used [4, 5, 6] is provided. By introduction of new unknown functions being a linear combination of solutions values on characteristics, the problem (1.1)–(1.3) is reduced to an equivalent problem consisting of the Goursat problem for hyperbolic equations with functional parameters and the boundary value problems with integral conditions for ordinary differential equations with respect to the parameters entered. Algorithm for finding the approximate solution to the investigated problem is constructed. The algorithm consists of two parts: in the first part, we solve two boundary value problems with integral condition for ordinary differential equations, and in the second part, we solve the Goursat problem for hyperbolic equation with parameters. Boundary value problems with integral condition for ordinary differential equations are intensively studied in recent years, and they find numerous applications in the applied problems [1, 2, 3]. In Section 3, the conditions for the existence of unique solution to the boundary value problems with integral condition for ordinary differential equations are presented. In Section 4, the convergence of algorithm is proved, and the conditions for the unique solvability of problem (1.1)–(1.3) are given in the terms of initial data.

2. Method's scheme and algorithm

Notation: $\mu(t) = u(t,0) - \frac{1}{2}u(0,0)$, $\lambda(x) = u(0,x) - \frac{1}{2}u(0,0)$, $\widetilde{u}(t,x)$, where $\widetilde{u}(t,x)$ is a new unknown function. We make a following replacement of desired function $u(t,x)$ in problem (1.1)–(1.3): $u(t,x) = \widetilde{u}(t,x) + \mu(t) + \lambda(x)$ and transit to the problem

$$\begin{aligned}\frac{\partial^2 \widetilde{u}}{\partial t \partial x} = A(t,x)\frac{\partial \widetilde{u}}{\partial x} + B(t,x)\frac{\partial \widetilde{u}}{\partial t} + C(t,x)\widetilde{u} + A(t,x)\dot{\lambda}(x) \\ + B(t,x)\dot{\mu}(t) + C(t,x)\lambda(x) + C(t,x)\mu(t) + f(t,x),\end{aligned} \tag{2.1}$$

$$\widetilde{u}(t,0) = 0, \quad t \in [0,T], \tag{2.2}$$

$$\widetilde{u}(0,x) = 0, \quad x \in [0,\omega], \tag{2.3}$$

$$\begin{gathered}\int_0^a K(t,\xi)d\xi\mu(t) + \int_0^a K(t,\xi)\widetilde{u}(t,\xi)d\xi + \int_0^a K(t,\xi)\lambda(\xi)d\xi = \psi(t), \\ t \in [0,T],\end{gathered} \tag{2.4}$$

$$\begin{gathered}\int_0^b M(\tau,x)d\tau\lambda(x) + \int_0^b M(\tau,x)\widetilde{u}(\tau,x)d\tau + \int_0^b M(\tau,x)\mu(\tau)d\tau = \varphi(x), \\ x \in [0,\omega].\end{gathered} \tag{2.5}$$

A triplet of functions $(\widetilde{u}(t,x), \mu(t), \lambda(x))$, satisfying the hyperbolic equation (2.1), the conditions on characteristics (2.2), (2.3), and the functional relations (2.4) and (2.5) at $\mu(0) = \lambda(0)$, will be called a solution to problem (2.1)–(2.5) if the function $\widetilde{u}(t,x) \in C(\Omega, R)$ has the partial derivatives

$$\frac{\partial \widetilde{u}(t,x)}{\partial x} \in C(\Omega,R), \quad \frac{\partial \widetilde{u}(t,x)}{\partial t} \in C(\Omega,R), \quad \frac{\partial^2 \widetilde{u}(t,x)}{\partial t \partial x} \in C(\Omega,R),$$

the functions $\mu(t)$ and $\lambda(x)$ are continuously differentiable on $[0,T]$ and $[0,\omega]$, respectively.

Relation $\mu(0) = \lambda(0)$ is a compatibility condition of data.

Problem (2.1)–(2.5) is equivalent to problem (1.1)–(1.3). If the function $u^*(t,x)$ is a solution to problem (1.1)–(1.3), then the triplet of functions $(\widetilde{u}^*(t,x),\ \mu^*(t),\ \lambda^*(x))$, where $\widetilde{u}^*(t,x) = u^*(t,x) - \mu^*(t) - \lambda^*(x)$, $\mu^*(t) = u^*(t,0) - \frac{1}{2}u^*(0,0)$, $\lambda^*(x) = u^*(0,x) - \frac{1}{2}u^*(0,0)$, is a solution to problem (2.1)–(2.5). The converse is also true. If the triplet of functions $(\widetilde{u}^{**}(t,x), \mu^{**}(t),\ \lambda^{**}(x))$ is a solution to problem (2.1)–(2.5), then the function $u^{**}(t,x)$ defined by the equality

$$u^{**}(t,x) = \widetilde{u}^{**}(t,x) + \mu^{**}(t) + \lambda^{**}(x),$$

where $u^{**}(t,0) - \frac{1}{2}u^{**}(0,0) = \mu^{**}(t)$, $u^{**}(0,x) - \frac{1}{2}u^{**}(0,0) = \lambda^{**}(x)$ is a solution to problem (1.1)–(1.3).

At fixed $\mu(t)$, $\lambda(x)$, problem (2.1)–(2.3) is the Goursat problem with respect to the function $\widetilde{u}(t,x)$ on the domain Ω. Relations (2.4) and (2.5) allow us to determine the unknown parameters $\mu(t)$, $\lambda(x)$, where the functions $\mu(t)$, $\lambda(x)$ satisfy condition $\mu(0) = \lambda(0)$.

By conditions (2.2), (2.3), relations (2.4) at $t = 0$ and (2.5) at $x = 0$ yield

$$\int_0^a K(0,\xi)d\xi\mu(0) + \int_0^a K(0,\xi)\lambda(\xi)d\xi = \psi(0), \tag{2.6}$$

$$\int_0^b M(\tau,0)d\tau\lambda(0) + \int_0^b M(\tau,0)\mu(\tau)d\tau = \varphi(0). \tag{2.7}$$

Taking into account $\mu(0) = \lambda(0)$, we obtain

$$\int_0^a K(0,\xi)d\xi\lambda(0) + \int_0^a K(0,\xi)\lambda(\xi)d\xi = \psi(0), \tag{2.8}$$

$$\int_0^b M(\tau,0)d\tau\mu(0)+\int_0^b M(\tau,0)\mu(\tau)d\tau=\varphi(0). \tag{2.9}$$

Let us define the following condition.

Condition (i). Assume

$$B_1(t)=\int_0^a K(t,\xi)d\xi\neq 0\quad\text{for all } t\in[0,T],$$
$$B_2(x)=\int_0^b M(\tau,x)\,d\tau\neq 0\quad\text{for all } x\in[0,\omega].$$

From relation (2.4) we determine the parameter

$$\mu(t)=-\frac{1}{B_1(t)}\Big\{\int_0^a K(t,\xi)\widetilde{u}(t,\xi)d\xi+\int_0^a K(t,\xi)\lambda(\xi)d\xi-\psi(t)\Big\}, \tag{2.10}$$

for $t\in[0,T]$. Similarly, from relation (2.5) we determine parameter

$$\lambda(x)=-\frac{1}{B_2(x)}\Big\{\int_0^b M(\tau,x)\widetilde{u}(\tau,x)d\tau+\int_0^b M(\tau,x)\mu(\tau)d\tau-\varphi(x)\Big\}, \tag{2.11}$$

for $x\in[0,\omega]$. Assumptions on the data of problem (1.1)–(1.3) allow us to differentiate (2.10) and (2.11) by t and x, respectively. Then we obtain

$$\begin{aligned}\dot{\mu}(t)&=\frac{\dot{B}_1(t)}{B_1^2(t)}\int_0^a K(t,\xi)\widetilde{u}(t,\xi)d\xi-\frac{1}{B_1(t)}\Big\{\int_0^a\frac{\partial K(t,\xi)}{\partial t}\widetilde{u}(t,\xi)d\xi\\&\quad+\int_0^a K(t,\xi)\frac{\partial\widetilde{u}(t,\xi)}{\partial t}d\xi\Big\}+\frac{\dot{B}_1(t)}{B_1^2(t)}\int_0^a K(t,\xi)\lambda(\xi)d\xi\\&\quad-\frac{1}{B_1(t)}\int_0^a\frac{\partial K(t,\xi)}{\partial t}\lambda(\xi)d\xi-\frac{\dot{B}_1(t)}{B_1^2(t)}\psi(t)+\frac{1}{B_1(t)}\dot{\psi}(t),\quad t\in[0,T],\end{aligned} \tag{2.12}$$

$$\begin{aligned}\dot{\lambda}(x)&=\frac{\dot{B}_2(x)}{B_2^2(x)}\int_0^b M(\tau,x)\widetilde{u}(\tau,x)d\tau-\frac{1}{B_2(x)}\Big\{\int_0^b\frac{\partial M(\tau,x)}{\partial x}\widetilde{u}(\tau,x)d\tau\\&\quad+\int_0^b M(\tau,x)\frac{\partial\widetilde{u}(\tau,x)}{\partial x}d\tau\Big\}+\frac{\dot{B}_2(x)}{B_2^2(x)}\int_0^b M(\tau,x)\mu(\tau)d\tau\\&\quad-\frac{1}{B_2(x)}\int_0^b\frac{\partial M(\tau,x)}{\partial x}\mu(\tau)d\tau-\frac{\dot{B}_2(x)}{B_2^2(x)}\varphi(x)+\frac{1}{B_2(x)}\dot{\varphi}(x),\end{aligned} \tag{2.13}$$

for $x\in[0,\omega]$. We introduce the new unknown functions

$$\widetilde{v}(t,x)=\frac{\partial\widetilde{u}(t,x)}{\partial x},\quad \widetilde{w}(t,x)=\frac{\partial\widetilde{u}(t,x)}{\partial t},$$

and the following notation

$$\begin{aligned}G_1(t,\widetilde{u},\widetilde{w})&=\frac{\dot{B}_1(t)}{B_1^2(t)}\int_0^a K(t,\xi)\widetilde{u}(t,\xi)d\xi\\&\quad-\frac{1}{B_1(t)}\Big\{\int_0^a\frac{\partial K(t,\xi)}{\partial t}\widetilde{u}(t,\xi)d\xi+\int_0^a K(t,\xi)\widetilde{w}(t,\xi)d\xi\Big\},\\L_1(t,\lambda)&=\frac{\dot{B}_1(t)}{B_1^2(t)}\int_0^a K(t,\xi)\lambda(\xi)d\xi-\frac{1}{B_1(t)}\int_0^a\frac{\partial K(t,\xi)}{\partial t}\lambda(\xi)d\xi,\end{aligned}$$

$$G_2(x,\widetilde{u},\widetilde{v}) = \frac{\dot{B}_2(x)}{B_2^2(x)}\int_0^b M(\tau,x)\widetilde{u}(\tau,x)d\tau \\ - \frac{1}{B_2(x)}\Big\{\int_0^b \frac{\partial M(\tau,x)}{\partial x}\widetilde{u}(\tau,x)d\tau + \int_0^b M(\tau,x)\widetilde{v}(\tau,x)d\tau\Big\}.$$

Then equations (2.12) and (2.13) can be written in the form

$$\dot{\mu}(t) = G_1(t,\widetilde{u},\widetilde{w}) + L_1(t,\lambda) + \frac{1}{B_1(t)}\dot{\psi}(t) - \frac{\dot{B}_1(t)}{B_1^2(t)}\psi(t), \quad t\in[0,T], \tag{2.14}$$

$$\dot{\lambda}(x) = G_2(x,\widetilde{u},\widetilde{v}) + L_2(x,\mu) + \frac{1}{B_2(x)}\dot{\varphi}(x) - \frac{\dot{B}_2(x)}{B_2^2(x)}\varphi(x), \quad x\in[0,\omega]. \tag{2.15}$$

Thus, we have a closed system of equations (2.1)–(2.3), (2.14) (2.9), (2.15) (2.8) for determining the unknown functions $\widetilde{v}(t,x)$, $\widetilde{w}(t,x)$, $\widetilde{u}(t,x)$, $\dot{\lambda}(x)$, $\lambda(x)$, $\dot{\mu}(t)$, $\mu(t)$.

Relation (2.14) in conjunction with (2.9) present a boundary value problem with integral condition for a differential equation with respect to $\mu(t)$, and the relation (2.15) in conjunction with (2.8) present a boundary value problem with integral condition for a differential equation with respect to $\lambda(x)$.

Boundary value problem with integral condition (2.14), (2.9) is equivalent to relation (2.4), and boundary value problem with integral condition (2.15), (2.8) is equivalent to relation (2.5) at $\mu(0) = \lambda(0)$.

If $\dot{\mu}(t)$, $\dot{\lambda}(x)$, $\mu(t)$, $\lambda(x)$ are known, then we find the functions $\widetilde{v}(t,x)$, $\widetilde{w}(t,x)$, $\widetilde{u}(t,x)$ from (2.1)–(2.3). Conversely, if we know the functions $\widetilde{v}(t,x)$, $\widetilde{w}(t,x)$, $\widetilde{u}(t,x)$, then we can find $\dot{\mu}(t)$, $\mu(t)$, $\dot{\lambda}(x)$, $\lambda(x)$ from boundary value problems (2.14), (2.9) and (2.15), (2.8). The unknowns are both $\widetilde{v}(t,x)$, $\widetilde{w}(t,x)$, $\widetilde{u}(t,x)$, and $\dot{\mu}(t)$, $\mu(t)$, $\dot{\lambda}(x)$, $\lambda(x)$. Therefore, to find solution of problem (2.1)–(2.5), we use an iterative method: determine the triplet $(\widetilde{u}^*(t,x), \mu^*(t), \lambda^*(x))$ as a limit of sequence $(\widetilde{u}^{(m)}(t,x), \mu^{(m)}(t)), \lambda^{(m)}(x))$, $m = 0,1,2,\dots$, according to the following algorithm:

Step 0. (1) Assuming $\widetilde{u}(t,x) = 0$, $\widetilde{w}(t,x) = 0$, $\lambda(x) = 0$, in the right-hand side of equation (2.14), we find initial approximations $\dot{\mu}^{(0)}(t)$, $\mu^{(0)}(t)$, $t\in[0,T]$ from the boundary value problem with integral condition (2.14), (2.9). Assuming $\widetilde{u}(t,x) = 0$, $\widetilde{v}(t,x) = 0$, $\mu(t) = 0$ in the right-hand side of equation (2.15), we find initial approximations $\dot{\lambda}^{(0)}(x)$, $\lambda^{(0)}(x)$, $x\in[0,\omega]$ from the boundary value problem with integral condition (2.15), (2.8).

(2) Find $\widetilde{v}^{(0)}(t,x)$, $\widetilde{w}^{(0)}(t,x)$, $\widetilde{u}^{(0)}(t,x)$, $(t,x)\in\Omega$ from the Goursat problem (2.1)–(2.3) for $\dot{\lambda}(x) = \dot{\lambda}^{(0)}(x)$, $\dot{\mu}(t) = \dot{\mu}^{(0)}(t)$, $\lambda(x) = \lambda^{(0)}(x)$, $\mu(t) = \mu^{(0)}(t)$.

Step 1. (1) Assuming $\widetilde{u}(t,x) = \widetilde{u}^{(0)}(t,x)$, $\widetilde{w}(t,x) = \widetilde{w}^{(0)}(t,x)$, $\lambda(x) = \lambda^{(0)}(x)$ in the right-hand side of equation (2.14), we find $\dot{\mu}^{(1)}(t)$, $\mu^{(1)}(t)$, $t\in[0,T]$ from the boundary value problem with integral condition (2.14), (2.9). Assuming $\widetilde{u}(t,x) = \widetilde{u}^{(0)}(t,x)$, $\widetilde{v}(t,x) = \widetilde{v}^{(0)}(t,x)$, $\mu(t) = \mu^{(0)}(t)$ in the right-hand side of equation (2.15), we find $\dot{\lambda}^{(1)}(x)$, $\lambda^{(1)}(x)$, $x\in[0,\omega]$ from the boundary value problem with integral condition (2.15), (2.8).

(2) Find $\widetilde{v}^{(1)}(t,x), \widetilde{w}^{(1)}(t,x), \widetilde{u}^{(1)}(t,x)$, $(t,x)\in\Omega$ from the Goursat problem (2.1)–(2.3) for $\dot{\lambda}(x) = \dot{\lambda}^{(1)}(x)$, $\dot{\mu}(t) = \dot{\mu}^{(1)}(t)$, $\lambda(x) = \lambda^{(1)}(x)$, $\mu(t) = \mu^{(1)}(t)$.
And so on.

Step *m*. (1) Assuming $\widetilde{u}(t,x) = \widetilde{u}^{(m-1)}(t,x)$, $\widetilde{w}(t,x) = \widetilde{w}^{(m-1)}(t,x)$, $\lambda(x) = \lambda^{(m-1)}(x)$ in the right-hand side of equation (2.14), we find $\dot{\mu}^{(m)}(t)$, $\mu^{(m)}(t)$, $t\in$

$[0,T]$ from the boundary value problem with integral condition (2.14), (2.9). Assuming $\widetilde{u}(t,x) = \widetilde{u}^{(m-1)}(t,x)$, $\widetilde{v}(t,x) = \widetilde{v}^{(m-1)}(t,x)$, $\mu(t) = \mu^{(m-1)}(t)$ in the right-hand side of equation (2.15), we find $\dot{\lambda}^{(m)}(x)$, $\lambda^{(m)}(x)$, $x \in [0,\omega]$ from the boundary value problem with integral condition (2.15), (2.8).

(2) Find $\widetilde{v}^{(m)}(t,x)$, $\widetilde{w}^{(m)}(t,x)$, $\widetilde{u}^{(m)}(t,x)$, $(t,x) \in \Omega$, from the Goursat problem (2.1)–(2.3) for $\dot{\lambda}(x) = \dot{\lambda}^{(m)}(x)$, $\dot{\mu}(t) = \dot{\mu}^{(m)}(t)$, $\lambda(x) = \lambda^{(m)}(x)$, $\mu(t) = \mu^{(m)}(t)$, $m = 1, 2, \dots$.

The constructed algorithm consists of two parts: we solve the boundary value problems with integral condition for the ordinary differential equations (2.14), (2.9) and (2.15), (2.8) in the first part, and we solve the Goursat problem for hyperbolic equations with functional parameters in the second part.

3. Boundary value problems with integral condition for the differential equations

Consider the boundary value problem with integral condition for the ordinary differential equations

$$\dot{\mu}(t) = \dot{g}_1(t), \quad t \in [0,T], \tag{3.1}$$

$$\int_0^b M(\tau,0)d\tau\mu(0) + \int_0^b M(\tau,0)\mu(\tau)d\tau = \varphi(0), \tag{3.2}$$

where the function $g_1(t)$ is continuously differentiable on $[0,T]$, the function $M(t,x)$ is continuous on Ω, $\varphi(0)$ is a constant, $0 < b \le T$.

The function $\mu(t) \in C([0,T],\mathbb{R})$ having the derivative $\dot{\mu}(t) \in C([0,T],\mathbb{R})$, is called a solution to problem (3.1), (3.2), if it satisfies ordinary differential equation (3.1) and boundary condition (3.2).

We also consider the boundary value problem with integral condition for the ordinary differential equation of the type

$$\dot{\lambda}(x) = \dot{g}_2(x), \quad x \in [0,\omega], \tag{3.3}$$

$$\int_0^a K(0,\xi)d\xi\lambda(0) + \int_0^a K(0,\xi)\lambda(\xi)d\xi = \psi(0), \tag{3.4}$$

where the function $g_2(x)$ is continuously differentiable on $[0,\omega]$, the function $K(t,x)$ is continuous on Ω, $\psi(0)$ is a constant, $0 < a \le \omega$.

The function $\lambda(x) \in C([0,\omega],\mathbb{R})$ having the derivative $\dot{\lambda}(x) \in C([0,\omega],\mathbb{R})$, is called a solution to problem (3.3), (3.4), if it satisfies ordinary differential equation (3.3) and boundary condition (3.4).

General solution to equation (3.1) has the form

$$\mu(t) = g_1(t) + C_1, \quad t \in [0,T],$$

where C_1 is a constant.

Since Condition (i) holds, the constant C_1 is uniquely determined by (3.2):

$$C_1 = \frac{1}{2B_2(0)}\varphi(0) - \frac{1}{2B_2(0)}\int_0^b M(\tau,0)\big\{g_1(\tau) + g_1(0)\big\}d\tau.$$

Then the unique solution to problem (3.1), (3.2) has the form

$$\mu(t) = g_1(t) + \frac{1}{2B_2(0)}\varphi(0) - \frac{1}{2B_2(0)}\int_0^b M(\tau,0)\big\{g_1(\tau) + g_1(0)\big\}d\tau, \tag{3.5}$$

for $t \in [0,T]$. Analogously, the general solution to equation (3.3) has the form

$$\lambda(x) = g_2(x) + C_2, \quad x \in [0,\omega],$$

where C_2 is a constant.

Since Condition (i) holds, the constant C_2 is uniquely determined by (3.4):

$$C_2 = \frac{1}{2B_1(0)}\psi(0) - \frac{1}{2B_1(0)}\int_0^a K(0,\xi)\big\{g_2(\xi) + g_2(0)\big\}d\xi.$$

Then unique solution to problem (3.3), (3.4) has the form

$$\lambda(x) = g_2(x) + \frac{1}{2B_1(0)}\psi(0) - \frac{1}{2B_1(0)}\int_0^a K(0,\xi)\big\{g_2(\xi) + g_2(0)\big\}d\xi, \tag{3.6}$$

for $x \in [0,\omega]$. Below we give conditions for the unique solvability of boundary value problems with integral condition (3.1), (3.2) and (3.3), (3.4).

Theorem 3.1. *Suppose Condition (i) is holds. Then problem* (3.1), (3.2) *has a unique solution $\mu^*(t) \in C([0,T],\mathbb{R})$ representable in the form* (3.5), *and*

$$\max_{t\in[0,T]} |\mu^*(t)| \le \mathcal{K}_1 \max\Big(\max_{t\in[0,T]} |g_1(t)|, |\varphi(0)|\Big), \tag{3.7}$$

where

$$\mathcal{K}_1 = 1 + \frac{1}{2|B_2(0)|}\Big[1 + 2b \max_{t\in[0,b]} |M(t,0)|\Big].$$

Theorem 3.2. *Suppose Condition (i) holds. Then problem* (3.3), (3.4) *has a unique solution $\lambda^*(x) \in C([0,\omega],\mathbb{R})$ representable in the form* (3.6), *and*

$$\max_{x\in[0,\omega]} |\lambda^*(x)| \le \mathcal{K}_2 \max\Big(\max_{x\in[0,\omega]} |g_2(x)|, |\psi(0)|\Big), \tag{3.8}$$

where

$$\mathcal{K}_2 = 1 + \frac{1}{2|B_1(0)|}\Big[1 + 2a \max_{x\in[0,a]} |K(0,x)|\Big].$$

4. Algorithm's convergence and main result

In Section 2, an algorithm for finding a solution to problem (2.1)–(2.5), which is equivalent to problem (1.1)–(1.3), is constructed. To formulate the main result, we let us give few assumptions and notation. Let Condition (i) hold, and introduce the notation:

$$\alpha = \max_{(t,x)\in\Omega} |A(t,x)|, \quad \beta = \max_{(t,x)\in\Omega} |B(t,x)|, \quad \gamma = \max_{(t,x)\in\Omega} |C(t,x)|,$$

$$H = \alpha + \beta + \gamma, \quad \kappa_1 = \max_{(t,x)\in\Omega} |K(t,x)|, \quad \kappa_2 = \max_{(t,x)\in\Omega} \Big|\frac{\partial K(t,x)}{\partial t}\Big|,$$

$$\sigma_1 = \max_{(t,x)\in\Omega} |M(t,x)|, \quad \sigma_2 = \max_{(t,x)\in\Omega} \Big|\frac{\partial M(t,x)}{\partial x}\Big|,$$

$$\beta_1 = \max_{t\in[0,T]} |[B_1(t)]^{-1}|, \quad \beta_2 = \max_{x\in[0,\omega]} |[B_2(x)]^{-1}|,$$

$$\delta_1 = \max_{t\in[0,T]} |\dot{B}_1(t)|, \quad \delta_2 = \max_{x\in[0,\omega]} |\dot{B}_2(x)|,$$

$$l_{11}(a) = a\beta_1\kappa_1\Big\{1 + \max(T,\omega)He^{H(T+\omega)}\Big\},$$

$$l_{21}(b) = b\beta_2\sigma_1\Big\{1 + \max(T,\omega)He^{H(T+\omega)}\Big\},$$

$$l_{12}(a) = a\beta_1\Big\{\delta_1\beta_1\kappa_1 + \kappa_2 + (\delta_1\beta_1\kappa_1 + \kappa_1 + \kappa_2)\max(T,\omega)He^{H(T+\omega)}\Big\},$$
$$l_{22}(b) = b\beta_2\Big\{\delta_2\beta_2\sigma_1 + \sigma_2 + (\delta_2\beta_2\sigma_1 + \sigma_1 + \sigma_2)\max(T,\omega)He^{H(T+\omega)}\Big\}.$$

In Section 3, the conditions for unique solvability of boundary value problems with integral condition (3.1), (3.2) and (3.3), (3.4) are established. At fixed $\widetilde{v}(t,x)$, $\widetilde{w}(t,x)$, $\widetilde{u}(t,x)$ on each step of the algorithm we solve the boundary value problems with integral condition (2.12), (2.9) and (2.13), (2.8). Besides, in problem (2.12), (2.9), we consider $\lambda(x)$ is also known. Same for $\mu(t)$ in problem (2.13), (2.8). At fixed $\dot{\lambda}(x)$, $\dot{\mu}(t)$, $\lambda(x)$, $\mu(t)$, we solved the Goursat problem (2.1)–(2.3).

The following statement gives the conditions for the convergence of proposed algorithm and the existence of unique solution to problem (2.1)–(2.5).

Theorem 4.1. *Let:*

(1) *the functions $A(t,x)$, $B(t,x)$, $C(t,x)$, and $f(t,x)$ be continuous on Ω;*
(2) *the functions $K(t,x)$ and $\psi(t)$ be continuously differentiable by t on Ω and $[0,T]$, respectively; the functions $M(t,x)$ and $\varphi(x)$ be continuously differentiable by x on Ω and $[0,\omega]$, respectively;*
(3) *Condition (i) hold;*
(4) *the inequality*
$$q = \max\Big(\mathcal{K}_1 l_{11}(a) + \mathcal{K}_2 l_{21}(b), l_{12}(a), l_{22}(b)\Big) < 1$$
be fulfilled.

Then problem (2.1)–(2.5) has a unique solution.

Proof. Let conditions (1)–(3) be fulfilled. We use step 0 of algorithm and consider the boundary value problem with integral condition

$$\dot{\mu}(t) = \frac{1}{B_1(t)}\dot{\psi}(t) - \frac{\dot{B}_1(t)}{B_1^2(t)}\psi(t), \quad t \in [0,T], \tag{4.1}$$
$$\int_0^b M(\tau,0)d\tau\mu(0) + \int_0^b M(\tau,0)\mu(\tau)d\tau = \varphi(0). \tag{4.2}$$
$$\dot{\lambda}(x) = \frac{1}{B_2(x)}\dot{\varphi}(x) - \frac{\dot{B}_2(x)}{B_2^2(x)}\varphi(x), \quad x \in [0,\omega], \tag{4.3}$$
$$\int_0^a K(0,\xi)d\xi\lambda(0) + \int_0^a K(0,\xi)\lambda(\xi)d\xi = \psi(0). \tag{4.4}$$

Condition (3) and the conditions of Theorems 3.1 and 3.2 yield the unique solvability of problems (4.1), (4.2) and (4.3), (4.4). We find initial approximations $\mu^{(0)}(t)$ and $\lambda^{(0)}(x)$ from the boundary value problems (4.1), (4.2) and (4.3), (4.4). Then, similar to the estimates (3.7) and (3.8), for the functions $\mu^{(0)}(t)$, $\lambda^{(0)}(x)$ and their derivatives $\dot{\mu}^{(0)}(t)$, $\dot{\lambda}^{(0)}(x)$ the estimates hold:

$$\max_{t\in[0,T]}|\mu^{(0)}(t)| \le \mathcal{K}_1 \max\Big(\beta_1 \max_{t\in[0,T]}|\psi(t)|, |\varphi(0)|\Big), \tag{4.5}$$
$$\max_{t\in[0,T]}|\dot{\mu}^{(0)}(t)| \le \beta_1 \max_{t\in[0,T]}|\dot{\psi}(t)| + \delta_1\beta_1^2 \max_{t\in[0,T]}|\psi(t)|. \tag{4.6}$$
$$\max_{x\in[0,\omega]}|\lambda^{(0)}(x)| \le \mathcal{K}_2 \max\Big(\beta_2 \max_{x\in[0,\omega]}|\varphi(x)|, |\psi(0)|\Big), \tag{4.7}$$

$$\max_{x\in[0,\omega]}|\dot{\lambda}^{(0)}(x)| \le \beta_2 \max_{x\in[0,\omega]}|\dot{\varphi}(x)| + \delta_2\beta_2^2 \max_{x\in[0,\omega]}|\varphi(x)|. \tag{4.8}$$

Solving the Goursat problem (2.1)–(2.3) for the found values of parameters, we find $\widetilde{v}^{(0)}(t,x)$, $\widetilde{w}^{(0)}(t,x)$, $\widetilde{u}^{(0)}(t,x)$ for all $(t,x)\in\Omega$.

The following inequalities are valid:

$$\begin{gathered}
|\widetilde{v}^{(0)}(t,x)| \le \max(T,\omega)e^{H(T+\omega)} \max_{(t,x)\in\Omega}|\widetilde{f}(t,x)|,\\
|\widetilde{w}^{(0)}(t,x)| \le \max(T,\omega)e^{H(T+\omega)} \max_{(t,x)\in\Omega}|\widetilde{f}(t,x)|,\\
|\widetilde{u}^{(0)}(t,x)| \le \max(T,\omega)e^{H(T+\omega)} \max_{(t,x)\in\Omega}|\widetilde{f}(t,x)|,
\end{gathered}$$

where

$$\widetilde{f}(t,x) = A(t,x)\dot{\lambda}^{(0)}(x) + B(t,x)\dot{\mu}^{(0)}(t) + C(t,x)\Big[\lambda^{(0)}(x)+\mu^{(0)}(t)\Big] + f(t,x).$$

Then, we determine the functions $\mu^{(m)}(t)$, $\lambda^{(m)}(x)$, $\dot{\mu}^{(m)}(t)$, $\dot{\lambda}^{(m)}(x)$, $\widetilde{v}^{(m)}(t,x)$, $\widetilde{w}^{(m)}(t,x)$, $\widetilde{u}^{(m)}(t,x)$ from the mth step of the algorithm, and we obtain $\mu^{(m+1)}(t)$, $\lambda^{(m+1)}(x)$, $\dot{\mu}^{(m+1)}(t)$, $\dot{\lambda}^{(m+1)}(x)$, $\widetilde{v}^{(m+1)}(t,x)$, $\widetilde{w}^{(m+1)}(t,x)$, $\widetilde{u}^{(m+1)}(t,x)$, from step $(m+1)$, $m=1,2,\dots$.

Evaluating the corresponding differences of successive approximations, we obtain

$$\begin{aligned}
&\max_{t\in[0,T]}|\mu^{(m+1)}(t)-\mu^{(m)}(t)|\\
&\le \mathcal{K}_1\beta_1 \max_{t\in[0,T]}\Big[\int_0^a |K(t,\xi)(\widetilde{u}^{(m)}(t,\xi)-\widetilde{u}^{(m-1)}(t,\xi))|d\xi \\
&\quad + \int_0^a |K(t,\xi)(\lambda^{(m)}(\xi)-\lambda^{(m-1)}(\xi))|d\xi\Big],
\end{aligned}\tag{4.9}$$

$$\begin{aligned}
&\max_{x\in[0,\omega]}|\lambda^{(m+1)}(x)-\lambda^{(m)}(x)|\\
&\le \mathcal{K}_2\beta_2 \max_{x\in[0,\omega]}\Big[\int_0^b |M(\tau,x)(\widetilde{u}^{(m)}(\tau,x)-\widetilde{u}^{(m-1)}(\tau,x))|d\tau \\
&\quad + \int_0^b |M(\tau,x)(\mu^{(m)}(\tau)-\mu^{(m-1)}(\tau))|d\tau\Big],
\end{aligned}\tag{4.10}$$

$$\begin{aligned}
&\max_{t\in[0,T]}|\dot{\mu}^{(m+1)}(t)-\dot{\mu}^{(m)}(t)|\\
&\le \max_{t\in[0,T]}\Big[|L_1(t,\lambda^{(m)}-\lambda^{(m-1)})| + |G_1(t,\widetilde{u}^{(m)}-\widetilde{u}^{(m-1)},\widetilde{w}^{(m)}-\widetilde{w}^{(m-1)})|\Big],
\end{aligned}\tag{4.11}$$

$$\begin{aligned}
&\max_{x\in[0,\omega]}|\dot{\lambda}^{(m+1)}(x)-\dot{\lambda}^{(m)}(x)|\\
&\le \max_{x\in[0,\omega]}\Big[|L_2(x,\mu^{(m)}-\mu^{(m-1)})| + |G_2(x,\widetilde{u}^{(m)}-\widetilde{u}^{(m-1)},\widetilde{v}^{(m)}-\widetilde{v}^{(m-1)})|\Big],
\end{aligned}\tag{4.12}$$

$$\begin{aligned}
&|\widetilde{v}^{(m+1)}(t,x) - \widetilde{v}^{(m)}(t,x)| \\
&\le \max(T,\omega) e^{H(T+\omega)} \Big\{ \alpha \max_{x\in[0,\omega]} |\dot{\lambda}^{(m+1)}(x) - \dot{\lambda}^{(m)}(x)| \\
&\quad + \beta \max_{t\in[0,T]} |\dot{\mu}^{(m+1)}(t) - \dot{\mu}^{(m)}(t)| + \gamma \Big[\max_{x\in[0,\omega]} |\lambda^{(m+1)}(x) - \lambda^{(m)}(x)| \\
&\quad + \max_{t\in[0,T]} |\mu^{(m+1)}(t) - \mu^{(m)}(t)| \Big] \Big\},
\end{aligned} \tag{4.13}$$

$$\begin{aligned}
&|\widetilde{w}^{(m+1)}(t,x) - \widetilde{w}^{(m)}(t,x)| \\
&\le \max(T,\omega) e^{H(T+\omega)} \Big\{ \alpha \max_{x\in[0,\omega]} |\dot{\lambda}^{(m+1)}(x) - \dot{\lambda}^{(m)}(x)| \\
&\quad + \beta \max_{t\in[0,T]} |\dot{\mu}^{(m+1)}(t) - \dot{\mu}^{(m)}(t)| + \gamma \Big[\max_{x\in[0,\omega]} |\lambda^{(m+1)}(x) - \lambda^{(m)}(x)| \\
&\quad + \max_{t\in[0,T]} |\mu^{(m+1)}(t) - \mu^{(m)}(t)| \Big] \Big\},
\end{aligned} \tag{4.14}$$

$$\begin{aligned}
&|\widetilde{u}^{(m+1)}(t,x) - \widetilde{u}^{(m)}(t,x)| \\
&\le \max(T,\omega) e^{H(T+\omega)} \Big\{ \alpha \max_{x\in[0,\omega]} |\dot{\lambda}^{(m+1)}(x) - \dot{\lambda}^{(m)}(x)| \\
&\quad + \beta \max_{t\in[0,T]} |\dot{\mu}^{(m+1)}(t) - \dot{\mu}^{(m)}(t)| + \gamma \Big[\max_{x\in[0,\omega]} |\lambda^{(m+1)}(x) - \lambda^{(m)}(x)| \\
&\quad + \max_{t\in[0,T]} |\mu^{(m+1)}(t) - \mu^{(m)}(t)| \Big] \Big\}.
\end{aligned} \tag{4.15}$$

Suppose that

$$\begin{aligned}
\Delta_{m+1} = \max\Big(&\max_{x\in[0,\omega]} |\lambda^{(m+1)}(x) - \lambda^{(m)}(x)| + \max_{t\in[0,T]} |\mu^{(m+1)}(t) - \mu^{(m)}(t)|, \\
&\max_{x\in[0,\omega]} |\dot{\lambda}^{(m+1)}(x) - \dot{\lambda}^{(m)}(x)|, \max_{t\in[0,T]} |\dot{\mu}^{(m+1)}(t) - \dot{\mu}^{(m)}(t)| \Big).
\end{aligned}$$

Then, from relations (4.9)–(4.12), taking into account the notation introduced and estimations (4.13)–(4.15), we obtain the main inequality

$$\Delta_{m+1} \le q\Delta_m. \tag{4.16}$$

Condition (4) of the theorem leads to the convergence of sequence $\Delta_m \to 0$ as $m \to \infty$, i.e., $\Delta_* = 0$. This gives the uniform convergence of sequences $\lambda^{(m)}(x)$, $\dot{\lambda}^{(m)}(x)$, $\mu^{(m)}(t)$, $\dot{\mu}^{(m)}(t)$, to $\lambda^*(x)$, $\dot{\lambda}^*(x)$, $\mu^*(t)$, $\dot{\mu}^*(t)$,respectively, as $m \to \infty$. Functions $\lambda^*(x)$ and $\mu^*(t)$ are continuous and continuously differentiable on $[0,\omega]$ and $[0,T]$, respectively. Based on estimates (4.13)–(4.15), we establish the uniform convergence of sequences $\widetilde{v}^{(m)}(t,x)$, $\widetilde{w}^{(m)}(t,x)$, $\widetilde{u}^{(m)}(t,x)$ to the functions $\widetilde{v}^*(t,x)$, $\widetilde{w}^*(t,x)$, $\widetilde{u}^*(t,x)$, respectively, with respect to $(t,x) \in \Omega$. Obviously, the functions $\widetilde{u}^*(t,x)$, $\widetilde{v}^*(t,x)$, and $\widetilde{w}^*(t,x)$ are continuous on Ω. Solving the problems on the $(m+1)$th step of the algorithm and passing to the limit as $m \to \infty$, we obtain that the functions $\widetilde{u}^*(t,x), \lambda^*(x)$, $\mu^*(t)$ together with their derivatives satisfy the Goursat problem (2.1)–(2.3) and boundary value problems with integral condition (2.14), (2.9) and (2.15), (2.8).

We carry out the inverse transition from problem (2.14), (2.9) to relation (2.4), and pass from problem (2.15), (2.8) to relation (2.5). Then the triplet of functions $(\widetilde{u}^*(t,x), \lambda^*(x), \mu^*(t))$ is solution to problem (2.1)–(2.5).

Next we prove the uniqueness of solution to (2.1)–(2.5). Let the triplet of functions $(\widetilde{u}^*(t,x), \lambda^*(x), \mu^*(t))$ and another triplet of functions $(\widetilde{u}^{**}(t,x), \lambda^{**}(x), \mu^{**}(t))$ be two solutions to the problem. We introduce the notation

$$\widetilde{\Delta} = \max\Big(\max_{x\in[0,\omega]} |\lambda^*(x) - \lambda^{**}(x)| + \max_{t\in[0,T]} |\mu^*(t) - \mu^{**}(t)|, \\ \max_{x\in[0,\omega]} |\dot{\lambda}^*(x) - \dot{\lambda}^{**}(x)|, \max_{t\in[0,T]} |\dot{\mu}^*(t) - \dot{\mu}^{**}(t)|, \Big).$$

After calculation, analogous to (4.9)–(4.15), we obtain

$$\widetilde{\Delta} \le q\widetilde{\Delta}. \tag{4.17}$$

By condition (4) of the theorem, we have $q < 1$. Then inequality (4.17) takes place only for $\widetilde{\Delta} \equiv 0$. This gives us $\lambda^*(x) = \lambda^{**}(x)$, $\mu^*(t) = \mu^{**}(t)$ and $\widetilde{u}^*(t,x) = \widetilde{u}^{**}(t,x)$. Therefore, the solution to problem (2.1)–(2.5) is unique. □

The next assertion follows from the equivalence of problem (1.1)–(1.3) and problem (2.1)–(2.5).

Theorem 4.2. *Let conditions (1)–(4) of Theorem 4.1 be fulfilled. Then problem* (1.1)–(1.3) *has a unique classical solution.*

Proof. Conditions (1)–(4) of Theorem 4.1 imply the existence of a unique solution to (2.1)–(2.5), the triplet of functions $(\widetilde{u}^*(t,x), \lambda^*(x), \mu^*(t))$. According to the algorithm presented above, for each $m = 0, 1, 2, \dots$, this triplet is determined as a limit of sequence triplets $(\widetilde{u}^{(m)}(t,x),\ \mu^{(m)}(t)),\ \lambda^{(m)}(x))$ as $m \to \infty$.

Then solution to problem (1.1)–(1.3), the function $u^*(t,x)$, exists and is determined by the equality $u^*(t,x) = \widetilde{u}^*(t,x) + \lambda^*(x) + \mu^*(t)$. □

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