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UNIQUENESS OF A POSITIVE SOLUTION FOR QUASILINEAR ELLIPTIC EQUATIONS IN HEISENBERG GROUP

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Abstract. In this note we address the question of uniqueness of the Brezis-Oswald problem for the p-Laplacian operator in Heisenberg Group. The non-availability of $C^{1,\alpha}$ regularity for all $1 < p < \infty$ is the problem to extend the proof of Diaz-Saa [10] in the Heisenberg Group case. We overcome the problem by proving directly a generalized version of Diaz-Saá inequality in the Heisenberg Group.

1. Introduction

The well-known paper of Brézis and Oswald [6] provides the necessary and sufficient condition for the existence and uniqueness of positive solutions for the equation

$$\begin{gathered} -\Delta u = f(x,u) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega \end{gathered}$$

where Ω is a bounded open domain in $\mathbb{R}^n$ and under some hypothesis on f. Almost immediately the result was extended by Diáz and Saá [10] to the p-Laplacian case by introducing a new inequality which came to be later known as the Díaz-Saá inequality.

The purpose of this paper is to extend the result of [10] in the context of Heisenberg Group, which we will denote by $\mathcal{H}^n$. Consider the problem:

$$\begin{gathered} -\Delta_{H,p} u = f(x,u) \quad \text{in } \Omega \\ u \geq 0, \quad u \not\equiv 0 \quad \text{in } \Omega \\ u = 0 \quad \text{on } \partial\Omega \end{gathered} \tag{1.1}$$

where Ω is an open bounded domain of $\mathcal{H}^n$ and $1 < p < \infty$.

We consider $f : \Omega \times [0,\infty) \to (0,\infty)$ satisfying the following hypothesis:

- (I) The function $r \mapsto f(x,r)$ is continuous on $[0,\infty)$ for a.e. $x \in \Omega$ and for every $r \geq 0$, the function $x \to f(x,r)$ is in $L^\infty(\Omega)$.
- (II) The function $r \mapsto \frac{f(x,r)}{r^{p-1}}$ is strictly decreasing on $(0,\infty)$ for a.e $x \in \Omega$.
- (III) There exists $C > 0$ such that $f(x,r) \leq C(r^{p-1} + 1)$ for a.e. $x \in \Omega$ and for all r.

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By weak solution of (1.1) we mean $u \in D_0^{1,p}(\Omega) \cap L^\infty(\Omega)$ such that

$$\int_\Omega |\nabla_H u|^{p-2} \langle \nabla_H u(x), \phi(x) \rangle dx = \int_\Omega f(x,u)\phi(x) dx \tag{1.2}$$

holds for all $\phi \in C_c^\infty(\Omega)$. In this note we aim to establish the uniqueness of the weak solution to (1.1).

Before we start with our results let us briefly recall some basic notions regarding the Heisenberg Group $\mathcal{H}^n$ along with some literature which is available on the study of elliptic equation on Heisenberg group.

The Heisenberg group $\mathcal{H}^n = (\mathcal{R}^{2n+1}, \cdot)$ is nilpotent Lie group endowed with the group structure:

$$(x,y,t) \cdot (x',y',t') = (x + x', y + y', t + t' + 2(\langle y, x' \rangle - \langle x, y' \rangle)) \tag{1.3}$$

where $x, y, x', y' \in \mathcal{R}^n$, $t, t' \in \mathcal{R}$ and $\langle , \rangle$ denotes the standard inner product in $\mathcal{R}^n$.

The left invariant vector field generating the Lie algebra is given by

$$\mathcal{T} = \frac{\partial}{\partial t}, \quad \mathcal{X}_i = \frac{\partial}{\partial x_i} + 2y_i \frac{\partial}{\partial t}, \quad \mathcal{Y}_i = \frac{\partial}{\partial y_i} - 2x_i \frac{\partial}{\partial t}, \quad i = 1, 2, \dots, n. \tag{1.4}$$

and satisfy the relationship

$$[\mathcal{X}_i, \mathcal{Y}_i] = -4\delta_{ij} T, \quad [\mathcal{X}_i, \mathcal{X}_j] = [\mathcal{Y}_i, \mathcal{Y}_j] = [\mathcal{X}_i, \mathcal{T}] = [\mathcal{Y}_i, \mathcal{T}] = 0. \tag{1.5}$$

The generalized gradient is given by $\nabla_H = (\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n, \mathcal{Y}_1, \mathcal{Y}_2, \dots, \mathcal{Y}_n)$. Hence the sub-Laplacian Δ_H and the p-sub-Laplacian $\Delta_{H,p}$ are denoted by

$$\Delta_H = \sum_{i=1}^n \mathcal{X}_i^2 + \mathcal{Y}_i^2 = \nabla_H \cdot \nabla_H,$$
$$\Delta_{H,p} = \nabla_H \cdot (|\nabla_H|^{p-2} \nabla_H), \quad p > 1\,.$$

We also denote the space $D^{1,p}(\Omega)$ and $D_0^{1,p}(\Omega)$ as $\{u : \Omega \to \mathcal{R}; u, |\nabla_H u| \in L^p(\Omega)\}$ and the closure of $C_0^\infty(\Omega)$ with respect to the norm $\|u\|_{D_0^{1,p}(\Omega)} = (\int_\Omega |\nabla_H u|^p \, dx\, dy\, dt)^{1/p}$ respectively. Some results on the Laplacian and the p-Laplacian has been generalized to the Heisenberg Group with various degree of success. Consider the problem

$$\begin{gathered} -\Delta_{H,p} u = f(u) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega \end{gathered} \tag{1.6}$$

Some of the very first results obtained regarding the above problem for $p = 2$ is by Garofalo-Lanconelli [12], where existence and nonexistence results were derived using integral identities of Rellich-Pohozaev type. In Birindelli et al [4], Liouville theorems for semilinear equations are proved. One can also find monotonicity and symmetry results in Birindelli and Prajapat [5]. As for the p-sub-Laplacian case, Niu et al [14] considered the question of non-uniqueness of the (1.6) using the Picone Identity and the Pohozaev Identities for the p-sub-Laplacian on Heisenberg Group. Results on p-sub-Laplacian involving singular indefinite weight can be found in Dou [11] and Tyagi [15] and the reference therein. For more details about Heisenberg Group the reader may consult [9].

One of the problems when dealing with p-sub-Laplacian is the non-availability of the $C^{1,\alpha}$ regularity for $1 < p < \infty$, although it has been proved in Marchi [13] to exist for p near 2. It is worth mentioning that the methods of Díaz-Sáa [10] can not be directly applied here due to the non-availability of $C^{1,\alpha}$ regularity in the

Heisenberg Group. In this work we bypass that problem by using a generalized version of Díaz-Sáa Inequality in Heisenberg Group.

2. Preliminary Results

We start this section with the generalized Picone's Identity for p-sub-Laplacian in Heisenberg Group, which is extension of the main result in Euclidean space obtained in [2]. In what follows we assume $g:(0,\infty)\to(0,\infty)$ is a locally Lipchitz function that satisfies the differential inequality

$$g'(x)\geq(p-1)[g(x)]^{\frac{p-2}{p-1}}\quad \text{a.e. in } (0,\infty) \tag{2.1}$$

Remark 2.1. Example of functions satisfying (2.1) are $g(x)=x^{p-1}$ (where the equality holds) and $e^{(p-1)x}$.

In what follows we use ∇ to denote ∇_H and Δ_p to denote $\Delta_{H,p}$.

Theorem 2.2 (Generalized Picone identity)**.** *Let $1<p\leq Q$ and Ω be any domain in $\mathcal{H}^n$. Let u and v be differentiable functions with $v>0$ a.e in Ω. Also assume g satisfies* (2.1)*. Define*

$$L(u,v)=|\nabla u|^p-p\frac{|u|^{p-2}u}{g(v)}\nabla u\cdot\nabla v|\nabla v|^{p-2}+\frac{g'(v)|u|^p}{[g(v)]^2}|\nabla v|^p \quad \textit{a.e in } \Omega.$$

$$R(u,v)=|\nabla u|^p-\nabla(\frac{|u|^p}{g(v)})|\nabla v|^{p-2}\nabla v \quad \textit{a.e. in } \Omega.$$

Then $L(u,v)=R(u,v)\geq 0$. Moreover $L(u,v)=0$ a.e. in Ω if and only if $\nabla(\frac{u}{v})=0$ a.e. in Ω.

Remark 2.3. Note that there is no restriction on the sign of u, as one can find in [7, Proposition 3]. When $g(x)=x^{p-1}$ and $u\geq 0$, we obtain the Picone identity [11, Lemma 2.1].

Proof of Theorem 2.2. Expanding $\nabla(\frac{|u|^p}{g(v)})$ we have

$$\begin{aligned}\nabla(\frac{|u|^p}{g(v)})&=\frac{pg(v)|u|^{p-2}u\nabla u-g'(v)|u|^p\nabla v}{[g(v)]^2}\\&=p\frac{|u|^{p-2}u\nabla u}{g(v)}-\frac{g'(v)|u|^p\nabla v}{[g(v)]^2}.\end{aligned}$$

Plugging it in $R(u,v)$ we have $R(u,v)=L(u,v)$.

To show positivity of $L(u,v)$ we proceed as follows,

$$\frac{|u|^{p-2}u}{g(v)}\nabla u.\nabla v|\nabla v|^{p-2}\leq\frac{|u|^{p-1}}{g(v)}|\nabla v|^{p-1}|\nabla u|. \tag{2.2}$$

By Young's inequality we have

$$p\frac{|u|^{p-1}}{g(v)}|\nabla v|^{p-1}|\nabla u|\leq|\nabla u|^p+(p-1)\frac{|u|^p|\nabla v|^p}{[g(v)]^{\frac{p}{p-1}}}\,. \tag{2.3}$$

Using (2.2) and (2.3) we have

$$L(u,v)\geq-(p-1)\frac{|u|^p|\nabla v|^p}{[g(v)]^{\frac{p}{p-1}}}+\frac{g'(v)|u|^p}{[g(v)]^2}|\nabla v|^p$$

Now since g satisfies (2.1) i.e., $g'(x)\geq(p-1)[g(x)]^{\frac{p-2}{p-1}}$ we have $L(u,v)\geq 0$.

Equality holds when the following occurs simultaneously:

$$g'(x) = (p-1)[g(x)]^{\frac{p-2}{p-1}}, \tag{2.4}$$

$$\frac{|u|^{p-2}u}{g(v)} \nabla u . \nabla v |\nabla v|^{p-2} = \frac{|u|^{p-1}}{g(v)} |\nabla v|^{p-1} |\nabla u|, \tag{2.5}$$

$$|\nabla u| = \frac{|u \nabla v|}{g(v)^{\frac{1}{p-1}}} \tag{2.6}$$

Set

$$\mathcal{X} = \{x \in \Omega : \frac{|u \nabla v|}{g(v)^{\frac{1}{p-1}}} = 0\} \tag{2.7}$$

By (2.6) we have

$$\frac{|u \nabla v|}{g(v)^{\frac{1}{p-1}}} = |\nabla u| = 0 \quad \text{a.e. on } \mathcal{X}. \tag{2.8}$$

From (2.8) and (2.4), for $g(x) = x^{p-1}$ we have

$$\frac{u}{v} \nabla v = \nabla u = 0 \quad \text{a.e. on } \mathcal{X}. \tag{2.9}$$

On $\mathcal{X}^c$, let

$$w = \frac{|\nabla u| [g(v)]^{\frac{1}{p-1}}}{|u \nabla v|} \tag{2.10}$$

Hence from the fact that $L(u, v) = 0$ a.e. in Ω we have

$$w^p - pw + p - 1 = 0 \tag{2.11}$$

which holds if and only if $w = 1$.

Again taking into account (2.4), for $g(x) = x^{p-1}$ we have

$$\nabla u \cdot \big(\nabla u - \nabla v \frac{u}{v}\big) = 0 \quad \text{a.e. in } \mathcal{X}^c \tag{2.12}$$

Combining (2.9) and (2.12) we can easily conclude that $L(u, v) = 0$ if and only if $\nabla\big(\frac{u}{v}\big) = 0$ a.e. in Ω. $\square$

With the generalized Picone's identity in our hands we can now proceed to prove the Picone's inequality which is the vital ingredient for the proof of Díaz-Saá Inequality. We will present a non-linear version of the inequality and will closely follow the proof of Abdellaoui-Peral [1].

Theorem 2.4 (Generalized Picone inequality). *Let $1 < p \leq Q$ and Ω be a bounded domain in $\mathcal{H}^n$. If $u, v \in D_0^{1,p}(\Omega)$ such that $-\Delta_p v = \mu$ where μ is a positive, bounded and measurable function satisfying the hypothesis (III) with $v|_{\partial\Omega} = 0$, $v(\not\equiv 0) \geq 0$ and g satisfies* (2.1)*. Then we have*

$$\int_\Omega |\nabla u|^p \geq \int_\Omega \Big(\frac{|u|^p}{g(v)}\Big)(-\Delta_p v). \tag{2.13}$$

Remark 2.5. When $g(u) = u^{p-1}$, we get Picone's inequality in Heisenberg Group in Dou [11].

Before we prove our theorem we need the following lemma.

Lemma 2.6. *Let $p>1$ and Ω be any domain in $\mathcal{H}^n$ and let $v\in D^{1,p}(\Omega)$ be such that $v\geq\delta>0$. Then for all $u\in C_c^\infty(\Omega)$ we have*
$$\int_\Omega |\nabla u|^p \geq \int_\Omega \Big(\frac{|u|^p}{g(v)}\Big)(-\Delta_p v). \tag{2.14}$$

Proof. Since $v\in D^{1,p}(\Omega)$, we can choose $v_n\in C^1(\Omega)$ such that the following holds:
$$v_n>\frac{\delta}{2} \text{ in } \Omega, \quad v_n\to v \text{ in } D^{1,p}(\Omega), \quad v_n\to v \text{ a.e. in } \Omega. \tag{2.15}$$
Employing Theorem 2.2 with v_n and u we have
$$\int_\Omega R(u,v_n)\geq 0 \quad \text{since } R(u,v_n)\geq 0 \text{ a.e. in } \Omega \text{ and for all } n\in\mathbf{N}. \tag{2.16}$$
i.e.,
$$\int_\Omega |\nabla u|^p \geq \int_\Omega \nabla\Big(\frac{|u|^p}{g(v_n)}\Big)|\nabla v_n|^{p-2}\nabla v_n = \int_\Omega \frac{|u|^p}{g(v_n)}(-\Delta_p v_n).$$

Note that since $-\Delta_p$ is a continuous function from $D^{1,p}(\Omega)$ to $D^{-1,p'}(\Omega)$ for $p'=\frac{p}{p-1}$, we have $-\Delta_p v_n\to -\Delta_p v$ in $D^{1,p}(\Omega)$ and for g locally Lipchitz continuous in $(0,\infty)$ we have $g(v_n)\to g(v)$ a.e. Hence using Lebesgue dominated convergence theorem and the fact that g is increasing on $(0,\infty)$ we have
$$\int_\Omega |\nabla u|^p \geq \int_\Omega \frac{|u|^p}{g(v)}(-\Delta_p v) \tag{2.17}$$
for any $u\in C_c^\infty(\Omega)$. □

Before we proceed with the proof of Theorem 2.4, we state the Strong Maximum Principle from [11] which was proved using the Harnack Inequality of [8].

Lemma 2.7 (Strong maximum principle)**.** *Let $p>1$ and $\Omega\subset\mathcal{H}^n$ be a bounded domain and $u\in D_0^{1,p}(\Omega)$ be nonnegative solution of the equation*
$$-\Delta_p u = h(x,u) \quad \text{in } \Omega;\ u|_{\partial\Omega}=0 \tag{2.18}$$
where $h:\Omega\times\mathbb{R}\to\mathbb{R}$ is a measurable function such that $|h(x,u)|\leq C(u^{p-1}+1)$. Then $u\equiv 0$ or $u>0$ in Ω.

Proof of Theorem 2.4. Using the Strong Maximum Principle we have $v>0$ in Ω. Denote, $v_n(x)=v(x)+\frac{1}{n}$, $n\in\mathbb{N}$. Thus we have the following:

- $\Delta_p v_n = \Delta_p v$.
- $v_n\to v$ a.e in Ω and in $D^{1,p}(\Omega)$.
- $g(v_n)\to g(v)$ a.e in Ω.

Hence using Lemma 2.6 for $u\in C_c^\infty(\Omega)$, we have
$$\int_\Omega |\nabla u|^p \geq \int_\Omega \frac{|u|^p}{g(v)}(-\Delta_p v) \tag{2.19}$$
Now to conclude our theorem for $u\in D_0^{1,p}(\Omega)$, we use $u_n\in C_c^\infty(\Omega)$ such that $u_n\to u$ in $D_0^{1,p}(\Omega)$. Choosing u_n and v_n in Lemma 2.6, we have
$$\int_\Omega |\nabla u_n|^p \geq \int_\Omega \Big(\frac{|u_n|^p}{g(v_n)}\Big)(-\Delta_p v_n)\,. \tag{2.20}$$

Now using the fact that g satisfies (2.1) by Fatou's lemma, we have

$$\int_\Omega |\nabla u|^p \geq \int_\Omega \Big(\frac{|u|^p}{g(v)}\Big)(-\Delta_p v) \tag{2.21}$$

which completes our proof. □

We conclude this section with the Díaz-Saá Inequality in Heisenberg Group. For this part we use $g(u) = u^{p-1}$

Theorem 2.8 (Díaz-Saá inequality)**.** *Let $1 < p \leq Q$ and Ω be a bounded domain in $\mathcal{H}^n$. If $u_i \in D_0^{1,p}(\Omega)$ s.t $-\Delta_p u_i = \mu_i$, where μ_i is a positive, bounded and measurable function satisfying the hypothesis (III) with $u_i|_{\partial\Omega} = 0$ and $u_i(\not\equiv 0) \geq 0$ a.e. in Ω for $i = 1, 2$. Then we have*

$$\int_\Omega \Big(-\frac{\Delta_p u_1}{u_1^{p-1}} + \frac{\Delta_p u_2}{u_2^{p-1}}\Big)(u_1^p - u_2^p) \geq 0. \tag{2.22}$$

Note that above theorem is not true for a general g satisfying (2.1).

Prof of Theorem 2.8. Choosing u_i for $i = 1, 2$ satisfying the hypothesis of Theorem 2.8 and then plugging the couple (u_1, u_2) into Theorem 2.4 we obtain

$$\int_\Omega |\nabla u_1|^p \geq \int_\Omega \Big(-\frac{\Delta_p u_2}{u_2^{p-1}}\Big)u_1^p \tag{2.23}$$

Using integration by parts on right-hand side, we have

$$\int_\Omega \Big(-\frac{\Delta_p u_1}{u_1^{p-1}} + \frac{\Delta_p u_2}{u_2^{p-1}}\Big)u_1^p \geq 0. \tag{2.24}$$

Now interchanging the couple (u_1, u_2) with (u_2, u_1) in Theorem 2.4 we obtain

$$\int_\Omega |\nabla u_2|^p \geq \int_\Omega \Big(-\frac{\Delta_p u_1}{u_1^{p-1}}\Big)u_2^p \tag{2.25}$$

Again using integration by parts on the right-hand side, we have

$$\int_\Omega \Big(-\frac{\Delta_p u_2}{u_2^{p-1}} + \frac{\Delta_p u_1}{u_1^{p-1}}\Big)u_2^p \geq 0. \tag{2.26}$$

Adding (2.24) and (2.26) we have

$$\int_\Omega \Big(-\frac{\Delta_p u_1}{u_1^{p-1}} + \frac{\Delta_p u_2}{u_2^{p-1}}\Big)(u_1^p - u_2^p) \geq 0. \tag{2.27}$$

Hence the proof is complete. □

3. Main results

In this section we state and prove our main result.

Theorem 3.1 (Uniqueness of a solution)**.** *There exists at most one positive weak solution to* (2.1) *in $D_0^{1,p}(\Omega) \cap L^\infty(\Omega)$ for $1 < p \leq Q$.*

Proof. Let u and v be two non-negative solutions of (1.1). Then using Lemma 2.7 we have, $u, v > 0$ in Ω. Moreover since $f(x, u)$ is positive and satisfy hypothesis (I) and (III), we have for $u \neq v$ by Theorem 2.8,

$$0 \leq \int_\Omega \Big(-\frac{\Delta_p u}{u^{p-1}} + \frac{\Delta_p v}{v^{p-1}}\Big)(u^p - v^p) = \int_\Omega \Big(\frac{f(x,u)}{u^{p-1}} - \frac{f(x,v)}{v^{p-1}}\Big)(u^p - v^p) < 0. \tag{3.1}$$

Hence we arrive at a contradiction. □

We conclude this article with a few comments:

- Because of the lack of regularity we are forced to put the positivity condition on f, which was not present in the assumptions of Diaz-Saa [10]. It will be interesting to know if one can conclude the same results for uniqueness without the positivity condition on f.
- The restriction on p is due to the fact that Lemma 2.7 (Strong Maximum Principle) is valid for $1 < p \leq Q$.
- The statements proved in Theorems 2.2 and 2.4 were valid for a wide range of functions satisfying (2.1). This results are new in the contexts of Heisenberg group and one can actually obtain Nonexistence results and Comparison Principles for p-sub-Laplacian similar to those in [2] and the reference therein.

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