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NON-LOCAL ELLIPTIC SYSTEMS ON THE HEISENBERG GROUP

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ABSTRACT. We present Liouville type results for certain systems of nonlinear elliptic equations containing fractional powers of the Laplacian on the Heisenberg group. Our method of proof is based on the test function method and a recent inequality proved by Alsaedi, Ahmad, and Kirane, leading to the derivation of sufficient conditions in terms of space dimension and systems parameters.

1. Introduction

This article concerns Liouville type results for two nonlinear systems of elliptic equations with nonlocal diffusions posed on the Heisenberg group. We start with the system

$$\begin{aligned} (-\Delta_{\mathbb{H}})^{\mu/2}u &= |v|^q, \quad q>1, \\ (-\Delta_{\mathbb{H}})^{\nu/2}v &= |u|^p, \quad p>1, \end{aligned} \tag{1.1}$$

posed in $\mathbb{R}^{2N+1}$, and where the fractional power of the Laplacian on the Heisenberg group $(-\Delta_{\mathbb{H}})^{\delta/2}$ $(0<\delta<2)$ accounts for anomalous diffusion and is to be defined later. Using the test function method and a variant of Cordoba-Cordoba's inequality [3] for the Heisenberg group proved in [1], we find a relation relating N,μ,ν,p and q leading to Liouville type results. Let us point out that we overcome a difficulty raised by the test function by using the inequality proved in [1] for $(-\Delta_{\mathbb{H}})^{\frac{\mu}{2}}$. Then we consider the system

$$\begin{aligned} (-\Delta_{\mathbb{H}})^{\mu_1/2}|u| + (-\Delta_{\mathbb{H}})^{\mu_2/2}|v| &= |v|^q, \quad q>1, \\ (-\Delta_{\mathbb{H}})^{\nu_1/2}|v| + (-\Delta_{\mathbb{H}})^{\nu_2/2}|u| &= |u|^p, \quad p>1, \end{aligned} \tag{1.2}$$

where $0<\mu_i,\nu_i\le 2$ $(i=1,2)$ are constants. Here the positivity condition on the solutions is omitted and replaced by the absolute value of u and v.

2. Preliminaries

For the reader's convenience, let us briefly recall the definition and basic properties of the Heisenberg group and the inequality in [1].

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2.1. **Heisenberg group.** The Heisenberg group $\mathbb{H}$, whose points will be denoted by $\eta = (x, y, \tau)$, is the Lie group $(\mathbb{R}^{2N+1}, \circ)$ with the non-commutative group operation $\circ$ defined by

$$\eta \circ \tilde{\eta} = (x + \tilde{x}, y + \tilde{y}, \tau + \tilde{\tau} + 2(x \cdot \tilde{y} - \tilde{x} \cdot y)),$$

where "$\cdot$" is the usual inner product in $\mathbb{R}^N$. The Laplacian $\Delta_{\mathbb{H}}$ over $\mathbb{H}$ is obtained from the vector fields $X_i = \frac{\partial}{\partial x_i} + 2y_i \frac{\partial}{\partial \tau}$ and $Y_i = \frac{\partial}{\partial y_i} - 2x_i \frac{\partial}{\partial \tau}$, by

$$\Delta_{\mathbb{H}} = \sum_{i=1}^{N} (X_i^2 + Y_i^2). \tag{2.1}$$

Explicit computation gives the expression

$$\Delta_{\mathbb{H}} = \sum_{i=1}^{N} \Big(\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + 4y_i \frac{\partial^2}{\partial x_i \partial \tau} - 4x_i \frac{\partial^2}{\partial y_i \partial \tau} + 4(x_i^2 + y_i^2) \frac{\partial^2}{\partial \tau^2} \Big). \tag{2.2}$$

A natural group of dilations on $\mathbb{H}$ is given by

$$\delta_\lambda(\eta) = (\lambda x, \lambda y, \lambda^2 \tau), \quad \lambda > 0,$$

whose Jacobian determinant is λ^Q, where

$$Q = 2N + 2 \tag{2.3}$$

is the homogeneous dimension of $\mathbb{H}$.

The operator $\Delta_{\mathbb{H}}$ is a degenerate elliptic operator. It is invariant with respect to the left translation of $\mathbb{H}$ and homogeneous with respect to the dilations δ_λ. More precisely, we have

$$\begin{gathered} \Delta_{\mathbb{H}}(u(\eta \circ \tilde{\eta})) = (\Delta_{\mathbb{H}} u)(\eta \circ \tilde{\eta}), \\ \Delta_{\mathbb{H}}(u \circ \delta_\lambda) = \lambda^2 (\Delta_{\mathbb{H}} u) \circ \delta_\lambda, \quad \eta, \tilde{\eta} \in \mathbb{H}. \end{gathered} \tag{2.4}$$

The natural distance from η to the origin is

$$|\eta|_{\mathbb{H}} = \Big(\tau^2 + \Big(\sum_{i=1}^{N} (x_i^2 + y_i^2) \Big)^2 \Big)^{1/4}. \tag{2.5}$$

2.2. **Fractional powers of sub-elliptic Laplacians.** The representation of the fractional power of $(-\Delta_{\mathbb{H}})^s$ is given by the following theorem.

Theorem 2.1. *The operator $\Delta_{\mathbb{H}}$ is a positive self-adjoint operator with domain $W_{\mathbb{H}}^{2,2}(\mathbb{H})$. Denote now by $\{E(\lambda)\}$ the spectral resolution of $\Delta_{\mathbb{H}}$ in $L^2(\mathbb{H})$. If $\alpha > 0$, then*

$$(-\Delta_{\mathbb{H}})^{\alpha/2} = \int_0^{+\infty} \lambda^{\alpha/2} \, dE(\lambda),$$

with domain

$$W_{\mathbb{H}}^{\alpha,2}(\mathbb{H}) := \{ v \in L^2(\mathbb{H}); \int_0^{+\infty} \lambda^\alpha \, d\langle E(\lambda)v, v\rangle < \infty \},$$

endowed with graph norm.

Proposition 2.2 ([1])**.** *Assume that the function $\varphi \in C_0^\infty(\mathbb{R}^{2N+1})$. Then*

$$\sigma \varphi^{\sigma-1} (-\Delta_{\mathbb{H}})^{\sigma/2} \varphi \geq (-\Delta_{\mathbb{H}})^{\sigma/2} \varphi^\sigma \tag{2.6}$$

holds point-wise.

A proof of the above proposition can be found in [1].

3. Main results

The definition of solutions we adopt for system (1.1) is as follows.

Definition 3.1. We say that the pair (u, v) is a weak solution of (1.1), if

$$
\begin{aligned}
&(u, v) \in L^p_{\rm loc}(\mathbb{R}^{2N+1}) \times L^q_{\rm loc}(\mathbb{R}^{2N+1}),\\
&\int_{\mathbb{R}^{2N+1}} u(-\Delta)^{\mu/2}\psi\,dx = \int_{\mathbb{R}^{2N+1}} |v|^q\psi\,dx, \qquad (3.1)\\
&\int_{\mathbb{R}^{2N+1}} v(-\Delta)^{\nu/2}\psi\,dx = \int_{\mathbb{R}^{2N+1}} |u|^p\psi\,dx, \qquad (3.2)
\end{aligned}
$$

for any nonnegative test function $\psi \in \mathcal{C}^\infty_0(\mathbb{R}^{2N+1})$.

Before we present our results, let us mention some important works on Liouville type theorems for the classical nonlinear elliptic equations/systems on the Heisenberg group. Véron and Pohozaev [9] improved the study of Birindelli, Capuzzo Dolcetta and Cutri [2] concerning the equation

$$
\Delta_{\mathbb{H}}(au) + |u|^p \le 0 \qquad (3.3)
$$

with a bounded function a and $1 < p$; they proved that (3.3) admits only trivial solution whenever $1 < p \le \frac{Q}{Q-2}$. Their work has been improved recently by Xu [10] who proved that $u \equiv 0$ provided $1 < p < \frac{Q(Q+2)}{(Q-1)^2}$.

For nonlinear equations we refer to the paper of Garofalo and Lanconelli [5] as well as the one of Uguzzoni [8]. Recently, Quas and Xi [7] emphasis that, the condition $1 < p, q \le \frac{N}{N-\alpha}$ covered by Dahmani-Karami-Kerbal [4, theorem 2] was not considered in [7, Theorem 1.3]. Here, we are considering the first system 1.1 for fractional Laplacian operators on Heisenberg Group, while in [7] and [4] the authors treated the system in $\mathbb{R}^N$ for classical Laplacian with the same fractional exponents and classical Laplacian with different fractional exponents respectively. The main result for system (1.1) is as follows.

Theorem 3.2. *Let (u, v) be a weak solution of system (1.1). If Q, the homogeneous dimension of $\mathbb{H}$, satisfies the inequality*

$$
Q < \big(\frac{pq}{pq-1}\big)\max\big\{\frac{\nu}{p} + \mu, \frac{\mu}{q} + \nu\big\}, \qquad (3.4)
$$

then (u, v) is trivial.

Our second main results concerns system (1.2).

Theorem 3.3. *Let (u, v) be a weak solution to system (1.2). If*

$$
Q < \max\big\{\gamma, \theta\big\} \qquad (3.5)
$$

where

$$
\begin{aligned}
\gamma &= \min\Big\{\frac{\nu_2 p}{p-1}, \nu_1 + \frac{\mu_2}{q-1}, \big(\frac{\mu_1}{q} + \nu_1\big)\frac{pq}{pq-1}\Big\},\\
\theta &= \min\Big\{\frac{\mu_2 q}{q-1}, \mu_1 + \frac{\nu_2}{p-1}, \big(\frac{\nu_1}{p} + \mu_1\big)\frac{pq}{pq-1}\Big\},
\end{aligned}
$$

then (u, v) is trivial.

4. Proofs of main results

Note that for a function $\psi \in \mathcal{C}_0^\infty(\mathbb{R}^N)$, $\delta \in (0,2]$ and $\beta > p'$ $((\beta-1)p' - \beta\frac{p'}{p} > 0)$ we have

$$\int_{\mathbb{R}^{2N+1}} \psi^{(\beta-1)p'-\beta\frac{p'}{p}} |(-\Delta_{\mathbb{H}})^{\delta/2}\psi|^{p'}\, d\eta = \int_K \psi^{(\beta-1)p'-\beta\frac{p'}{p}} |(-\Delta_{\mathbb{H}})^{\delta/2}\psi|^{p'}\, d\eta < \infty,$$

where $K := \mathrm{supp}(\psi)$ stands for support of ψ, and $p + p' = pp'$. For the proof of our main results, we consider a cut-off function $\varphi \in \mathcal{C}_0^\infty(\mathbb{R})$ such that $0 \le \varphi \le 1$, $|\varphi'(r)| \le \frac{C}{r}$, and for any $r > 0$,

$$\varphi(r) = \begin{cases} 1 & \text{if } r \le 1, \\ 0 & \text{if } r \ge 2. \end{cases}$$

Proof of Theorem 3.2. From (3.1) and (3.2) we have

$$\begin{gathered} \int_{\mathbb{R}^{2N+1}} u(-\Delta_{\mathbb{H}})^{\mu/2}\psi^\beta d\eta = \int_{\mathbb{R}^{2N+1}} |v|^q \psi^\beta\, d\eta, \\ \int_{\mathbb{R}^{2N+1}} v(-\Delta_{\mathbb{H}})^{\nu/2}\psi^\beta d\eta = \int_{\mathbb{R}^{2N+1}} |u|^p \psi^\beta\, d\eta, \end{gathered}$$

for any nonnegative test function $\psi^\beta \in \mathcal{C}_0^\infty(\mathbb{R}^N)$ with $\beta > \max(p', q')$.

Using the convexity inequality in Proposition 2.2 and the Hölder inequality, we estimate the first integral over K as follows,

$$\begin{aligned} &\int_{\mathbb{R}^{2N+1}} u(-\Delta_{\mathbb{H}})^{\mu/2}\psi^\beta\, d\eta \\ &\le \beta \int_K u\psi^{\beta/p}\psi^{-\beta/p}\psi^{\beta-1}(-\Delta_{\mathbb{H}})^{\mu/2}\psi\, d\eta \\ &\le \beta\Big(\int_K |u|^p\psi^\beta\, d\eta\Big)^{1/p}\Big(\int_K \psi^{(\beta-1)p'-\beta\frac{p'}{p}}|(-\Delta_{\mathbb{H}})^{\mu/2}\psi|^{p'}\, d\eta\Big)^{1/p'}, \end{aligned}$$

where $K := \mathrm{supp}(\psi)$ and $p + p' = pp'$.

Similarly, we obtain the estimate for the second integral

$$\begin{aligned} &\int_{\mathbb{R}^{2N+1}} v(-\Delta_{\mathbb{H}})^{\nu/2}\psi^\beta\, d\eta \\ &\le \beta \int_K v\psi^{\beta/q}\psi^{-\beta/q}\psi^{\beta-1}(-\Delta_{\mathbb{H}})^{\nu/2}\psi\, d\eta \\ &\le \beta\Big(\int_K |v|^q\psi^\beta\, d\eta\Big)^{1/q}\Big(\int_K \psi^{(\beta-1)q'-\beta\frac{q'}{q}}|(-\Delta_{\mathbb{H}})^{\nu/2}\psi|^{q'}\, d\eta\Big)^{1/q'}, \end{aligned}$$

where $q + q' = qq'$. If we set

$$\mathcal{A}(r,\delta) := \Big(\int_K \psi^{(\beta-1)r'-\beta\frac{r'}{r}}|(-\Delta_{\mathbb{H}})^{\delta/2}\psi|^{r'}\, d\eta\Big)^{1/r'},$$

then we can write

$$\int_{\mathbb{R}^{2N+1}} |u|^p\psi^\beta d\eta \le \beta\mathcal{A}(q,\nu)\Big(\int_{\mathbb{R}^{2N+1}} |v|^q\psi^\beta d\eta\Big)^{1/q}, \tag{4.1}$$

$$\int_{\mathbb{R}^{2N+1}} |v|^q\psi^\beta d\eta \le \beta\mathcal{A}(p,\mu)\Big(\int_{\mathbb{R}^{2N+1}} |u|^p\psi^\beta d\eta\Big)^{1/p}. \tag{4.2}$$

Therefore,

$$\Big(\int_{\mathbb{R}^{2N+1}}|v|^q\psi^\beta\,d\eta\Big)^{1/q}\le\beta^{1/q}\Big(\int_{\mathbb{R}^{2N+1}}|u|^p\psi^\beta\,d\eta\Big)^{1/pq}\Big(\mathcal{A}(p,\mu)\Big)^{1/q}. \tag{4.3}$$

Using (4.1) and (4.3), we obtain

$$\int_{\mathbb{R}^{2N+1}}|u|^p\psi^\beta\,d\eta\le\beta^{1+1/q}\Big(\int_{\mathbb{R}^{2N+1}}|u|^p\psi^\beta\,d\eta\Big)^{1/pq}\Big(\mathcal{A}(q,\nu)\Big)\Big(\mathcal{A}(p,\mu)\Big)^{1/q},$$

and consequently,

$$\Big(\int_{\mathbb{R}^{2N+1}}|u|^p\psi^\beta\,d\eta\Big)^{1-1/(pq)}\le\beta^{1+1/q}\Big(\mathcal{A}(q,\nu)\Big)\Big(\mathcal{A}(p,\mu)\Big)^{1/q}.$$

Similarly, we obtain

$$\Big(\int_{\mathbb{R}^{2N+1}}|v|^q\psi^\beta\,d\eta\Big)^{1-1/(pq)}\le\beta^{1+1/p}\Big(\mathcal{A}(p,\mu)\Big)\Big(\mathcal{A}(q,\nu)\Big)^{1/p}.$$

Now, we take

$$\psi(\eta)=\varphi\Big(\frac{\tau^2+|x|^4+|y|^4}{R^4}\Big),$$

and change variables from $\eta=(x,y,\tau)$ to $\tilde\eta=(\tilde x,\tilde y,\tilde\tau)$ as follows:

$$\tau=R^2\tilde\tau,\quad x=R\tilde x,\quad y=R\tilde y.$$

Using

$$|(-\Delta_{\mathbb{H}})^{\nu/2}\psi|^{p'}=R^{-p'\mu}|(-\Delta_{\mathbb{H}})^{\nu/2}\varphi(\tilde\eta)|^{p'}$$

and $d\eta=R^Q d\tilde\eta$, we obtain

$$\mathcal{A}(p,\mu)\le CR^{-\mu+\frac{Q}{p'}} \tag{4.4}$$

where

$$C=\beta^{1+1/p}\Big(\int_\Omega\varphi^{(\beta-1)p'-\beta\frac{p'}{p}}|(-\Delta_{\mathbb{H}})^{\mu/2}\varphi|^{p'}\,d\tilde\eta\Big)^{1/p'},$$
$$\Omega=\Big\{(\tilde x,\tilde y,\tilde\tau)\in\mathbb{R}^{2N+1}:\tilde\tau^2+|\tilde x|^4+|\tilde y|^4\le 2\Big\}.$$

So, we have

$$\Big(\int_{\mathbb{R}^{2N+1}}|u|^p\psi^\beta\,d\eta\Big)^{1-1/(pq)}\le\ CR^{\theta_1},$$
$$\Big(\int_{\mathbb{R}^{2N+1}}|v|^q\psi^\beta\,d\eta\Big)^{1-1/(pq)}\le\ CR^{\theta_2},$$

where

$$\theta_1=(-\mu p'+Q)\frac{1}{p'q}+(-\nu q'+Q)\frac{1}{q'},$$
$$\theta_2=(-\nu q'+Q)\frac{1}{pq'}+(-\mu p'+Q)\frac{1}{p'}.$$

Now, using (3.4), we can see that if

$$\theta_1<0\quad\Longleftrightarrow\quad Q<(\frac{pq}{pq-1})(\frac{\mu}{q}+\nu)$$

or

$$\theta_2<0\quad\Longleftrightarrow\quad Q<(\frac{pq}{pq-1})(\frac{\nu}{p}+\mu);$$

that is,

$$Q < (\frac{pq}{pq-1}) \max\Big\{\frac{\nu}{p}+\mu, \frac{\mu}{q}+\nu\Big\},$$

then, we have

$$\lim_{R\to\infty}\int_{\mathbb{R}^{2N+1}} |u|^p\psi^\beta\,d\eta = \int_{\mathbb{R}^{2N+1}} |u|^p\,d\eta = 0$$

or

$$\lim_{R\to\infty}\int_{\mathbb{R}^{2N+1}} |v|^q\psi^\beta\,d\eta = \int_{\mathbb{R}^{2N+1}} |v|^q\,d\eta = 0;$$

therefore $(u,v)\equiv(0,0)$. This completes the proof. □

In the case of a single equation

$$(-\Delta_{\mathbb{H}})^{\mu/2}u = |u|^p, \quad u\ge 0 \quad \text{in } \mathbb{R}^N$$

using the scaled variables as in the proof of Theorem 3.2, one can verify that if $1<p<\frac{Q}{Q-\mu}$ then the solution is trivial.

Proof of Theorem 3.3. Let (u,v) be a weak solution of system (1.2). Following the same method as in the proof of Theorem 3.2 for system (1.1), one obtains

$$\int_{\mathbb{R}^{2N+1}} |u|^p\psi^\beta\,d\eta \le \beta\mathcal{A}(q,\nu_1)\Big(\int_K |v|^q\psi^\beta\,d\eta\Big)^{1/q} + \beta\mathcal{A}(p,\nu_2)\Big(\int_K |u|^p\psi^\beta\,d\eta\Big)^{1/p},$$

and

$$\int_{\mathbb{R}^{2N+1}} |v|^q\psi^\beta\,d\eta \le \beta\mathcal{A}(p,\mu_1)\Big(\int_K |u|^p\psi^\beta\,d\eta\Big)^{1/p} + \beta\mathcal{A}(q,\mu_2)\Big(\int_K |v|^q\psi^\beta\,d\eta\Big)^{1/q}.$$

Similarly, we have

$$\begin{aligned}\Big(\int_{\mathbb{R}^{2N+1}} |u|^p\psi^\beta\,d\eta\Big)^{pq} &\le C\Big\{\Big(\mathcal{A}(p,\nu_2)\Big)^{\frac{pq}{p-1}} + \Big(\mathcal{A}(q,\nu_1)\Big)^{q}\Big(\mathcal{A}(q,\mu_2)\Big)^{\frac{q}{q-1}} \\ &\quad + \Big(\big(\mathcal{A}(q,\nu_1)\big)^q\mathcal{A}_\beta(p,\mu_1)\Big)^{\frac{pq}{pq-1}}\Big\},\end{aligned}$$

and

$$\begin{aligned}\Big(\int_{\mathbb{R}^{2N+1}} |v|^q\psi^\beta\,dx\Big)^{pq} &\le C\Big\{\Big(\mathcal{A}(q,\mu_2)\Big)^{\frac{pq}{q-1}} + \Big(\mathcal{A}(p,\mu_1)\Big)^{p}\Big(\mathcal{A}(p,\nu_2)\Big)^{\frac{p}{p-1}} \\ &\quad + \Big(\big(\mathcal{A}(p,\mu_1)\big)^p\mathcal{A}(q,\nu_1)\Big)^{\frac{pq}{pq-1}}\Big\}.\end{aligned}$$

Also, using the arguments of the previous theorem, we obtain

$$\Big(\int_{\mathbb{R}^{2N+1}} |u|^p\psi^\beta\,dx\Big)^{pq} \le C(R^{\gamma_1'}+R^{\gamma_2'}+R^{\gamma_3'}),$$

where

$$\begin{gathered}\gamma_1' = \big(-\nu_2+\frac{Q}{p'}\big)\frac{pq}{p-1},\\ \gamma_2' = \big(-\nu_1+\frac{Q}{q'}\big)q + \big(-\mu_2+\frac{Q}{q'}\big)\frac{q}{q-1},\\ \gamma_3' = \Big(\big(-\nu_1+\frac{Q}{q'}\big)q + \big(-\mu_1+\frac{Q}{p'}\big)\Big)\frac{pq}{pq-1},\\ \Big(\int_{\mathbb{R}^{2N+1}} |v|^q\psi^\beta\,d\eta\Big)^{pq} \le C(R^{\theta_1'}+R^{\theta_2'}+R^{\theta_3'}),\end{gathered}$$

where

$$\theta_1' = (-\mu_2 + \frac{Q}{q'})\frac{pq}{q-1},$$
$$\theta_2' = (-\mu_1 + \frac{Q}{p'})p + (-\nu_2 + \frac{Q}{p'})\frac{p}{p-1},$$
$$\theta_3' = \Big((-\mu_1 + \frac{Q}{p'})p + (-\nu_1 + \frac{Q}{q'})\Big)\frac{pq}{pq-1}.$$

Taking either $\max(\gamma_1', \gamma_2', \gamma_3') < 0$ or $\max(\theta_1', \theta_2', \theta_3') < 0$, and using the same arguments as in the previous proofs one can show that $u = v = 0$. $\square$

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