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EXISTENCE AND EXPONENTIAL STABILITY OF ANTI-PERIODIC SOLUTIONS IN CELLULAR NEURAL NETWORKS WITH TIME-VARYING DELAYS AND IMPULSIVE EFFECTS

CHANGJIN XU

Abstract. In this article we study a cellular neural network with impulsive effects. By using differential inequality techniques, we obtain verifiable criteria on the existence and exponential stability of anti-periodic solutions. An example is included to illustrate the feasibility and of our main results.

1. Introduction

Because of the wide range of applications in neurobiology, image processing, evolutionary theory, pattern recognition and optimization and so on, cellular neural networks have attracted much attention in recent years [9]. It is well known that impulsive differential equations are mathematical apparatus for simulation of process and phenomena observed in control theory, physics, chemistry, population dynamics, biotechnologies, industrial robotics, economics, etc. [3, 18, 35]. Therefore many results on the existence and stability of an equilibrium point of cellular neural networks with impulses have been reported (see [14, 16, 17, 29, 39, 41, 42, 44, 52]). In applied sciences, the existence of anti-periodic solutions plays a key role in characterizing the behavior of nonlinear differential equations [11, 21, 22, 36]. For example, high-order Hopfield neural networks can be analog voltage transmission, and voltage transmission process can be described as an anti-periodic process [30], anti-periodic trigonometric polynomials play an important role in interpolation problems [10], and anti-periodic wavelets were investigated in [7], in neural networks, the global stable anti-periodic solution can reveal the characteristic and stability of signal [37]. Recently, there are some papers that deal with the problem of existence and stability of anti-periodic solutions (see [1, 2, 8, 12, 13, 15, 19, 23, 24, 25, 26, 28, 30, 31, 32, 33, 34, 38, 40, 50, 51]). In addition, we know that many evolutionary processes exhibit impulsive effects which are usually subject to short time perturbations whose durations may be neglected in comparison with durations of the processes [38]. This motivates us to consider the existence and stability of anti-periodic solutions for cellular neural networks with impulses. To the best of our knowledge, very few

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authors have focused on the problems of anti-periodic solutions for such impulsive cellular neural networks. In this paper, we consider the anti-periodic solution of the following cellular neural network with delays and impulses

$$\begin{gathered}\dot{x}_i(t)=-c_i(t)x_i(t)+\sum_{j=1}^{n}a_{ij}(t)f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}(t)f_j(x_j(t-\tau_{ij}(t)))+u_i(t),\\ t\neq t_k,\\ x_i(t_k^+)=(1+\delta_{ik})x_i(t_k),\quad k=1,2,\dots,\end{gathered}\tag{1.1}$$

where $i=1,2,\dots,n$,$x_i(t)$ represent the state vector of the ith unit at time t, $c_i,a_{ij},b_{ij},f_jg_j,u_i,\tau_{ij}$ are continuous functions on R, $c_i>0$, a_{ij} are the connection weights between ith unit and jth unit at time t, b_{ij} is the connection weights between ith unit and jth unit at time $t-\tau_{ij}$, f_j and g_j are the activation function, u_i are external input to the ith unit, τ_{ij} are the time varying delay and satisfy $0\le\tau_{ij}\le\tau$, τ is a positive constant, t_k are the impulsive moments and satisfy $0<t_1<t_2<\dots<t_k<\dots,\lim_{k\to\infty}t_k=\infty$, δ_{ik} characterize the impulsive at jumps at time t_k for ith unit.

The main purpose of this article is to give the sufficient conditions of existence and exponential stability of anti-periodic solution of system (1.1). Some new sufficient conditions for the existence, unique and exponential stability of anti-periodic solutions of system (1.1) are established. Our results not only can be applied directly to many concrete examples of cellular neural networks, but also extend, to a certain extent, the results in some previously known ones. In addition, an example is presented to illustrate the effectiveness of our main results.

For convenience, we introduce the following notation

$$\begin{gathered}a_{ij}^+=\sup_{t\in R}|a_{ij}(t)|,\quad b_{ij}^+=\sup_{t\in R}|b_{ij}(t)|,\quad u_i^+=\sup_{t\in R}|u_i(t)|,\\ c_i^-=\min_{t\in R}|c_i(t)|,\quad \tau=\sup_{t\in R}\max_{1\le i,j\le n}\{\tau_{ij}(t)\}.\end{gathered}$$

We assume the following hypothesis:

(H1) For $i,j=1,2,\dots,n$, $a_{ij},b_{ij},u_i,f_j,g_j:R\to R$, $c_i,\tau_{ij}:R\to[0,+\infty)$ are continuous functions, and there exist a constant $T>0$ such that

$$\begin{gathered}c_i(t+T)=c_i(t),\quad \tau_{ij}(t+T)=\tau_{ij}(t),\quad u_i(t+T)=-u_i(t),\\ a_{ij}(t+T)f_j(u)=-a_{ij}(t)f_j(-u),\quad b_{ij}(t+T)g_j(u)=-b_{ij}(t)g_j(-u),\end{gathered}$$

for all $t,u\in R$.

(H2) For each $j\in\{1,2,\dots,n\}$, the activation function $f_j:R\to R$ is continuous and there exists an nonnegative constant L_j^f such that

$$f_j(0)=0,\quad |f_j(u)-f_j(v)|\le L_j^f|u-v|$$

for all $u,v\in R$.

(H3) $\prod_{0\le t_k<t}(1+\delta_{ik})$ $(i=1,2,\dots,n,\ k=1,2,\dots)$ are periodic functions of period T and $\delta_{ik}>-1$.

(H4) For $i=1,2,\dots,n$, $k=1,2,\dots,$ there exist positive constants m and M such that $m\le\prod_{0\le t_k<t}(1+\delta_{ik})\le M$.

(H5) There exist constants $\eta > 0$, $\lambda > 0$, $i = 1, 2, \dots, n$, such that for all $t > 0$,
$$\lambda - c_i^- + \frac{M}{m}\Big[\sum_{j=1}^{n}(a_{ij}^+ + b_{ij}^+)L_j^f\Big]e^{\lambda\tau} < -\eta < 0.$$
Let $x = (x_1, x_2, \dots, x_n)^T \in R^n$, in which "$T$" denotes the transposition. We define $|x| = (|x_1|, |x_2|, \dots, |x_n|)^T$ and $\|x\| = \max_{1\le i\le n}|x_i|$. Obviously, the solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ of (1.1) has components $x_i(t)$ piece-wise continuous on $(-\tau, +\infty)$, $x(t)$ is differentiable on the open intervals (t_{k-1}, t_k) and $x(t_k^+)$ exists.

Definition 1.1. Let $u(t) : R \to R$ be piece-wise continuous function having countable number of discontinuous $\{t_k\}|_{k=1}^{+\infty}$ of the first kind. It is said to be T-anti-periodic on R if
$$\begin{gathered} u(t+T) = -u(t), \quad t \ne t_k, \\ u((t_k+T)^+) = -u(t_k^+), \quad k = 1, 2, \dots . \end{gathered}$$

Definition 1.2. Let $x^*(t) = \big(x_1^*(t), x_2^*(t), \dots, x_n^*(t)\big)^T$ be an anti-periodic solution of (1.1) with initial value $\varphi^* = (\varphi_1^*(t), \varphi_2^*(t), \dots, \varphi_n^*(t))^T$. If there exist constants $\lambda > 0$ and $M > 1$ such that for every solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ of (1.1) with an initial value $\varphi = (\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t))^T$,
$$|x_i(t) - x_i^*(t)| \le M\|\varphi - \varphi^*\|e^{-\lambda t}, \quad \text{for all } t > 0,\ i = 1, 2, \dots, n,$$
where
$$\|\varphi - \varphi^*\| = \sup_{-\tau\le s\le 0}\max_{1\le i\le n}|\varphi_i(s) - \varphi_i^*(s)|.$$
Then $x^*(t)$ is said to be globally exponentially stable.

The rest of this article is organized as follows. In the next section, we give some preliminary results. In Section 3, we derive the existence of T-anti-periodic solution, which is globally exponential stable. In Section 4, we present an example to illustrate the effectiveness of our main results.

2. Preliminaries

In this section, we firstly establish a fundamental theorem that enable us to reduce the existence of solution of system (1.1) to the corresponding problem for a delayed differential equation without impulses. Consider the following non-impulsive delayed differential system
$$\begin{aligned} \dot{y}_i(t) &= -c_i(t)y_i(t) + \prod_{0\le t_k<t}(1+\delta_{ik})^{-1}\Big[\sum_{j=1}^{n}a_{ij}(t)f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j(t)\Big) \\ &\quad + \sum_{j=1}^{n}b_{ij}(t)f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})y_j(t-\tau_{ij}(t))\Big)\Big] \\ &\quad + \prod_{0\le t_k<t}(1+\delta_{ik})^{-1}u_i(t), \quad t > 0 \end{aligned} \tag{2.1}$$
with initial condition $y_i(s) = \varphi_i(s)$, $s \in [-\tau, 0]$, $i = 1, 2, \dots, n$.

In this section, we present three important lemmas which are used to prove our main results in Section 3.

Lemma 2.1. *Assume that* (H3) *holds. (i) If* $y = (y_1, y_2, \dots, y_n)$ *is a solution of* (2.1)*, then*

$$x = \Big(\prod_{0\le t_k<t} (1+\delta_{ik}) y_1, \prod_{0\le t_k<t} (1+\delta_{ik}) y_2, \dots, \prod_{0\le t_k<t} (1+\delta_{ik}) y_n \Big)$$

is a solution of (2.1).

(ii) If $x = (x_1, x_2, \dots, x_n)$ *is a solution of* (1.1)*, then*

$$y = \Big(\prod_{0\le t_k<t} (1+\delta_{ik})^{-1} x_1, \prod_{0\le t_k<t} (1+\delta_{ik})^{-1} x_2, \dots, \prod_{0\le t_k<t} (1+\delta_{ik})^{-1} x_n \Big)$$

is a solution of (2.1).

The proof of the above lemma is similar to that in Li et al [20]. We omit it here.

Lemma 2.2. *Let* (H1)–(H4) *hold. Suppose that* $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T$ *is a solution of* (2.1) *with initial conditions*

$$y_i(s) = \varphi_i(s), \quad |\varphi_i(s)| < \gamma, \quad s \in [-\tau, 0], \quad i = 1, 2, \dots, n. \tag{2.2}$$

Then

$$|y_i(t)| < \gamma, \quad \forall t \ge 0,\ i = 1, 2, \dots, n, \tag{2.3}$$

where

$$\gamma > \frac{u_i^+}{mc_i^- - M\big[\sum_{j=1}^n (a_{ij}^+ - b_{ij}^+) L_j^f\big]}, \tag{2.4}$$
$$c_i^- > \frac{M}{m}\Big[\sum_{j=1}^n (a_{ij}^+ - b_{ij}^+) L_j^f\Big].$$

Proof. For any given initial condition, hypotheses (H2) and (H4) guarantee the existence and unique of $y(t)$, the solution to (2.1) in $[-\tau, +\infty)$. By way of contradiction, we assume that (2.3) does not hold. Then there must exist $i \in \{1, 2, \dots, n\}$ and $\theta_0 > 0$ such that

$$|y_i(\theta_0)| = \gamma, \quad |y_j(\theta_0)| < \gamma \quad \text{for all } t \in (-\tau, \theta_0),\ j = 1, 2, \dots, n. \tag{2.5}$$

By computing the upper left derivative of $|y_i(t)|$, together with the assumptions (2.3), (2.4), (2.5), (H2) and (H4), we have

$$\begin{aligned}
0 &\le D^+(|y_i(\theta_0)|) \\
&\le -c_i(\theta_0)|y_i(\theta_0)| + \Big| \prod_{0\le t_k<\theta_0} (1+\delta_{ik})^{-1} \Big[\sum_{j=1}^n a_{ij}(\theta_0) f_j\Big(\prod_{0\le t_k<\theta_0} (1+\delta_{jk}) y_j(\theta_0) \Big) \\
&\quad + \sum_{j=1}^n b_{ij}(\theta_0) f_j\Big(\prod_{0\le t_k<\theta_0-\tau_{ij}(\theta_0)} (1+\delta_{jk}) y_j(\theta_0 - \tau_{ij}(\theta_0)) \Big) \Big] \\
&\quad + \prod_{0\le t_k<\theta_0} (1+\delta_{ik})^{-1} u_i(\theta_0) \Big| \\
&\le -c_i^-|y_i(\theta_0)| + \prod_{0\le t_k<\theta_0} (1+\delta_{ik})^{-1} \Big[\sum_{j=1}^n |a_{ij}(\theta_0)| \Big| f_j\Big(\prod_{0\le t_k<\theta_0} (1+\delta_{jk}) y_j(\theta_0) \Big) \Big| \\
&\quad + \sum_{j=1}^n |b_{ij}(\theta_0)| \Big| f_j\Big(\prod_{0\le t_k<\theta_0-\tau_{ij}(\theta_0)} (1+\delta_{jk}) y_j(\theta_0 - \tau_{ij}(\theta_0)) \Big) \Big| \Big]
\end{aligned}$$

$$
\begin{aligned}
&+\prod_{0\le t_k<\theta_0}(1+\delta_{ik})^{-1}|u_i(\theta_0)|\\
&\le -c_i^-|y_i(\theta_0)|+\prod_{0\le t_k<\theta_0}(1+\delta_{ik})^{-1}\Big[\sum_{j=1}^n a_{ij}^+L_j^f\prod_{0\le t_k<\theta_0}(1+\delta_{jk})|y_j(\theta_0)|\\
&\quad+\sum_{j=1}^n b_{ij}^+L_j^f\prod_{0\le t_k<\theta_0-\tau_{ij}(\theta_0)}(1+\delta_{jk})|y_j(\theta_0-\tau_{ij}(\theta_0))|\Big]+\prod_{0\le t_k<\theta_0}(1+\delta_{ik})^{-1}u_i^+\\
&\le -c_i^-\gamma+\prod_{0\le t_k<\theta_0}(1+\delta_{ik})^{-1}\Big[\sum_{j=1}^n a_{ij}^+L_j^f\prod_{0\le t_k<\theta_0}(1+\delta_{jk})\gamma\\
&\quad+\sum_{j=1}^n b_{ij}^+L_j^f\prod_{0\le t_k<\theta_0-\tau_{ij}(\theta_0)}(1+\delta_{jk})\gamma\Big]+\prod_{0\le t_k<\theta_0}(1+\delta_{ik})^{-1}u_i^+\\
&\le -\Big[c_i^- -\frac{M}{m}\Big(\sum_{j=1}^n(a_{ij}^+-b_{ij}^+)L_j^f\Big)\Big]\gamma+\frac{1}{m}u_i^+<0, \qquad (2.6)
\end{aligned}
$$

which is a contradiction and implies that (2.3) holds. This completes the proof. $\square$

Lemma 2.3. *Suppose that* (H1)–(H5) *hold. Let* $y^*(t)=(y_1^*(t),y_2^*(t),\dots,y_n^*(t))^T$ *be the solution of* (2.1) *with initial value* $\varphi^*=(\varphi_1^*(t),\varphi_2^*(t),\dots,\varphi_n^*(t))^T$*, and let* $y(t)=(y_1(t),y_2(t),\dots,y_n(t))^T$ *be the solution of* (2.1) *with initial value* $\varphi=(\varphi_1(t),\varphi_2(t),\dots,\varphi_n(t))^T$*. Then there exist constants* $\lambda>0$ *and* $M>1$ *such that*

$$
|y_i(t)-y_i^*(t)|\le M\|\varphi-\varphi^*\|e^{-\lambda t},\quad \textit{for all } t>0,\ i=1,2,\dots,n.
$$

Proof. Let $u(t)=\{u_i(t)\}=\{y_i(t)-y_i^*(t)\}=y(t)-y^*(t)$. Then

$$
\begin{aligned}
&u_i'(t)\\
&=-c_i(t)u_i(t)+\Big(\prod_{0\le t_k<t}(1+\delta_{ik})^{-1}\Big)\Big\{\sum_{j=1}^n a_{ij}(t)\Big[f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j(t)\Big)\\
&\quad-f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j^*(t)\Big)\Big]+\sum_{j=1}^n b_{ij}(t)\Big[f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})y_j(t-\tau_{ij}(t))\Big)\\
&\quad-f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})y_j^*(t-\tau_{ij}(t))\Big)\Big]\Big\}, \qquad (2.7)
\end{aligned}
$$

where $i=1,2,\dots,n$. Next, we define a Lyapunov functional

$$
V_i(t)=|u_i(t)|e^{\lambda t},\quad i=1,2,\dots,n. \qquad (2.8)
$$

It follows from (2.7) and (2.8) that

$$
\begin{aligned}
&D^+(V_i(t)) \qquad (2.9)\\
&\le D^+(|u_i(t)|)e^{\lambda t}+\lambda|u_i(t)|e^{\lambda t}\\
&\le(\lambda-c_i^-)|u_i(t)|e^{\lambda t}+\Big(\prod_{0\le t_k<t}(1+\delta_{ik})^{-1}\Big)\Big\{\sum_{j=1}^n|a_{ij}(t)|\Big|f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j(t)\Big)\\
&\quad-f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j^*(t)\Big)\Big|
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}|b_{ij}(t)|\Big|f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})y_j(t-\tau_{ij}(t))\Big)\\
&-f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})y_j^*(t-\tau_{ij}(t))\Big)\Big|\Big\}e^{\lambda t}\\
&\le(\lambda-c_i^-)|u_i(t)|e^{\lambda t}+\Big(\prod_{0\le t_k<t}(1+\delta_{ik})^{-1}\Big)\Big[\sum_{j=1}^{n}a_{ij}^+L_j^f\Big(\prod_{0\le t_k<t}(1+\delta_{jk})\Big)|u_j(t)|\\
&+\sum_{j=1}^{n}b_{ij}^+L_j^f\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})\Big)|u_j(t-\tau_{ij}(t))|\Big]e^{\lambda t},
\end{aligned}
\tag{2.10}
$$

where $i=1,2,\dots,n$. Let $M>1$ denote an arbitrary real number and set

$$
\|\varphi-\varphi^*\|=\sup_{-\tau\le s\le 0}\max_{1\le j\le n}|\varphi_j(s)-\varphi_j^*(s)|>0,\quad j=1,2,\dots,n.
$$

Then by (2.10), we have

$$
V_i(t)=|u_i(t)|e^{\lambda t}<M\|\varphi-\varphi^*\|\quad\text{for all } t\in[-\infty,0],\ i=1,2,\dots,n.
$$

Thus we can claim that

$$
V_i(t)=|u_i(t)|e^{\lambda t}<M\|\varphi-\varphi^*\|,\quad\text{for all } t>0, i=1,2,\dots,n. \tag{2.11}
$$

Otherwise, there must exist $i\in\{1,2,\dots,n\}$ and $t_i>0$ such that

$$
V_i(t_i)=M\|\varphi-\varphi^*\|,\quad V_j(t)<M\|\varphi-\varphi^*\|\quad\text{for all } t\in[-\tau,t_i),\ j=1,2,\dots,n. \tag{2.12}
$$

Combining (2.10) with (2.12), we obtain

$$
\begin{aligned}
&0\le D^+(V_i(t_i)-M\|\varphi-\varphi^*\|)=D^+(V_i(t_i))\\
&\le(\lambda-c_i^-)|u_i(t_i)|e^{\lambda t_i}\\
&\quad+\Big(\prod_{0\le t_k<t_i}(1+\delta_{ik})^{-1}\Big)\Big[\sum_{j=1}^{n}a_{ij}^+L_j^f\Big(\prod_{0\le t_k<t_i}(1+\delta_{jk})\Big)|u_j(t_i)|e^{\lambda\tau_0}\\
&\quad+\sum_{j=1}^{n}b_{ij}^+L_j^f\Big(\prod_{0\le t_k<t_i-\tau_{ij}(t_i)}(1+\delta_{jk})\Big)|u_j(t_i-\tau_{ij}(t_i))|e^{\lambda t_i}\Big]\\
&=(\lambda-c_i^-)|u_i(t_i)|e^{\lambda t_i}\\
&\quad+\Big(\prod_{0\le t_k<t_i}(1+\delta_{ik})^{-1}\Big)\Big[\sum_{j=1}^{n}a_{ij}^+L_j^f\Big(\prod_{0\le t_k<t_i}(1+\delta_{jk})\Big)|u_j(t_i)|e^{\lambda t_i}\\
&\quad+\sum_{j=1}^{n}b_{ij}^+L_j^f\Big(\prod_{0\le t_k<\tau_0-\tau_{ij}(t_i)}(1+\delta_{jk})\Big)|u_j(t_i-\tau_{ij}(t_i))|e^{\lambda(t_i-\tau_{ij}(t_i))}e^{\lambda\tau_{ij}(t_i)}\Big]\\
&\le(\lambda-c_i^-)M\|\varphi-\varphi^*\|\\
&\quad+\Big(\prod_{0\le t_k<t_i}(1+\delta_{ik})^{-1}\Big)\Big[\sum_{j=1}^{n}a_{ij}^+L_j^f\Big(\prod_{0\le t_k<t_i}(1+\delta_{jk})\Big)M\|\varphi-\varphi^*\|\\
&\quad+\sum_{j=1}^{n}b_{ij}^+L_j^f\Big(\prod_{0\le t_k<t_i-\tau_{ij}(t_i)}(1+\delta_{jk})\Big)e^{\lambda\tau}M\|\varphi-\varphi^*\|\Big]
\end{aligned}
$$

$$
\begin{aligned}
&= \Big\{(\lambda - c_i^-) + \Big(\prod_{0\le t_k<t_i}(1+\delta_{ik})^{-1}\Big)\Big[\sum_{j=1}^n a_{ij}^+ L_j^f\Big(\prod_{0\le t_k<t_i}(1+\delta_{jk})\Big) \\
&\quad + \sum_{j=1}^n b_{ij}^+ L_j^f\Big(\prod_{0\le t_k<t_i-\tau_{ij}(t_i)}(1+\delta_{jk})\Big)e^{\lambda\tau}\Big]\Big\} M\|\varphi-\varphi^*\| \\
&\le \Big\{(\lambda - c_i^-) + \frac{M}{m}\Big[\sum_{j=1}^n (a_{ij}^+ + b_{ij}^+)L_j^f\Big]e^{\lambda\tau}\Big\} M\|\varphi-\varphi^*\|.
\end{aligned}
$$

Then

$$
\lambda - c_i^- + \frac{M}{m}\Big[\sum_{j=1}^n (a_{ij}^+ + b_{ij}^+)L_j^f\Big]e^{\lambda\tau} > 0,
$$

which contradicts (H5), then (2.12) holds. In view of (2.11), we know that

$$
V_i(t) = |u_i(t)|e^{\lambda t} < M\|\varphi-\varphi^*\|, i = 1,2,\dots,n.
$$

Namely,

$$
|y_i(t) - y_i^*(t)| = |u_i(t)| < M\|\varphi-\varphi^*\| \quad \text{for all } t>0,\ i=1,2,\dots,n.
$$

This completes the proof. □

Remark 2.4. If $y^*(t) = (y_1^*(t), y_2^*(t), \dots, y_n^*(t))^T$ is a T-anti-periodic solution of (2.1), it follows from Lemma 2.2 and Definition 1.2 that $y^*(t)$ is globally exponentially stable.

3. Main results

In this section, we present our main result that there exists the exponentially stable anti-periodic solution of (1.1).

Theorem 3.1. *Assume that* (H1)–(H5) *are satisfied. Then* (1.1) *has exactly one T-anti-periodic solution $x^*(t)$. Moreover, this solution is globally exponentially stable.*

Proof. Let $v(t) = (v_1(t), v_2(t), \dots,\ v_n(t))^T$ be a solution of (2.1) with initial conditions

$$
v_i(s) = \varphi_i^v(s), |\varphi_i^v(s)| < \gamma, \quad s\in(-\tau,0],\ i=1,2,\dots,n. \tag{3.1}
$$

Thus according to Lemma 2.2, the solution $v(t)$ is bounded and

$$
|v_i(t)| < \gamma \quad \text{for all } t\in R,\ i=1,2,\dots,n. \tag{3.2}
$$

From (2.1), we obtain

$$
\begin{aligned}
&\big((-1)^{p+1}v_i(t+(p+1)T)\big)' \\
&= (-1)^{p+1}\Big\{-c_i(t+(p+1)T)v_i(t+(p+1)T) \\
&\quad + \prod_{0\le t_k<t+(p+1)T}(1+\delta_{ik})^{-1}\Big[\sum_{j=1}^n a_{ij}(t+(p+1)T) \\
&\quad \times f_j\Big(\prod_{0\le t_k<t+(p+1)T}(1+\delta_{jk})v_j(t+(p+1)T)\Big) \\
&\quad + \sum_{j=1}^n b_{ij}(t+(p+1)T)f_j\Big(\prod_{0\le t_k<t+(p+1)T-\tau_{ij}(t+(p+1)T)}(1+\delta_{jk})v_j(t+(p+1)T
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \tau_{ij}(t+(p+1)T))\Big)\Big]\\
&\quad + \prod_{0\le t_k<t+(p+1)T}(1+\delta_{ik})^{-1}u_i(t+(p+1)T)\Big\}\\
&= -c_i(t)(-1)^{p+1}v_i(t+(p+1)T)\\
&\quad + \prod_{0\le t_k<t}(1+\delta_{ik})^{-1}\Big[\sum_{j=1}^{n}a_{ij}(t)f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})(-1)^{p+1}v_j(t+(p+1)T)\Big)\\
&\quad + \sum_{j=1}^{n}b_{ij}(t)f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})(-1)^{p+1}v_j(t+(p+1)T-\tau_{ij}(t))\Big)\Big]\\
&\quad + \prod_{0\le t_k<t}(1+\delta_{ik})^{-1}u_i(t),
\end{aligned}
\tag{3.3}
$$

where $i=1,2,\dots,n$. Thus $(-1)^{p+1}v(t+(p+1)T)$ are the solutions of (2.1) on R for any natural number p. Then, from Lemma 2.3, there exists a constant $M>1$ such that

$$
\begin{aligned}
&|(-1)^{p+1}v_i(t+(p+1)T)-(-1)^k v_i(t+pT)|\\
&\le Me^{-\lambda(t+pT)}\sup_{-\tau\le s\le 0}\max_{1\le i\le n}|v_i(s+T)+v_i(s)|\\
&\le 2e^{-\lambda(t+pT)}M\gamma,
\end{aligned}
\tag{3.4}
$$

where $i=1,2,\dots,n$. Thus, for any natural number q, we have

$$
(-1)^{q+1}v_i(t+(q+1)T)=v_i(t)+\sum_{k=0}^{q}[(-1)^{k+1}v_i(t+(k+1)T)-(-1)^k v_i(t+kT)]. \tag{3.5}
$$

Hence

$$
\begin{aligned}
&|(-1)^{q+1}v_i(t+(q+1)T)|\\
&\le |v_i(t)|+\sum_{k=0}^{q}|(-1)^{k+1}v_i(t+(k+1)T)-(-1)^k v_i(t+kT)|,
\end{aligned}
\tag{3.6}
$$

where $i=1,2,\dots,n$. From (3.4), (3.6) it follows that $(-1)^{q+1}v_i(t+(q+1)T)$ is a fundamental sequence on any compact set of R. Obviously, $\{(-1)^q v(t+qT)\}$ converges uniformly to a piece-wise continuous function $y^*(t)=(y_1^*(t),y_2^*(t),\dots,y_n^*(t))^T$ on any compact set of R.

Now we show that $y^*(t)$ is T-anti-periodic solution of (2.1). Firstly, $y^*(t)$ is T-anti-periodic, since

$$
\begin{aligned}
y^*(t+T)&=\lim_{q\to\infty}(-1)^q v(t+T+qT)\\
&=-\lim_{(q+1)\to\infty}(-1)^{q+1}v(t+(q+1)T)=-y^*(t).
\end{aligned}
\tag{3.7}
$$

In the sequel, we prove that $y^*(t)$ is a solution of (2.1). Noting that the right-hand side of (2.1) is piece-wise continuous, (3.3) implies that $\{((-1)^{q+1}v(t+(q+1)T))'\}$ uniformly converges to a piece-wise continuous function on any compact subset of

R. Thus, letting $q \to \infty$, we can easily obtain

$$
\begin{aligned}
\dot{y}_i^*(t) = -c_i(t)y_i^*(t) + \prod_{0\le t_k<t}(1+\delta_{ik})^{-1}\Big[\sum_{j=1}^n a_{ij}(t)f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j^*(t)\Big) \\
+\sum_{j=1}^n b_{ij}(t)f_j\Big(\prod_{0\le t_k<t-\tau_{ij}(t)}(1+\delta_{jk})y_j^*(t-\tau_{ij}(t))\Big)\Big] \\
+\prod_{0\le t_k<t}(1+\delta_{ik})^{-1}u_i(t), \quad t>0,
\end{aligned} \tag{3.8}
$$

where $i = 1, 2, \dots, n$. Therefore, $y^*(t)$ is a solution of (2.1). Applying Lemma 2.1, Definition 1.2 and Lemma 2.3, we can easily check that $x^*(t)$ is globally exponentially stable. The proof is complete. □

Shi and Dong [38] investigated the following Hopfield neural networks with impulses:

$$
\begin{gathered}
\dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^n b_{ij}(t)f_j(x_j(t)) + I_i(t), \quad t\ne t_k, \\
x_i(t_k^+) = (1+d_{ik})x_i(t_k), \quad k=1,2,\dots,
\end{gathered} \tag{3.9}
$$

where $i = 1, 2, \dots, n$. About the manning of the parameters, one can see [38]. By some analytical technique and by upper left derivative of the Lyapunov functional with $t \ne t_k$ and $t = t_k$, Shi and Dong [38] obtained some sufficient conditions which ensure the existence and the global exponential stability of anti-periodic solution of system (3.9). In this paper, we consider a more general neural networks with delays and impulses. Moreover, the research technique is different from that of [38]. By transforming the neural networks with impulses into an equivalent form without impulses and constructing the Lyapunov functional, we obtain the sufficient conditions which ensure the existence and global exponential stability of anti-periodic solution of the model. From this viewpoint, we say that the results obtained in this paper complement the previous results in [38].

In [13, 22, 23, 30, 33, 34, 36, 47, 48, 51], authors considered the anti-periodic solution of neural networks without impulses. In [31, 37, 45, 46, 49], authors investigated the global exponential stability of anti-periodic solution of neural networks with impulses by upper left derivative of the Lyapunov functional with $t \ne t_k$ and $t = t_k$. In [21], author studied the existence and global exponential stability anti-periodic solution of neural networks with impulses by the method of coincidence degree theory and Lyapunov functions. In this paper, we firstly transform the neural networks with impulses into an equivalent neural networks without impulses, then consider the existence and global exponential stability of anti-periodic solution of the equivalent model by constructing a suitable Lyapunov functional. To the best of our knowledge, there are very few papers that deal with this aspect. Moreover, all the results in [13, 22, 23, 30, 31, 33, 34, 36, 37, 45, 46, 47, 48, 49, 51] and the references therein cannot applicable to system (1.1) to obtain the existence and global exponential stability of anti-periodic solutions. Therefore the results obtained in this paper are essentially new and complement the previous publications.

4. An example

In this section, we illustrate the results obtained in previous sections. Let $n = 2$, consider the cellular neural networks with time-varying delays and impulsive effects

$$
\begin{aligned}
\dot{x}_1(t) &= -c_1(t)x_1(t) + \sum_{j=1}^{2} a_{1j}(t)f_j(x_j(t)) \\
&\quad + \sum_{j=1}^{2} b_{1j}(t)f_j(x_j(t-\tau_{1j}(t))) + u_1(t), \quad t \neq t_k, \\
\dot{x}_2(t) &= -c_2(t)x_2(t) + \sum_{j=1}^{2} a_{2j}(t)f_j(x_j(t)) \\
&\quad + \sum_{j=1}^{2} b_{2j}(t)f_j(x_j(t-\tau_{2j}(t))) + u_2(t), \quad t \neq t_k, \\
&\quad x_1(t_k^+) = (1+\delta_{1k})x_i(t_k), \quad k = 1, 2, \dots, \\
&\quad x_2(t_k^+) = (1+\delta_{2k})x_i(t_k), \quad k = 1, 2, \dots,
\end{aligned}
\tag{4.1}
$$

which is equivalent to

$$
\begin{aligned}
\dot{y}_1(t) &= -c_1(t)y_1(t) + \prod_{0\le t_k<t}(1+\delta_{1k})^{-1}\Big[\sum_{j=1}^{2} a_{ij}(t)f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j(t)\Big) \\
&\quad + \sum_{j=1}^{2} b_{1j}(t)f_j\Big(\prod_{0\le t_k<t-\tau_{1j}(t)}(1+\delta_{jk})y_j(t-\tau_{1j}(t))\Big)\Big] \\
&\quad + \prod_{0\le t_k<t}(1+\delta_{1k})^{-1}u_1(t), \quad t>0 \\
\dot{y}_2(t) &= -c_2(t)y_2(t) + \prod_{0\le t_k<t}(1+\delta_{2k})^{-1}\Big[\sum_{j=1}^{2} a_{2j}(t)f_j\Big(\prod_{0\le t_k<t}(1+\delta_{jk})y_j(t)\Big) \\
&\quad + \sum_{j=1}^{2} b_{2j}(t)f_j\Big(\prod_{0\le t_k<t-\tau_{2j}(t)}(1+\delta_{jk})y_j(t-\tau_{2j}(t))\Big)\Big] \\
&\quad + \prod_{0\le t_k<t}(1+\delta_{2k})^{-1}u_2(t), \quad t>0,
\end{aligned}
\tag{4.2}
$$

where $f_j(u) = \frac{1}{2}(|u+1| - |u-1|)$ $(j = 1, 2)$, $u_1(t) = 0.1\sin t$, $u_2(t) = 0.2\cos t$ and

$$
\begin{gathered}
\begin{bmatrix} c_1(t) & c_2(t) \\ u_1(t) & u_2(t) \end{bmatrix} = \begin{bmatrix} 3+|\cos t| & 3+|\sin t| \\ 2\sin t & 3\sin t \end{bmatrix} \\
\begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{5}|\sin t| & \frac{1}{5}|\cos t| \\ \frac{1}{4}|\cos t| & \frac{1}{4}|\sin t| \end{bmatrix}, \\
\begin{bmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{4}|\sin t| & \frac{1}{4}|\cos t| \\ \frac{1}{5}|\cos t| & \frac{1}{5}|\sin t| \end{bmatrix}, \\
\begin{bmatrix} \tau_{11}(t) & \tau_{12}(t) \\ \tau_{21}(t) & \tau_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.05|\sin t| & 0.05|\sin t| \\ 0.04|\cos t| & 0.04|\cos t| \end{bmatrix}
\end{gathered}
$$

Then $L_j^f = 1, c_1^- = c_2^- = 2$, $a_{1j}^+ = 0.2$, $a_{2j}^+ = 0.25$, $b_{1j}^+ = 0.25$, $b_{2j}^+ = 0.2$, $\tau = 0.05$. Let $\eta = 0.6$, $\lambda = 0.5$, $m = 1$ and $M = 2$. Then

$$\lambda - c_i^- + \frac{M}{m}\Big[\sum_{j=1}^{2}(a_{ij}^+ + b_{ij}^+)L_j^f\Big]e^{\lambda\tau} < 0.5 - 3 + 0.9 \times 2 \times e^{0.05\times 0.5} = -0.6544 < -0.6 < 0,$$

which implies that system (4.2) satisfies all the conditions in Theorem 3.1. Thus we can conclude that (4.1) has exactly one π-anti-periodic solution. Moreover, this solution is globally exponentially stable. The results are verified by the numerical simulations in Figure 1.

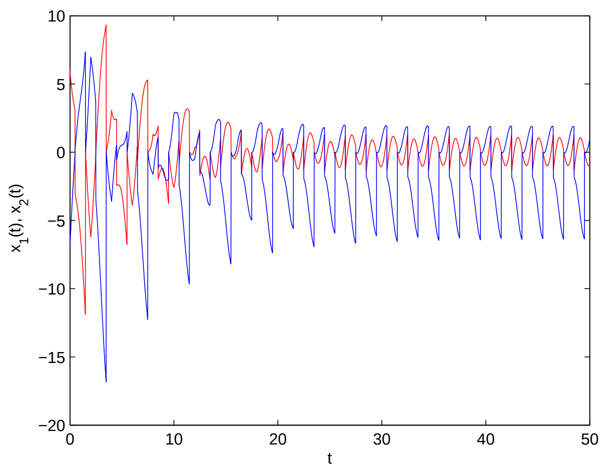

FIGURE 1. Time response of state variables $x_1(t)$ (red) and $x_2(t)$ (blue).

Conclusion. In this paper, we investigated the asymptotical behavior of a cellular neural networks with time-varying delays and impulsive effects. Applying the fundamental theorem, we reduce the existence of solution of system (1.1) to the corresponding problem for a delayed differential equation without impulses and derive a series of new sufficient conditions to guarantee the existence and global exponential stability of an anti-periodic solution for the cellular neural networks with time-varying delays and impulsive effects. The obtained conditions are easily to check in practice. Finally, an example is given to illustrative the feasibility and effectiveness. To the best of our knowledge, there are only few papers that focus on the anti-periodic solution problem of cellular neural networks with impulsive effects by reducing the impulsive cellular neural networks to the cellular neural networks without impulse. Thus our work is new and an excellent complement of previously known results.

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Changjin Xu

Guizhou Key Laboratory of Economics System Simulation, School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang 550004, China

E-mail address: xcj403@126.com