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COMPACTNESS OF THE DIFFERENCE BETWEEN THE POROUS THERMOELASTIC SEMIGROUP AND ITS DECOUPLED SEMIGROUP

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Abstract. Under suitable assumptions, we prove the compactness of the difference between the porous thermoelastic semigroup and its decoupled one. This will be achieved by proving the norm continuity of this difference and the compactness of the difference between the resolvents of their generators. Applications to porous thermoelastic systems are given.

1. Introduction

An increasing interest to determine the decay behavior of solutions of several porous elastic and thermoelastic problems has been discovered recently. The theory of porous elastic material was established first by Cowin and Nunziato [5, 6, 7]. In a recent paper the authors of [25] proved a slow decay of solution of porous elastic system with boundary Dirichlet conditions in one dimensional case. After, Casas and Quintanilla [8], proved the exponential decay of a porous thermoelastic system. This problem has recently been the focus of interest of Glowinsky and Lada [13, 14, 15]. In this work, we consider the abstract porous thermoelastic model

$$\ddot{w}_1(t) + A_1 w_1(t) + C_1 w_2(t) + C_2\theta(t) = 0, \quad t \geq 0, \tag{1.1}$$

$$\ddot{w}_2(t) + A_2 w_2(t) - C_1^* w_1(t) - C_3\theta(t) + DD^*\dot{w}_2(t) = 0, \quad t \geq 0, \tag{1.2}$$

$$\dot{\theta}(t) + A_3\theta(t) - C_2^*\dot{w}_1(t) + C_3^*\dot{w}_2(t) = 0, \quad t \geq 0, \tag{1.3}$$

$$w_1(0) = w_1^0, \quad \dot{w}_1(0) = w_1^1, \quad w_2(0) = w_2^0, \quad \dot{w}_2(0) = w_2^1, \quad \theta(0) = \theta^0, \tag{1.4}$$

with its decoupled system

$$\begin{aligned}&\ddot{w}_1(t) + A_1 w_1(t) + C_1 w_2(t) + C_2 A_3^{-1} C_2^*\dot{w}_1(t) - C_2 A_3^{-1} C_3^*\dot{w}_2(t)\\ &= 0, \quad t \geq 0,\end{aligned} \tag{1.5}$$

$$\begin{aligned}&\ddot{w}_2(t) + A_2 w_2(t) - C_1^* w_1(t) - C_3 A_3^{-1} C_2^*\dot{w}_1(t) + (C_3 A_3^{-1} C_3^* + DD^*)\dot{w}_2(t)\\ &= 0, \quad t \geq 0,\end{aligned} \tag{1.6}$$

$$\dot{\theta}(t) = -A_3\theta(t) + C_2^*\dot{w}_1(t) - C_3^*\dot{w}_2(t), \quad t \geq 0, \tag{1.7}$$

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$$w_1(0) = w_1^0, \quad \dot{w}_1(0) = w_1^1, \quad w_2(0) = w_2^0, \quad \dot{w}_2(0) = w_2^1, \quad \theta(0) = \theta^0. \tag{1.8}$$

The corresponding porous elastic system is given by the first and second equations in the decoupled system (1.5)-(1.8),

$$\begin{aligned}&\ddot{w}_1(t) + A_1 w_1(t) + C_1 w_2(t) + C_2 A_3^{-1} C_2^* \dot{w}_1(t) - C_2 A_3^{-1} C_3^* \dot{w}_2(t)\\ &= 0, \quad t \geq 0,\end{aligned} \tag{1.9}$$

$$\begin{aligned}&\ddot{w}_2(t) + A_2 w_2(t) - C_1^* w_1(t) - C_3 A_3^{-1} C_2^* \dot{w}_1(t) + (C_3 A_3^{-1} C_3^* + DD^*) \dot{w}_2(t)\\ &= 0, \quad t \geq 0,\end{aligned} \tag{1.10}$$

$$w_1(0) = w_1^0, \quad \dot{w}_1(0) = w_1^1, \quad w_2(0) = w_2^0, \quad \dot{w}_2(0) = w_2^1. \tag{1.11}$$

In this article, we first show the existence of solution of problems determined by systems (1.1)-(1.4), (1.5)-(1.8) and (1.9)-(1.11) using the Lumer-Phillips theorem from the theory of semigroups [9, Corollary 3.20]]. Second we address the problem of compactness of difference between the porous-thermoelasticity C_0-semigroup $(\mathcal{T}(t))_{t\geq 0}$ generated by the system (1.1)-(1.4) and the C_0-semigroup $(\mathcal{T}_d(t))_{t\geq 0}$ generated by its decoupled system (1.5)-(1.8). As in [1], we prove the norm continuity of $t \longmapsto \mathcal{T}(t) - \mathcal{T}_d(t)$ for $t > 0$, and we show the compactness of the difference $R(\lambda, \mathcal{A}) - R(\lambda, \mathcal{A}_d)$ for every λ in $\rho(\mathcal{A}) \cap \rho(\mathcal{A}_d)$, where $\mathcal{A}$ and $\mathcal{A}_d$ are the generators of $(\mathcal{T}(t))_{t\geq 0}$ and $(\mathcal{T}_d(t))_{t\geq 0}$, respectively. These two results together with [20, Theorem 2.3] lead to the compactness of the difference $\mathcal{T}(t) - \mathcal{T}_d(t)$. This yields that the essential spectrums $\sigma_e(\mathcal{T}(t))$, and $\sigma_e(\mathcal{T}_d(t))$ coincide. In the case where the operators A_3^{-1} and $A_1^{-1/2} C_1 A_2^{-1}$ are compact, following a similar argument as in [11], we prove that $\sigma_e(\mathcal{S}(t)) = \sigma_e(\mathcal{T}_d(t))$, where $(\mathcal{S}(t))_{t\geq 0}$ is the C_0-semigroup generated by the system (1.9)-(1.11).

Consequently one can derive stability results on the first semigroup from the ones of the third semigroup. Finally two applications to a porous thermoelastic system are given. In the first application where A_i^{-1}, $i = 1, 2$ are compact but A_3^{-1} is not compact, we show that only the two essential spectrums $\sigma_e(\mathcal{T}(t))$, and $\sigma_e(\mathcal{T}_d(t))$ coincide. The second application is similar to the one given by Glowinsky and Lada in [15], where the exponential stability of porous thermoelastic system is derived from the corresponding decoupled system. In this application, following a different approach and using the compactness of A_i^{-1}, $i = 1, 2, 3$, we obtain the same stability result first for the simpler porous elastic system, then the property is derived for the original porous thermoelastic system.

2. Main results

In what follows, $A_i : \mathcal{D}(A_i) \subset H_i \to H_i$, $i = 1, 2, 3$, be self-adjoint positive operators with bounded inverses, and H_i be Hilbert spaces equipped with the norm $\|\cdot\|_{H_i}$, $i = 1, 2, 3$. The operator A_i can be extended (or restricted) to each $H_{i,\alpha}$, such that it becomes a bounded operator

$$A_i : H_{i,\alpha} \to H_{i,\alpha-1}, \quad \forall \alpha \in \mathbb{R}, \tag{2.1}$$

where for $\alpha \geq 0$, $H_{i,\alpha} = \mathcal{D}(A_i^\alpha)$, with the norm $\|z\|_{i,\alpha} = \|A_i^\alpha z\|_{H_i}$ and for $\alpha \leq 0$, $H_{i,\alpha} = H_{i,-\alpha}^*$, the dual of $H_{i,-\alpha}$ with respect to the pivot space H_i. The operator $D \in \mathcal{L}(H_2)$ and D^* its adjoint. The coupled operators C_i, i=1,2,3, satisfy

(C1) $D(C_1) \subset H_2 \to H_1$, with adjoint C_1^* such that $D(A_2^{1/2}) \hookrightarrow D(C_1)$ and $D(A_1^{1/2}) \hookrightarrow D(C_1^*)$.

(C2) $D(C_2) \subset H_3 \to H_1$ with adjoint C_2^* such that $D(A_3^{1/2}) \hookrightarrow D(C_2)$ and $D(A_1^{1/2}) \hookrightarrow D(C_2^*)$.

(C3) $D(C_3) \subset H_3 \to H_2$ with adjoint C_3^* such that

$$D(A_3^{1/2}) \hookrightarrow D(C_3) \quad \text{and} \quad D(A_2^{1/2}) \hookrightarrow D(C_3^*). \tag{2.2}$$

Set

$$\mathcal{H} := H_{1,1/2} \times H_{2,1/2} \times H_1 \times H_2 \times H_3,$$

in this Hilbert space we introduce the new inner product

$$\Big\langle \begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ \theta \end{pmatrix}, \begin{pmatrix} \widetilde{w}_1 \\ \widetilde{w_2} \\ \widetilde{v}_1 \\ \widetilde{v}_2 \\ \widetilde{\theta} \end{pmatrix} \Big\rangle = \langle w_1, \widetilde{w}_1 \rangle_{H_{1,1/2}} + \langle w_2, \widetilde{w_2} \rangle_{H_{2,1/2}} + \langle v_1, \widetilde{v}_1 \rangle_{H_1} + \langle v_2, \widetilde{v}_2 \rangle_{H_2}$$
$$+ \langle \theta, \widetilde{\theta} \rangle_{H_3} + \Re(\langle C_1^* w_1, \widetilde{w}_2 \rangle_{H_2} - \langle w_2, C_1^* \widetilde{w}_1 \rangle_{H_2}).$$

The associated norm of this inner product coincides with the canonical norm of $\mathcal{H}$.

We can rewrite (1.1)-(1.4) and (1.5)-(1.8) as the first order evolution equations in $\mathcal{H}$,

$$\frac{d\eta}{dt} = \mathcal{A}\eta, \quad \eta \in \mathcal{H},$$
$$\eta(0) = (w_1^0, w_2^0, w_1^1, w_2^1, \theta^0),$$

and

$$\frac{d\overline{\eta}}{dt} = \mathcal{A}_d \overline{\eta}, \quad \overline{\eta} \in \mathcal{H},$$
$$\overline{\eta}(0) = (w_1^0, w_2^0, w_1^1, w_2^1, \theta^0),$$

respectively, where $\mathcal{A}$ is the unbounded linear operator defined by

$$\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}, \quad \mathcal{A} = \begin{pmatrix} 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ -A_1 & -C_1 & 0 & 0 & -C_2 \\ C_1^* & -A_2 & 0 & -DD^* & C_3 \\ 0 & 0 & C_2^* & -C_3^* & -A_3 \end{pmatrix}, \tag{2.3}$$

with

$$\mathcal{D}(\mathcal{A}) = \mathcal{D}(A_1) \times \mathcal{D}(A_2) \times \mathcal{D}(A_1^{1/2}) \times \mathcal{D}(A_2^{1/2}) \times \mathcal{D}(A_3), \tag{2.4}$$

and the operator $\mathcal{A}_d$ associated to the decoupled system

$$\begin{aligned} \mathcal{A}_d : \mathcal{D}(\mathcal{A}_d) = \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H},\ \mathcal{A}_d \\ = \begin{pmatrix} 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ -A_1 & -C_1 & -C_2A_3^{-1}C_2^* & C_2A_3^{-1}C_3^* & 0 \\ C_1^* & -A_2 & C_3A_3^{-1}C_2^* & -C_3A_3^{-1}C_3^* - DD^* & 0 \\ 0 & 0 & C_2^* & -C_3^* & -A_3 \end{pmatrix}. \end{aligned} \tag{2.5}$$

We rewrite the coupled second order system (1.9)-(1.11) on the Hilbert space

$$\mathcal{H}_c := H_{1,1/2} \times H_{2,1/2} \times H_1 \times H_2,$$

as the first order evolution equation

$$\frac{d\widetilde{\eta}}{dt} = \mathcal{M}\widetilde{\eta}, \quad \widetilde{\eta} \in \mathcal{H}_c,$$
$$\widetilde{\eta}^0 = (w_1^0, w_2^0, w_1^1, w_2^1),$$

and $\mathcal{M} : \mathcal{D}(\mathcal{M}) \subset \mathcal{H}_c \to \mathcal{H}_c$, is the unbounded linear operator defined by

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -A_1 & -C_1 & -C_2A_3^{-1}C_2^* & C_2A_3^{-1}C_3^* \\ C_1^* & -A_2 & C_3A_3^{-1}C_2^* & -C_3A_3^{-1}C_3^* - DD^* \end{pmatrix}, \tag{2.6}$$

with

$$\mathcal{D}(\mathcal{M}) = \mathcal{D}(A_1) \times \mathcal{D}(A_2) \times \mathcal{D}(A_1^{1/2}) \times \mathcal{D}(A_2^{1/2}). \tag{2.7}$$

Now we formulate the main results of this paper.

Theorem 2.1. *The operators $\mathcal{A}$, $\mathcal{A}_d$ and $\mathcal{M}$ generate strongly continuous contraction semigroups $(\mathcal{T}(t))_{t\ge 0}$, $(\mathcal{T}_d(t))_{t\ge 0}$ on $\mathcal{H}$ and $(\mathcal{S}(t))_{t\ge 0}$ on $\mathcal{H}_c$.*

Theorem 2.2. *Assume that*

$$A_1^{-1/2}C_2A_3^{-1}, \quad A_1^{-1/2}C_1A_2^{-1}, \quad A_2^{-1/2}C_3A_3^{-1}, \tag{2.8}$$

are compact operators from H_3 to H_1, from H_2 to H_1 and from H_3 to H_2 respectively. Then $\mathcal{T}(t) - \mathcal{T}_d(t)$ is compact for every $t \ge 0$.

As a consequence of Theorem 2.2, we have the following particular results.

Corollary 2.3. *Assume that the operators A_i^{-1}, $i = 1, 2$, are compact. Then $\mathcal{T}(t) - \mathcal{T}_d(t)$ is compact for every $t \ge 0$.*

Corollary 2.4. *Assume that the operators A_3^{-1} and $A_1^{-1/2}C_1A_2^{-1}$ are compact. Then $\sigma_e(\mathcal{T}(t)) = \sigma_e(\mathcal{S}(t))$ for $t \ge 0$.*

3. Well-posedness results

In this section we use Lumer-Phillips theorem (see [9, Corollary 3.20]) for the proof of Theorem 2.1.

3.1. **Porous thermoelastic system.** To show that the operator $(\mathcal{A}, \mathcal{D}(\mathcal{A}))$ defined by (2.3)-(2.4) generates a contraction semigroup on the Hilbert $\mathcal{H}$, we need the following technical lemma.

Lemma 3.1. *The operator $\mathcal{A}$ is invertible in $\mathcal{H}$ and $\mathcal{A}^{-1}$ is bounded on $\mathcal{H}$.*

Proof. Given a vector $\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} \in \mathcal{H}$, we need $\begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} \in \mathcal{D}(\mathcal{A})$, such that

$$\mathcal{A}\begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix}.$$

We have

$$\mathcal{A}\begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} \Leftrightarrow \begin{cases} v_1 = f_1, \\ v_2 = f_2, \\ A_1w_1 + C_1w_2 + C_2w_3 = -f_3, \\ -C_1^*w_1 + A_2w_2 + DD^*v_2 - C_3w_3 = -f_4, \\ -C_2^*v_1 + C_3^*v_2 + A_3w_3 = -f_5. \end{cases}$$

Hence

$$\mathcal{A}\begin{pmatrix} w_1\\ w_2\\ v_1\\ v_2\\ w_3\end{pmatrix}=\begin{pmatrix} f_1\\ f_2\\ f_3\\ f_4\\ f_5\end{pmatrix}\Leftrightarrow\begin{cases} v_1=f_1,\\ v_2=f_2,\\ A_1w_1+C_1w_2+C_2w_3=-f_3,\\ -C_1^*w_1+A_2w_2-C_3w_3=-f_4-DD^*f_2,\\ w_3=-A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2),\end{cases}$$

$$\Leftrightarrow\begin{cases} v_1=f_1,\\ v_2=f_2,\\ A_1w_1+C_1w_2=C_2A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2)-f_3=K_1,\\ -C_1^*w_1+A_2w_2=-C_3A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2)-f_4-DD^*f_2=K_2,\\ w_3=-A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2),\end{cases}$$

$$\Leftrightarrow\begin{cases} v_1=f_1,\\ v_2=f_2,\\ w_1=-A_1^{-1}C_1w_2+A_1^{-1}K_1,\\ (C_1^*A_1^{-1}C_1+A_2)w_2=K_2+C_1^*A_1^{-1}K_1,\\ w_3=-A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2).\end{cases}$$

We have

$$v_1=f_1\in H_{1,1/2},\quad v_2=f_2\in H_{2,1/2},\quad w_3=-A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2)\in\mathcal{D}(A_3).$$

Suppose that we have found w_2 with the appropriate regularity. Then,

$$w_1=-A_1^{-1}C_1w_2+A_1^{-1}K_1\in\mathcal{D}(A_1).$$

We now solve the equation

$$(C_1^*A_1^{-1}C_1+A_2)w_2=K_2+C_1^*A_1^{-1}K_1. \tag{3.1}$$

To find w_2 we introduce a bilinear form Λ on $\mathcal{D}(A_2^{1/2})$, defined by

$$\Lambda(\eta,\zeta)=\langle A_1^{-1/2}C_1\eta,A_1^{-1/2}C_1\zeta\rangle+\langle A_2^{\frac12}\eta,A_2^{\frac12}\zeta\rangle.$$

Since Λ is a bilinear continuous and coercive form on $\mathcal{D}(A_2^{1/2})$, the Lax-Milgram Lemma leads to the existence and uniqueness of $w_2\in\mathcal{D}(A_2^{1/2})$ solution to the equation (3.1).

Moreover $K_2+C_1^*A_1^{-1}K_1-C_1^*A_1^{-1}C_1w_2\in H_2$ and $[(A_2)_{-1}]^{-1}H_2=\mathcal{D}(A_2)$, (where $(A_2)_{-1}$ is an extension of A_2), then $w_2\in\mathcal{D}(A_2)$, (see [3, Proposision 5]). Set $B_1=(C_1^*A_1^{-1}C_1+A_2)^{-1}$, then we have

$$\mathcal{A}\begin{pmatrix} w_1\\ w_2\\ v_1\\ v_2\\ w_3\end{pmatrix}=\begin{pmatrix} f_1\\ f_2\\ f_3\\ f_4\\ f_5\end{pmatrix}\Leftrightarrow\begin{cases} v_1=f_1,\\ v_2=f_2,\\ w_1=-A_1^{-1}C_1w_2+A_1^{-1}K_1,\\ w_2=B_1K_2+B_1C_1^*A_1^{-1}K_1,\\ w_3=-A_3^{-1}(f_5-C_2^*f_1+C_3^*f_2),\end{cases}$$

$$\Leftrightarrow \begin{cases} v_1 = f_1, \\ v_2 = f_2, \\ w_1 = (-A_1^{-1}C_1B_1C_3A_3^{-1}C_2^* + A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^* \\ \quad -A_1^{-1}C_2A_3^{-1}C_2^*)f_1 + (A_1^{-1}C_1B_1C_3A_3^{-1}C_3^* + A_1^{-1}C_1B_1DD^* \\ \quad -A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^* + A_1^{-1}C_2A_3^{-1}C_3^*)f_2 \\ \quad +(A_1^{-1}C_1B_1C_1^*A_1^{-1} - A_1^{-1})f_3 + A_1^{-1}C_1B_1f_4 \\ \quad +(A_1^{-1}C_1B_1C_3A_3^{-1} - A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1} + A_1^{-1}C_2A_3^{-1})f_5, \\ w_2 = (B_1C_3A_3^{-1}C_2^* - B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^*)f_1 \\ \quad +(-B_1C_3A_3^{-1}C_3^* - B_1DD^* + B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^*)f_2 \\ \quad -B_1C_1^*A_1^{-1}f_3 - B_1f_4 + (-B_1C_3A_3^{-1} + B_1C_1^*A_1^{-1}C_2A_3^{-1})f_5, \\ w_3 = -A_3^{-1}(f_5 - C_2^*f_1 + C_3^*f_2). \end{cases}$$

Thus,

$$\mathcal{A}^{-1} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & A_1^{-1}C_1B_1 & a_{15} \\ a_{21} & a_{22} & -B_1C_1^*A_1^{-1} & -B_1 & a_{25} \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ A_3^{-1}C_2^* & -A_3^{-1}C_3^* & 0 & 0 & -A_3^{-1} \end{pmatrix}, \tag{3.2}$$

where

$$\begin{aligned} a_{11} &= -A_1^{-1}C_1B_1C_3A_3^{-1}C_2^* + A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^* - A_1^{-1}C_2A_3^{-1}C_2^*, \\ a_{12} &= A_1^{-1}C_1B_1C_3A_3^{-1}C_3^* + A_1^{-1}C_1B_1DD^* - A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^* \\ &\quad + A_1^{-1}C_2A_3^{-1}C_3^*, \\ a_{13} &= A_1^{-1}C_1B_1C_1^*A_1^{-1} - A_1^{-1}, \\ a_{15} &= A_1^{-1}C_1B_1C_3A_3^{-1} - A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1} + A_1^{-1}C_2A_3^{-1}, \\ a_{21} &= B_1C_3A_3^{-1}C_2^* - B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^*, \\ a_{22} &= -B_1C_3A_3^{-1}C_3^* - B_1DD^* + B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^*, \\ a_{25} &= -B_1C_3A_3^{-1} + B_1C_1^*A_1^{-1}C_2A_3^{-1}. \end{aligned}$$

The boundedness of the operator $\mathcal{A}^{-1}$ follows by the assumptions (2.2). □

Now, to prove that the operator $\mathcal{A}$ generates a strongly continuous contraction semigroup $(\mathcal{T}(t))_{t\geq 0}$ on $\mathcal{H}$, we have only to show that $(\mathcal{A}, \mathcal{D}(\mathcal{A}))$ is a dissipative operator on $\mathcal{H}$ and $\lambda I - \mathcal{A}$ is surjective for some $\lambda > 0$.

For every $\begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} \in \mathcal{D}(\mathcal{A})$, by the Cauchy-Schwartz inequality, we have

$$\begin{aligned} &\Re\Big(\Big\langle \mathcal{A}\begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} \Big\rangle\Big) = \Re\Big(\Big\langle \begin{pmatrix} v_1 \\ v_2 \\ -A_1w_1 - C_1w_2 - C_2w_3 \\ C_1^*w_1 - A_2w_2 - DD^*v_2 + C_3w_3 \\ C_2^*v_1 - C_3^*v_2 - A_3w_3 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} \Big\rangle\Big) \\ &= \Re\Big(\langle v_1, w_1\rangle_{H_{1,1/2}} + \langle v_2, w_2\rangle_{H_{2,1/2}} - \langle A_1w_1, v_1\rangle_{H_1} - \langle C_1w_2, v_1\rangle_{H_1} \\ &\quad - \langle C_2w_3, v_1\rangle_{H_1} + \langle C_1^*w_1, v_2\rangle_{H_2} - \langle A_2w_2, v_2\rangle_{H_2} - \langle DD^*v_2, v_2\rangle_{H_2} \\ &\quad + \langle C_3w_3, v_2\rangle_{H_2} + \langle C_2^*v_1, w_3\rangle_{H_3} - \langle C_3^*v_2, w_3\rangle_{H_3} - \langle A_3w_3, w_3\rangle_{H_3} \\ &\quad + \langle C_1^*v_1, w_2\rangle_{H_2} - < v_2, C_1^*w_1\rangle_{H_2}\Big) \\ &= -\|D^*v_2\|_{H_2}^2 - \|A_3^{1/2}w_3\|_{H_3}^2 \leq 0. \end{aligned}$$

Finally, $\mathcal{A}$ is dissipative. By a standard argument, one shows that $(\lambda I - \mathcal{A})$ is surjective for $\lambda \in (0, \frac{1}{\|\mathcal{A}^{-1}\|})$. Thus, [9, Corollary 3.20] leads to the claim.

3.2. **Decoupled system.** We show that the operator $(\mathcal{A}_d, \mathcal{D}(\mathcal{A}_d))$, associated with the decoupled system (1.5)-(1.8), generates a contraction semigroup on the Hilbert space $\mathcal{H}$. For this, we first show the following lemma.

Lemma 3.2. *The operator $\mathcal{A}_d$ is boundedly invertible in $\mathcal{H}$.*

Proof. Following the argument of the proof of Lemma 3.1, we show that the operator $\mathcal{A}_d$ is invertible and

$$\mathcal{A}_d^{-1} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & A_1^{-1}C_1B_1 & 0 \\ b_{21} & b_{22} & -B_1C_1^*A_1^{-1} & -B_1 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ A_3^{-1}C_2^* & -A_3^{-1}C_3^* & 0 & 0 & -A_3^{-1} \end{pmatrix}, \tag{3.3}$$

where

$$\begin{gathered} b_{11} = -A_1^{-1}C_1B_1C_3A_3^{-1}C_2^* + A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^* - A_1^{-1}C_2A_3^{-1}C_2^*, \\ b_{12} = A_1^{-1}C_1B_1C_3A_3^{-1}C_3^* + A_1^{-1}C_1B_1DD^* - A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^* \\ + A_1^{-1}C_2A_3^{-1}C_3^*, \\ b_{13} = A_1^{-1}C_1B_1C_1^*A_1^{-1} - A_1^{-1}, \\ b_{21} = B_1C_3A_3^{-1}C_2^* - B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^*, \\ b_{22} = -B_1C_3A_3^{-1}C_3^* - B_1DD^* + B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^*. \end{gathered}$$

□

Now we show the dissipativity of the operator $(\mathcal{A}_d, \mathcal{D}(\mathcal{A}_d))$ on $\mathcal{H}$. Take $\begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} \in \mathcal{D}(\mathcal{A}_d)$, by the Cauchy-Schwarz inequality, we have

$$\begin{aligned} &\Re\Big(\Big\langle \mathcal{A}_d \begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix}, \begin{pmatrix} w_1 \\ w_2 \\ v_1 \\ v_2 \\ w_3 \end{pmatrix} \Big\rangle\Big) \\ &= \Re\Big(\langle v_1, w_1\rangle_{H_{1,1/2}} + \langle v_2, w_2\rangle_{H_{2,1/2}} - \langle A_1w_1, v_1\rangle_{H_1} \\ &\quad - \langle C_1w_2, v_1\rangle_{H_1} - \langle C_2A_3^{-1}C_2^*v_1, v_1\rangle_{H_1} + \langle C_2A_3^{-1}C_3^*v_2, v_1\rangle_{H_1} \\ &\quad + \langle C_1^*w_1, v_2\rangle_{H_2} - \langle A_2w_2, v_2\rangle_{H_2} + \langle C_3A_3^{-1}C_2^*v_1, v_2\rangle_{H_2} \\ &\quad - \langle (C_3A_3^{-1}C_3^* + DD^*)v_2, v_2\rangle_{H_2} + \langle C_2^*v_1, w_3\rangle_{H_3} - < C_3^*v_2, w_3\rangle_{H_3} \\ &\quad - \langle A_3w_3, w_3\rangle_{H_3} + \langle C_1^*v_1, w_2\rangle_{H_2} - < C_1^*w_1, v_2\rangle_{H_2}\Big) \\ &= \Re\Big(- \|D^*v_2\|_{H_2}^2 - \|A_3^{1/2}C_2^*v_1\|_{H_3}^2 + 2\langle A_3^{-1/2}C_3^*v_2, A_3^{-1/2}C_2^*v_1\rangle \\ &\quad - \|A_3^{-1/2}C_3^*v_2\|^2\Big) \\ &\le -\|D^*v_2\|_{H_2}^2 \le 0. \end{aligned}$$

The proof of $\lambda I - \mathcal{A}$ is surjective for some $\lambda > 0$, follows as in Theorem 2.1.

3.3. **Porous elastic system.** As above, we can compute the operator

$$\mathcal{M}^{-1} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & A_1^{-1}C_1B_1 \\ b_{21} & b_{22} & -B_1C_1^*A_1^{-1} & -B_1 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{pmatrix}, \tag{3.4}$$

where $B_1 = (C_1^*A_1^{-1}C_1 + A_2)^{-1}$, and

$$\begin{aligned} b_{11} &= -A_1^{-1}C_1B_1C_3A_3^{-1}C_2^* + A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^* - A_1^{-1}C_2A_3^{-1}C_2^*, \\ b_{12} &= A_1^{-1}C_1B_1C_3A_3^{-1}C_3^* + A_1^{-1}C_1B_1DD^* - A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^* \\ &\quad + A_1^{-1}C_2A_3^{-1}C_3^*, \\ b_{13} &= A_1^{-1}C_1B_1C_1^*A_1^{-1} - A_1^{-1}, \\ b_{21} &= B_1C_3A_3^{-1}C_2^* - B_1C_1^*A_1^{-1}C_2A_3^{-1}C_2^*, \\ b_{22} &= -B_1C_3A_3^{-1}C_3^* - B_1DD^* + B_1C_1^*A_1^{-1}C_2A_3^{-1}C_3^*, \end{aligned}$$

and show that the operator $\mathcal{M}$ generates a strongly continuous contraction semigroup $(\mathcal{S}(t))_{t\geq 0}$ on $\mathcal{H}_c$.

4. Compactness result

In this section we prove the compactness of the difference $\mathcal{T}(t)-\mathcal{T}_d(t)$, we use [20, Theorem 2.3], where it is sufficient to prove the norm continuity of the difference between the two semigroups, and the compactness of the difference between the resolvents of their generators. To show the first assertion, we need the following technical lemma, see [21, Theorem 1.4.3] .

Lemma 4.1. *The map $t \mapsto A_3^{\alpha}e^{-A_3t}$ is norm continuous on $(0,\infty)$ for all $\alpha \geq 0$.*

Now we can show the following norm continuity result.

Theorem 4.2. *The map $t \mapsto \mathcal{T}(t) - \mathcal{T}_d(t)$ is norm continuous on $(0,\infty)$.*

Proof. Let $t > 0$ and $x_0 = \begin{pmatrix} w_1^0 \\ w_2^0 \\ w_1^1 \\ w_2^1 \\ w_3^0 \end{pmatrix} \in \mathcal{D}(\mathcal{A})$ such that $\|x_0\| \leq 1$. Let us write

$$\mathcal{T}(t)x_0 - \mathcal{T}_d(t)x_0 = \begin{pmatrix} w_1(t)-\overline{w}_1(t) \\ w_2(t)-\overline{w}_2(t) \\ v_1(t)-\overline{v}_1(t) \\ v_2(t)-\overline{v}_2(t) \\ w_3(t)-\overline{w}_3(t) \end{pmatrix} = \int_0^t \mathcal{T}(t-s)\begin{pmatrix} 0 \\ 0 \\ f(s) \\ g(s) \\ 0 \end{pmatrix} ds,$$

where

$$\begin{aligned} f(s) &= C_2A_3^{-1}C_2^*\overline{v}_1(s) - C_2A_3^{-1}C_3^*\overline{v}_2(s) - C_2\overline{w}_3(s), \\ g(s) &= -C_3A_3^{-1}C_2^*\overline{v}_1(s) + C_3A_3^{-1}C_3^*\overline{v}_2(s) + C_3\overline{w}_3(s). \end{aligned}$$

Let $0 < h < 1$, we begin by checking that $\|f(s+h) - f(s)\| \to 0$ as $h \to 0$.

We have $\overline{w}_3(t) = e^{-A_3t}w_3^0 + \int_0^t e^{-A_3(t-\sigma)}C_2^*\overline{v}_1(\sigma)d\sigma - \int_0^t e^{-A_3(s-\sigma)}C_3^*\overline{v}_2(\sigma)d\sigma$. Then

$$\begin{aligned} f(s) &= C_2A_3^{-1}C_2^*\overline{v}_1(s) - C_2A_3^{-1}C_3^*\overline{v}_2(s) - C_2e^{-A_3s}w_3^0 \\ &\quad - C_2A_3^{-1/2}\int_0^s A_3^{1/2}e^{-A_3(s-\sigma)}C_2^*\overline{v}_1(\sigma)d\sigma \end{aligned}$$

$$
\begin{aligned}
&+ C_2A_3^{-1/2}\int_0^s A_3^{1/2}e^{-A_3(s-\sigma)}C_3^*\overline{v}_2(\sigma)d\sigma \\
&= (C_2A_3^{-1/2})(C_2A_3^{-1/2})^*\overline{v}_1(s) - (C_2A_3^{-1/2})(C_3A_3^{-1/2})^*\overline{v}_2(s) \\
&\quad - (C_2A_3^{-1/2})A_3^{1/2}e^{-A_3s}w_3^0 - (C_2A_3^{-1/2})\int_0^s A_3e^{-A_3(s-\sigma)}(C_2A_3^{-1/2})^*\overline{v}_1(\sigma)d\sigma \\
&\quad + (C_2A_3^{-1/2})\int_0^s A_3e^{-A_3(s-\sigma)}(C_3A_3^{-1/2})^*\overline{v}_2(\sigma)d\sigma.
\end{aligned}
$$

Since $C_2A_3^{-1/2}$ and $C_3A_3^{-1/2}$ are bounded operators from H_3 to H_1 and from H_3 to H_2 respectively, and $s\mapsto e^{-A_3s}$, $s\mapsto A_3^{1/2}e^{-A_3s}$ are norm continuous on $(0,\infty)$, the map $s\mapsto (C_2A_3^{-1/2})A_3^{1/2}e^{-A_3(s)}$ is norm continuous on $(0,\infty)$, and there exists a positive constant $\alpha(s)$ and $\beta(s)$ such that $\|(C_2A_3^{-1/2})^*\overline{v}_1(\sigma)\|\le\alpha(s)\|\overline{v}_1(\sigma)\|$ and $\|(C_3A_3^{-1/2})^*\overline{v}_2(\sigma)\|\le\beta(s)\|\overline{v}_2(\sigma)\|$, for every $\sigma\in[0,s)$. By the inequality

$$
\|(\overline{w}_1,\overline{w}_2,\overline{v}_1,\overline{v}_2,\overline{w}_3)\|_{\mathcal{H}}\le\|x_0\|_{\mathcal{H}},\quad \text{for all } t\ge 0,
$$

we deduce

$$
\begin{aligned}
\|(C_2A_3^{-1/2})^*\overline{v}_1(\sigma)\|\le\alpha(s)\|x_0\|,\\
\|(C_3A_3^{-1/2})^*\overline{v}_2(\sigma)\|\le\beta(s)\|x_0\|,
\end{aligned}
$$

for every $\sigma\in[0,s)$. Thus

$$
\begin{aligned}
s\mapsto\int_0^s A_3e^{-A_3(s-\sigma)}(C_2A_3^{-1/2})^*\overline{v}_1(\sigma)d\sigma,\\
s\mapsto\int_0^s A_3e^{-A_3(s-\sigma)}(C_3A_3^{-1/2})^*\overline{v}_2(\sigma)d\sigma
\end{aligned}
$$

are continuous on $(0,\infty)$ uniformly with respect to $\|x_0\|\le 1$.

Finally $\|f(s+h)-f(s)\|\to 0$, as $h\to 0$, uniformly in x_0. Using the same argument, we have $\|g(s+h)-g(s)\|\to 0$, as $h\to 0$, uniformly in x_0.

Let us write

$$
\begin{aligned}
&\left\|\begin{pmatrix} w_1(t+h)-\overline{w}_1(t+h)\\ w_2(t+h)-\overline{w}_2(t+h)\\ v_1(t+h)-\overline{v}_1(t+h)\\ v_2(t+h)-\overline{v}_2(t+h)\\ w_3(t+h)-\overline{w}_3(t+h)\end{pmatrix} - \begin{pmatrix} w_1(t)-\overline{w}_1(t)\\ w_2(t)-\overline{w}_2(t)\\ v_1(t)-\overline{v}_1(t)\\ v_2(t)-\overline{v}_2(t)\\ w_3(t)-\overline{w}_3(t)\end{pmatrix}\right\| \\
&= \Big\|\int_0^{t+h}\mathcal{T}(t+h-s)\begin{pmatrix}0\\0\\f(s)\\g(s)\\0\end{pmatrix}ds - \int_0^t\mathcal{T}(t-s)\begin{pmatrix}0\\0\\f(s)\\g(s)\\0\end{pmatrix}ds\Big\| \\
&= \Big\|\int_0^{t+h}\mathcal{T}(s)\begin{pmatrix}0\\0\\f(t+h-s)\\g(t+h-s)\\0\end{pmatrix}ds - \int_0^t\mathcal{T}(s)\begin{pmatrix}0\\0\\f(t-s)\\g(t-s)\\0\end{pmatrix}ds\Big\| \\
&= \Big\|\int_0^{t}\mathcal{T}(s)\begin{pmatrix}0\\0\\f(t+h-s)-f(t-s)\\g(t+h-s)-g(t-s)\\0\end{pmatrix}ds + \int_0^h\mathcal{T}(t+s)\begin{pmatrix}0\\0\\f(h-s)\\g(h-s)\\0\end{pmatrix}ds\Big\| \\
&\le \Big\|\int_0^{t}\begin{pmatrix}0\\0\\f(t+h-s)-f(t-s)\\g(t+h-s)-g(t-s)\\0\end{pmatrix}ds\Big\| + \Big\|\int_0^h\begin{pmatrix}0\\0\\f(h-s)\\g(h-s)\\0\end{pmatrix}ds\Big\|.
\end{aligned}
$$

In addition, there exists constants N_1 and N_2 such that

$$\sup_{s\in[0,t+1]} \|f(h-s)\| \le N_1, \quad \sup_{s\in[0,t+1]} \|g(h-s)\| \le N_2$$

uniformly with respect to x_0, and $0 < h < 1$.

Since $\|f(s+h) - f(s)\| \to 0$ as $h \to 0$ and $\|g(s+h) - g(s)\| \to 0$ as $h \to 0$ uniformly with respect x_0, we deduce that $\int_0^t \|f(t+h-s) - f(t-s)\|ds \to 0$ and $\int_0^t \|g(t+h-s) - g(t-s)\|ds \to 0$, as $h \to 0$ uniformly for $x_0 \in \mathcal{D}(\mathcal{A})$ such that $\|x_0\| \le 1$. Finally, $t \mapsto \mathcal{T}(t) - \mathcal{T}_d(t)$ is norm continuous on $(0,\infty)$. □

Proof of Theorem 2.2. Since the map $t \mapsto \mathcal{T}(t) - \mathcal{T}_d(t)$ is norm continuous on $(0,\infty)$, we need only to show the compactness of $R(\lambda, \mathcal{A}) - R(\lambda, \mathcal{A}_d)$, $\lambda \in \rho(\mathcal{A}) \cap \rho(\mathcal{A}_d)$. From the following result

$$\mathcal{R}(\lambda, \mathcal{A}_d) - \mathcal{R}(\lambda, \mathcal{A}) = \mathcal{A}\,\mathcal{R}(\lambda, \mathcal{A})[\mathcal{A}^{-1} - \mathcal{A}_d^{-1}]\mathcal{A}_d\mathcal{R}(\lambda, \mathcal{A}_d),$$

it is sufficient to prove that $\mathcal{A}^{-1} - \mathcal{A}_d^{-1}$ is compact. We have

$$\mathcal{A}^{-1} - \mathcal{A}_d^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & c_{15} \\ 0 & 0 & 0 & 0 & c_{25} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4.1}$$

where

$$c_{15} = A_1^{-1}C_1B_1C_3A_3^{-1} - A_1^{-1}C_1B_1C_1^*A_1^{-1}C_2A_3^{-1} + A_1^{-1}C_2A_3^{-1},$$
$$c_{25} = -B_1C_3A_3^{-1} + B_1C_1^*A_1^{-1}C_2A_3^{-1}.$$

From the assumption (2.8), it is clear that the operators c_{15} and c_{25} are compact, and this achieves the proof. □

Proof of Corollary 2.4. Since the operators A_3^{-1} and $A_1^{-1/2}C_1A_2^{-1}$ are compact, assumption (2.8) is satisfied. In view of Theorem 2.2, it is enough to show that for each $t > 0$,

$$\big\{\mathcal{T}_d(t)(w_1^0, w_2^0, w_1^1, w_2^1, w_3^0) - (\mathcal{S}(t)(w_1^0, w_2^0, w_1^1, w_2^1); 0) : \|(w_1^0, w_2^0, w_1^1, w_2^1, w_3^0)\| \le 1\big\}$$

is a compact set in $\mathcal{H}$, i.e. that

$$\Big\{e^{-A_3t}w_3^0 + \int_0^t e^{-A_3(t-\sigma)}C_2^*\overline{v}_1(\sigma)d\sigma - \int_0^t e^{-A_3(s-\sigma)}C_3^*\overline{v}_2(\sigma)d\sigma : \\ \|(w_1^0, w_2^0, w_1^1, w_2^1, w_3^0)\| \le 1\Big\}$$

is a compact set in H_3, where $(\overline{w}_1(\sigma), \overline{w}_2(\sigma), \overline{v}_1(\sigma), \overline{v}_2(\sigma)) = \mathcal{S}(\sigma)(w_1^0, w_2^0, w_1^1, w_2^1)$. Since

$$\begin{aligned} &(w_1^0, w_2^0, w_1^1, w_2^1, w_3^0) \\ &\to A_3^{1/2}e^{-A_3t}w_3^0 + \int_0^t A_3e^{-A_3(t-\sigma)}(C_2A_3^{-1/2})^*\overline{v}_1(\sigma)d\sigma \\ &\quad - \int_0^t A_3e^{-A_3(s-\sigma)}(C_3A_3^{-1/2})^*\overline{v}_2(\sigma)d\sigma \end{aligned}$$

is bounded with values in H_3 (we have used the Lemma 4.1 and Lebesgue's theorem) and $A_3^{-1/2}$ is compact, the result follows. □

Remark 4.3. (1) If we have the conditions $C_3A_3^{-\gamma}$, $C_2A_3^{-\gamma}$ and $C_1A_2^{-\gamma}$ are compact for some $\gamma < 1$ then the assumptions (2.8) are satisfied and we have the compactness of the difference between $\mathcal{T}(t) - \mathcal{T}_d(t)$ for every $t \geq 0$, which is similar to Henry's condition in [11].

(2) If we suppose that A_1^{-1} and A_2^{-1} are compacts we have $\sigma_e(\mathcal{T}(t)) = \sigma_e(\mathcal{T}_d(t))$ for $t \geq 0$ but to have $\sigma_e(\mathcal{T}_d(t)) = \sigma_e(\mathcal{S}(t))$ for $t \geq 0$ we need the condition A_3^{-1} is compact.

5. Applications

We give two illustrating examples of Theorem 2.2 and Corollary 2.4.

Application 1. We give one application of Theorem 2.2. Let Ω the bounded open Jelly Roll set proposed in [26],

$$\Omega = \{(x,y) \in \mathbb{R}^2 : \frac{1}{2} < r < 1\} \setminus \Gamma,$$

where Γ is the curve in $\mathbb{R}^2$ given in polar coordinates by

$$r(\phi) = \frac{\frac{3\pi}{2} + \arctan(\phi)}{2\pi}, \quad -\infty < \phi < \infty.$$

We consider the initial and boundary problem

$$\begin{gathered}
\ddot{u}(t,x) - \Delta_e u(t,x) - b\nabla\phi(t,x) + c\nabla\theta(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\
\ddot{\phi}(t,x) - (a\Delta - \alpha I)\phi(t,x) + b \operatorname{div}\, u(t,x) - d\theta(t,x) + r\dot{\phi}(t,x) = 0 \\
\text{in } (0,+\infty)\times\Omega, \\
\dot{\theta}(t,x) - (\Delta - kI)\theta(t,x) + c \operatorname{div} \dot{u}(t,x) + d\dot{\phi}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\
u = 0, \quad \phi = 0, \quad \frac{\partial\theta}{\partial n} = 0 \quad \text{on } (0,+\infty)\times\partial\Omega, \\
u(0) = u^0, \quad \dot{u}(0) = u^1, \quad \phi(0) = \phi^0, \quad \dot{\phi}(0) = \phi^1, \quad \theta(0) = \theta^0, \quad \text{in } \Omega,
\end{gathered} \tag{5.1}$$

where n denotes the outer uniter normal vector to $\partial\Omega$, $\Delta_e := \mu\Delta + (\mu+\lambda)\nabla \operatorname{div}$, and μ, λ, a, b, c, d, r, α, k are positive constants.

To fit this system into the abstract setting of (1.1)-(1.4), we take

$$\begin{gathered}
H_1 = L^2(\Omega)^2, \quad H_2 = H_3 = L^2(\Omega), \quad H_{1,\frac{1}{2}} = (H_0^1(\Omega))^2, \quad H_{2,\frac{1}{2}} = H_0^1(\Omega), \\
\mathcal{H} = \mathcal{H}_c \times L^2(\Omega), \quad \mathcal{H}_c = (H_0^1(\Omega))^2 \times H_0^1(\Omega) \times L^2(\Omega)^2 \times L^2(\Omega), \\
A_1 = -\Delta_e, \quad \mathcal{D}(A_1) = \mathcal{D}(-\Delta_D) = (H^2(\Omega) \cap H_0^1(\Omega))^2, \\
A_2 = -(a\Delta - \alpha I), \quad \mathcal{D}(A_2) = \mathcal{D}(-\Delta_D) = H^2(\Omega) \cap H_0^1(\Omega), \\
A_3 = -(\Delta - kI), \quad \mathcal{D}(A_3) = \mathcal{D}(-\Delta_N).
\end{gathered}$$

We recall that u, ϕ, θ are the displacement vector, the volume fraction and the temperature. The Dirichlet and Neumann Laplacian operators Δ_D and Δ_N are the unique positive self adjoint operators associated to the closed quadratic form on $H_0^1(\Omega)$ and $H^1(\Omega)$ respectively

$$\langle \Delta f, g \rangle = \int_\Omega \nabla f \nabla g dx.$$

The operator $DD^* = rI_{H_2}$, and the coupled operators

$$C_1 = -b\nabla, \quad C_2 = c\nabla, \quad C_1^* = b \operatorname{div}, \quad C_2^* = -c \operatorname{div}, \quad C_3 = d\, I_{H_3},$$

$$\mathcal{D}(C_1) = \mathcal{D}(C_2) = H^1(\Omega), \quad \mathcal{D}(C_2^*) = \mathcal{D}(C_1^*) = \{u \in H^1(\Omega)^2 : u \cdot \overrightarrow{n} = 0 \text{ in } \partial\Omega\}.$$

Note that the conditions (2.2) are verified and we have A_1^{-1} and A_2^{-1} are compact from H_1 and H_2 respectively, then the assumptions (2.8) are satisfied, consequently the Theorem 2.2 is satisfied. To show $\sigma_e(\mathcal{T}_d(t)) = \sigma_e(\mathcal{S}(t))$ for $t \geq 0$, we need the compactness of A_3^{-1}, but from [26], A_3^{-1} is not compact.

Application 2. We give an application of Corollary 2.4. Let $\Omega \subset \mathbb{R}^2$ a bounded open domain with boundary $\partial\Omega$ having regularity of class C^2, and satisfies the following condition:

(A1) If $\varphi \in (H_0^1(\Omega))^2$ such that

$$\begin{gathered} -\Delta\varphi = \gamma^2\varphi \quad \text{in } \Omega, \\ \operatorname{div}\varphi = 0 \quad \text{in } \Omega, \\ \varphi = 0 \quad \text{in } \partial\Omega. \end{gathered} \tag{5.2}$$

for some $\gamma \in \mathbb{R}$, then $\varphi = 0$.

We consider the initial and boundary problem

$$\begin{gathered} \ddot{u}(t,x) - \Delta_e\, u(t,x) - b\nabla\phi(t,x) + c\nabla\theta(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ \ddot{\phi}(t,x) - (a\,\Delta - \alpha I)\phi(t,x) + b\operatorname{div} u(t,x) + r\dot{\phi}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ \dot{\theta}(t,x) - \Delta\theta(t,x) + c\operatorname{div}\dot{u}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ u = 0, \quad \phi = 0, \quad \theta = 0 \quad \text{on } (0,+\infty)\times\partial\Omega, \\ u(0) = u^0, \quad \dot{u}(0) = u^1, \quad \phi(0) = \phi^0, \quad \dot{\phi}(0) = \phi^1, \theta(0) = \theta^0 \quad \text{in } \Omega, \end{gathered} \tag{5.3}$$

where $\Delta_e := \mu\Delta + (\mu+\lambda)\nabla\operatorname{div}$ is Lamé operator, μ, λ, a, b, c, r, α are positive constants, and the condition $(\lambda+\mu)\alpha > b^2$ is satisfied.

To fit this system into the abstract setting of (1.1)-(1.4), we take

$$\begin{gathered} H_1 = L^2(\Omega)^2, \quad H_2 = H_3 = L^2(\Omega), \quad H_{1,\frac{1}{2}} = (H_0^1(\Omega))^2, \quad H_{2,\frac{1}{2}} = H_0^1(\Omega), \\ \mathcal{H} = \mathcal{H}_c \times L^2(\Omega), \quad \text{where } \mathcal{H}_c = (H_0^1(\Omega))^2 \times H_0^1(\Omega) \times L^2(\Omega)^2 \times L^2(\Omega), \\ A_1 = -\Delta_e, \qquad \mathcal{D}(A_1) = \mathcal{D}(-\Delta_D) = (H^2(\Omega)\cap H_0^1(\Omega))^2, \\ A_2 = -(a\Delta - \alpha I), \quad \mathcal{D}(A_2) = H^2(\Omega)\cap H_0^1(\Omega), \\ A_3 = -\Delta, \quad \mathcal{D}(A_3) = \mathcal{D}(A_2). \end{gathered}$$

The operator $DD^* = rI_{H_2}$, and the coupled operators

$$\begin{gathered} C_1 = -b\nabla, \quad C_1^* = b\operatorname{div}, \quad C_2 = c\nabla, \quad C_2^* = -c\operatorname{div}, \quad C_3 = 0, \\ \mathcal{D}(C_1) = \mathcal{D}(C_2) = H^1(\Omega), \quad \mathcal{D}(C_2^*) = \mathcal{D}(C_1^*) = \{u \in H^1(\Omega)^2 : u \cdot \overrightarrow{n} = 0 \quad \text{on } \partial\Omega\}. \end{gathered}$$

The decoupled system corresponding to system (5.3) is given by

$$\begin{gathered} \ddot{u}(t,x) - \Delta_e\, u(t,x) - b\nabla\phi(t,x) + c^2P\dot{u}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ \ddot{\phi}(t,x) - (a\Delta - \alpha I)\phi(t,x) + b\operatorname{div} u(t,x) + r\dot{\phi}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ \dot{\theta}(t,x) - \Delta\theta(t,x) + c\operatorname{div}\dot{u}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ u = 0, \quad \phi = 0, \quad \theta = 0 \quad \text{on } (0,+\infty)\times\partial\Omega, \\ u(0) = u^0, \quad \dot{u}(0) = u^1, \quad \phi(0) = \phi^0, \quad \dot{\phi}(0) = \phi^1, \theta(0) = \theta^0 \quad \text{in } \Omega, \end{gathered} \tag{5.4}$$

where $P := \nabla(\Delta)^{-1}div$ the orthogonal projection operator from $L^2(\Omega)^2$ into the subspace $\{\nabla\varphi; \varphi \in H_0^1(\Omega)\}$. Now we write the porous elastic system given by the first and second equation in decoupled system (5.4)

$$\begin{gathered}
\ddot{u}(t,x) - \Delta_e\, u(t,x) - b\nabla\phi(t,x) + c^2 P\dot{u}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\
\ddot{\phi}(t,x) - (a\Delta - \alpha I)\phi(t,x) + b\operatorname{div} u(t,x) + r\dot{\phi}(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\
u = 0, \quad \phi = 0 \quad \text{on } (0,+\infty)\times\partial\Omega, \\
u(0) = u^0, \quad \dot{u}(0) = u^1, \quad \phi(0) = \phi^0, \quad \dot{\phi}(0) = \phi^1 \quad \text{in } \Omega.
\end{gathered} \tag{5.5}$$

Let $(\mathcal{T}(t))_{t\geq 0}$ the porous-thermoelastic C_0-semigroup generated by the system (5.3) and $(\mathcal{S}(t))_{t\geq 0}$ the porous elastic C_0-semigroup generated by the system (5.5). Note that the operators A_1^{-1}, A_2^{-1} and A_3^{-1} are compact, consequently the assumptions of Corollary 2.4 are satisfied, then

$$\sigma_e(\mathcal{T}(t)) = \sigma_e(\mathcal{S}(t)) \quad \text{for } t \geq 0.$$

The second aim of this application is to characterize the exponential energy decay of solution of system (5.5), and then deduce the one of the coupled systems (5.3).

Now we show that $(\mathcal{S}(t))_{t\geq 0}$ is exponentially stable in $\mathcal{H}_c$, by using a similar argument as in the proof of [15, theorem 4.4]. Let $\varepsilon(t) := \mathcal{S}(t)\varepsilon^0$, $t \geq 0$, be the solution of

$$\frac{d\varepsilon}{dt} = \mathcal{M}\varepsilon, \quad \varepsilon(0) = \varepsilon^0, \tag{5.6}$$

where $\varepsilon^0 := (u^0, \phi^0, u^1, \phi^1)$. We look for $\varepsilon(t)$ having the form $\overline{\varepsilon}(t) = \Sigma_{l=0}^{\infty} b^l \varepsilon_l(t)$, where $\varepsilon_l(t) \equiv \big(u_l(t), \phi_l(t), \dot{u}_l(t), \dot{\phi}_l(t)\big)$, $\overline{\varepsilon}(0) = \varepsilon^0$ and $l \in \{0\}\cup\mathbb{N}$. After the formal substitution into the equation (5.6) we derive equations for (u_l, ϕ_l), where $l \in \{0\}\cup\mathbb{N}$. For (u_0, ϕ_0) we obtain

$$\ddot{u}_0(t,x) - \Delta_e u_0(t,x) + c^2 P\dot{u}_0(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \tag{5.7}$$

$$\ddot{\phi}_0(t,x) - (a\Delta - \alpha I)\phi_0(t,x) + r\dot{\phi}_0(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \tag{5.8}$$

$$u_0 = 0, \quad \phi_0 = 0 \quad \text{on } (0,+\infty)\times\partial\Omega,$$

$$u_0(0) = u^0, \quad \dot{u}_0(0) = u^1, \quad \phi_0(0) = \phi^0, \quad \dot{\phi}_0(0) = \phi^1 \quad \text{in } \Omega.$$

For $k \in \{0\}\cup\mathbb{N}$, (u_{k+1}, ϕ_{k+1}) will be the solution of problem

$$\ddot{u}_{k+1}(t,x) - \Delta_e\, u_{k+1}(t,x) - b\nabla\phi_k(t,x) + c^2 P\dot{u}_{k+1} = 0 \quad \text{in } (0,+\infty)\times\Omega, \tag{5.9}$$

$$\begin{aligned}
&\ddot{\phi}_{k+1}(t,x) - (a\Delta - \alpha I)\phi_{k+1}(t,x) + b\operatorname{div} u_k(t,x) + r\dot{\phi}_{k+1}(t,x) \\
&= 0 \quad \text{in } (0,+\infty)\times\Omega,
\end{aligned} \tag{5.10}$$

$$u_{k+1} = 0, \quad \phi_{k+1} = 0 \quad \text{on } (0,+\infty)\times\partial\Omega,$$

$$u_{k+1}(0) = 0, \quad \dot{u}_{k+1}(0) = 0, \quad \phi_{k+1}(0) = 0, \quad \dot{\phi}_{k+1}(0) = 0 \quad \text{in } \Omega.$$

Let $\varepsilon = (u, \phi, v, \psi) \in \mathcal{H}_c$, and define the norms

$$\begin{gathered}
\|(u(t), v(t))\|_1^2 := \int_\Omega [\mu|\nabla u(x,t)|^2 + (\lambda+\mu)|\operatorname{div} u(x,t)|^2 + |v(x,t)|^2]dx, \\
\|(\phi(t), \psi(t))\|_2^2 := \int_\Omega [a|\nabla\phi(x,t)|^2 + \alpha|\phi(x,t)|^2 + |\psi(x,t)|^2]dx, \\
\|\varepsilon\|^2 := \|(u(t), v(t))\|_1^2 \; + \; \|(\phi(t), \psi(t))\|_2^2.
\end{gathered}$$

From [19], there exists $M_1, \gamma_1 > 0$, such that

$$\|(\phi_0(t), \dot{\phi}_0(t))\|_2^2 \le M_1 e^{-\gamma_1 t}\|(\phi^0, \phi^1)\|_2^2, \quad t \ge 0.$$

Note that the damped Lamé system (5.7) has been studied by Zuazua and Lebeau in [17] and they proved the exponential decay of solution of (5.7) if the following inequality of observability holds true for some $T, C > 0$, i.e,

$$\|\varphi^0\|_{(L^2(\Omega))^2} + \|\varphi^1\|_{(H^{-1}(\Omega))^2} \le C\int_0^T \|\operatorname{div}\varphi\|_{H^{-1}(\Omega)}dt, \tag{5.11}$$

where $\varphi(t)$ is solution of the Lamé system

$$\begin{gathered} \ddot{\varphi}(t,x) - \Delta_e\,\varphi(t,x) = 0 \quad \text{in } (0,+\infty)\times\Omega, \\ \varphi = 0 \quad \text{on } (0,+\infty)\times\partial\Omega, \\ \varphi(0) = \varphi^0, \dot{\varphi}(0) = \varphi^1 \quad \text{in } \Omega. \end{gathered} \tag{5.12}$$

Under the condition that (5.11) is satisfied, we have

$$\|(u_0(t), \dot{u}_0(t))\|_1^2 \le M_2 e^{-\gamma_2 t}\|(u^0, u^1)\|_1^2, \quad t \ge 0\,,$$

for positive constants M_2, γ_2. Let $\gamma = inf(\gamma_1, \gamma_2)$, we have

$$\|(u_0(t), \phi_0(t), \dot{u}_0(t), \dot{\phi}_0(t))\| \le Me^{-\frac{\gamma}{2}t}\|(u^0, \phi^0, u^1, \phi^1)\|, \quad t \ge 0. \tag{5.13}$$

Let $(\mathcal{G}(t))_{t\ge0}$ and $(\mathcal{K}(t))_{t\ge0}$ be the contraction C_0-semigroups generated by the equations (5.8) and (5.7) respectively, where $(\phi_0(t), \dot{\phi}_0(t)) = \mathcal{G}(t)(\phi^0, \phi^1)$, and $(u_0(t), \dot{u}_0(t)) = \mathcal{K}(t)(u^0, u^1)$. For the solution of system (5.10) we have

$$(\phi_{k+1}(t), \dot{\phi}_{k+1}(t)) = \int_0^t \mathcal{G}(t-s)(0, -\operatorname{div} u_k(s))ds.$$

Then

$$\|(\phi_{k+1}(t), \dot{\phi}_{k+1}(t))\|_2 \le \int_0^t M_1 e^{-\frac{\gamma_1}{2}(t-s)}\|(0, -\operatorname{div} u_k(s))\|_2 ds.$$

Since $\|(0, -\operatorname{div} u_k(s))\|_2 \le C_1\|(u_k(s), \dot{u}_k(s))\|_1$, we have

$$\|(\phi_{k+1}(t), \dot{\phi}_{k+1}(t))\|_2 \le \int_0^t C_1 M_1 e^{-\frac{\gamma_1}{2}(t-s)}\|(u_k(s), \dot{u}_k(s))\|_1 ds.$$

For the solution of system (5.9) we have

$$(u_{k+1}(t), \dot{u}_{k+1}(t)) = \int_0^t \mathcal{K}(t-s)(0, b\nabla\phi_k(s))ds.$$

Then

$$\|(u_{k+1}(t), \dot{u}_{k+1}(t))\|_1 \le \int_0^t M_2 e^{-\frac{\gamma_2}{2}(t-s)}\|(0, b\nabla\phi_k(s))\|_1 ds.$$

Since $\|(0, b\nabla\phi_k(s))\|_1 \le C_2 b\|(\phi_k(s), \dot{\phi}_k(s))\|_2$, we have

$$\|(u_{k+1}(t), \dot{u}_{k+1}(t))\|_1 \le \int_0^t bC_2M_2 e^{-\frac{\gamma_2}{2}(t-s)}\|(\phi_k(s), \dot{\phi}_k(s))\|_2 ds.$$

Then we have

$$\begin{aligned} &\|(u_{k+1}(t), \phi_{k+1}(t), \dot{u}_{k+1}(t), \dot{\phi}_{k+1}(t))\| \\ &\le \int_0^t M_3 e^{-\frac{\gamma}{2}(t-s)}\|(u_k(s), \phi_k(s), \dot{u}_k(s), \dot{\phi}_k(s))\|ds. \end{aligned} \tag{5.14}$$

From (5.13) and (5.14) we deduce that

$$\|(u_l(t), \phi_l(t), \dot{u}_l(t), \dot{\phi}_l(t))\| \leq MM_3^l \frac{t^l}{l!} e^{-\frac{\gamma}{2}t} \|(u^0, \phi^0, u^1, \phi^1)\|.$$

Let $0 < b < \frac{\gamma}{2M_3}$, the sequence $\Sigma_{l=0}^{\infty} b^l \varepsilon_l(t)$ is convergent in $C([0,\tau];\mathcal{H}_c)$ for every $\tau > 0$. Let $\overline{\varepsilon}^n(t) = \Sigma_{l=0}^{n} b^l \varepsilon_l(t),\ n \in \mathbb{N}$ where $\overline{\varepsilon}^n(t)$ is the solution of the problem

$$\frac{d\overline{\varepsilon}^n(t)}{dt} = \mathcal{M}\overline{\varepsilon}^n(t) + \beta_n(t); \quad \overline{\varepsilon}^n(0) = \overline{\varepsilon}^0,$$

where $\beta_n(t) := \big(0, b^n \nabla \phi_n(t), 0, -b^n \operatorname{div} u_n(t)\big)^T$. We have

$$\overline{\varepsilon}^n(t) = \mathcal{S}(t)\overline{\varepsilon}^0 + \int_0^t \mathcal{S}(t-s)\beta_n(s)ds,$$

and

$$\|\overline{\varepsilon}(t) - \mathcal{S}(t)\varepsilon^0\| = \|\Sigma_{l=n+1}^{\infty} b^l \varepsilon_l(t) + \int_0^t \mathcal{S}(t-s)\beta_n(s)ds\| \to 0, \quad \text{as } n \to \infty,\ \forall n \in \mathbb{N}.$$

This means that $\mathcal{S}(t)\varepsilon^0 = \overline{\varepsilon}(t)$ and

$$\|\mathcal{S}(t)\varepsilon^0\| \leq \Sigma_{l=0}^{\infty} b^l \|\varepsilon_l(t)\| \leq M\Sigma_{l=0}^{\infty} b^l M_3^l \frac{t^l}{l!} e^{-\frac{\gamma t}{2}} \|\varepsilon^0\| \leq Me^{-\varrho t}\|\varepsilon^0\|,$$

where $\varrho := \frac{\gamma}{2} - M_3 b$. Consequently $(\mathcal{S}(t))_{t\geq 0}$ is exponentially stable and $w_e(\mathcal{M}) < 0$. Since $\sigma_e(\mathcal{T}(t)) = \sigma_e(\mathcal{S}(t))$ for $t \geq 0$, then

$$w_e(\mathcal{A}) < 0.$$

Now we prove that $(\mathcal{T}(t))_{t\geq 0}$ is exponentially stable in $\mathcal{H}$, i.e, $\|\mathcal{T}(t)\| \leq Me^{-\delta t}$, $t \geq 0$, where $M, \delta > 0$. From [14, Theorem 2.9] the semigroup $(\mathcal{T}(t))_{t\geq 0}$ is asymptotically stable in $\mathcal{H}$ i.e, $\lim_{t\to\infty} \|\mathcal{T}(t)x\| = 0$, for every $x \in \mathcal{H}$. Then $s_1(\mathcal{A}) \leq 0$, where

$$s_1(\mathcal{A}) = \sup\{\Re\lambda / \lambda \in \sigma(\mathcal{A}) \backslash \sigma_e(\mathcal{A})\}.$$

To show that $w_0(\mathcal{A}) < 0$, it suffices to prove that $s_1(\mathcal{A}) < 0$. Suppose that $s_1(\mathcal{A}) = 0$, then there exists $\{\lambda_n\}_1^{\infty} \subset \sigma(\mathcal{A}) \backslash \sigma_e(\mathcal{A})$, such that $\Re\lambda_n \to 0$, as $n \to \infty$. $e^{\lambda_n t_0}$ is an eigenvalue of $\mathcal{T}(t_0)$, we have $|e^{\lambda_n t_0}| \leq 1$ and $|e^{\lambda_n t_0}| \to 1$ as $n \to \infty$. Let y be the accumulation point of $\{e^{\lambda_n t_0}\}_1^{\infty}$ in $\mathbb{C}$. Then $y \in \sigma_e(\mathcal{T}(t_0))$ and $|y| = 1$. Thus,

$$r_e(\mathcal{T}(t_0)) \geq 1,$$

furthermore

$$r_e(\mathcal{T}(t_0)) = e^{w_e(\mathcal{A})t_0} < 1.$$

This contradiction implies that $s_1(\mathcal{A}) < 0$, using $w_e(\mathcal{A}) < 0$, we obtain $w_0(\mathcal{A}) < 0$. Finally we have proved the uniform stabilization of the energy of solution of system (5.3).

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