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LOCAL SOLVABILITY AND BLOW-UP FOR BENJAMIN-BONA-MAHONY-BURGERS, ROSENAU-BURGERS AND KORTEWEG-DE VRIES-BENJAMIN-BONA-MAHONY EQUATIONS

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Abstract. In this article some well-known problems in mathematical physics for Benjamin-Bona-Mahony-Burgers, Rosenau-Burgers and Korteweg-de Vries-Benjamin-Bona-Mahony equations are considered. These equations describe important processes in different fields of physics, particulary in hydro- and electrodynamics. We study initial-boundary problems with natural physical boundary conditions. Sufficient conditions of local solvability and blow-up in finite time are obtained. For this the methods of contraction mapping and nonlinear capacity, developed by Galaktionov, Pokhozhaev and Mitidieri, are used.

1. Introduction

The Kortweg-de Vries equation is well known in different fields of science and technology,

$$u_t + uu_x + u_{xxx} = 0. \tag{1.1}$$

Recently Pokhozaev obtained sufficient conditions for the finite time blow-up of solutions of initial-boundary problems for KdV equation [23, 24, 25, 26]. He used the powerful method of nonlinear capacity, developed in [18]. Note that in these papers both classical, and weak solutions of the problems were considered.

By the method of nonlinear capacity, we study the following three equations which are important in different physical applications such as waves on shallow water, and processes in semiconductors with negative differential conductivity: Benjamin-Bona-Mahony-Burgers equation

$$\frac{\partial}{\partial t}(u_{xx} - u) + u_{xx} - uu_x = 0, \tag{1.2}$$

Rosenau-Burgers equation:

$$\frac{\partial}{\partial t}(u_{xxxx} + u) + u_{xx} - uu_x = 0, \tag{1.3}$$

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Korteweg-de Vries-Benjamin-Bona-Mahony equation:

$$\frac{\partial}{\partial t}(u_{xx} - u) - u_{xxx} - uu_x = 0. \tag{1.4}$$

The literature on these equations is very extensive. Among them, we mention the classical papers [1]–[27]. Also we mention our papers [12, 13, 30], where sufficient conditions for blow-up in one- and multidimensional (1.2), (1.3) and (1.4) equations were obtained, but for other boundary conditions and without solving of the local solvability question.

Finally, for completeness we recall that there are three main methods for studying blow-up: The method of nonlinear capacity, developed by Pokhozhaev and Mitidieri [17, 18, 22]; the energy method developed by Levine [2, 11, 14, 15, 20, 21, 29]; and the method, based on maximum principle, proposed by Samarskii, Galaktionov, Kurdyumov and Mikhailov [8, 28].

2. Blow-up for equation (1.2)

Let us consider the initial-boundary problem (1.2) and

$$\frac{\partial}{\partial t}(u_{xx} - u) + u_{xx} - uu_x = 0, \quad x \in (0, L),\ t > 0, \tag{2.1}$$

$$u(0,t) = u_x(0,t) = 0, \quad u(x,0) = u_0(x), \quad x \in [0, L], \quad t \geqslant 0, \tag{2.2}$$

where $L \in (0, +\infty)$. We are looking for the solution of problem (2.1)–(2.2) such that

$$u(x)(t) \in \mathbb{C}^{(1)}([0,T]; \mathbb{C}_0^{(2)}([0,L])) \tag{2.3}$$

for some $T > 0$. By definition, if $f \in \mathbb{C}_0^{(2)}$, then $f \in \mathbb{C}^{(2)}$ and $f(0) = f_x(0) = 0$. In this section we show the following result.

Theorem 2.1. *For any $u_0 \in \mathbb{C}_0^{(2)}([0,L])$ there exists one and only one solution of problem (2.1), (2.2) such that*

$$u(x)(t) \in \mathbb{C}^{(1)}([0,T_0); \mathbb{C}_0^{(2)}([0,L])),$$

where either $T_0 = +\infty$ or $T_0 < +\infty$ and the limiting equality holds

$$\limsup_{t \uparrow T_0} \sup_{x \in [0,L]} |u(x)(t)| = +\infty. \tag{2.4}$$

Proof. We begin with some notation. For any $f \in \mathbb{C}[0,L]$ we use the Volterra operator

$$G * f = \int_0^x dy \int_0^y f(z)\, dz = \int_0^x (x - z) f(z)\, dz \quad \text{for } x \in [0, L].$$

By (2.3) we can rewrite (2.1) as

$$\frac{\partial}{\partial t}[\mathbb{I} - G*]u + u = \frac{1}{2}\int_0^x u^2\, dx. \tag{2.5}$$

The spectral radius of the Volterra operator is zero. Thus multiplying both sides (2.5) by $[\mathbb{I} - G*]^{-1}$, we obtain

$$\frac{\partial u}{\partial t} + u = -\sum_{k=1}^{+\infty} [G*]^k u + \frac{1}{2}\sum_{k=0}^{+\infty} [G*]^k \int_0^x u^2\, dy. \tag{2.6}$$

With the new variables $w = e^t u$, integrating (2.6), we have

$$w = \mathbb{F}[w] \equiv u_0 + \int_0^t ds B[w](s), \quad B[w] = B_1[w] - B_2[w], \tag{2.7}$$

where

$$B_1[w] = -\sum_{k=1}^{+\infty} [G*]^k w, \quad B_2[w] = -\frac{e^{-t}}{2} \sum_{k=0}^{+\infty} [G*]^k \int_0^x w^2\, dy.$$

Let us consider the following closed, bounded and convex subset in the Banach space $\mathbb{C}([0,T] \times [0,L])$:

$$\mathbb{B}_R \equiv \big\{ w \in \mathbb{C}([0,T] \times [0,L]) : \|w\| \equiv \sup_{(t,x) \in [0,T] \times [0,L]} |w(x,t)| \leqslant R \big\}.$$

We prove that the operator $\mathbb{F}[w] : \mathbb{B}_R \to \mathbb{B}_R$ f or large enough $R > 0$ and small $T > 0$. Indeed, suppose that (t_1, x_1) and (t_2, x_2) belong to $[0,L] \times [0,T]$. Without loss of generality we suppose that $t_2 \leqslant t_1$. Then the following chain of inequalities holds

$$\begin{aligned} &\| \int_0^{t_1} ds B_1[w](s, x_1) - \int_0^{t_2} ds B_1[w](s, x_2) \| \\ &\leqslant \int_{t_1}^{t_2} \|B_1[w](s, x_1)\|\, ds + \int_0^{t_1} \|B_1[w](s, x_1) - B_1[w](s, x_2)\| \\ &\leqslant |t_2 - t_1| \sum_{k=1}^{+\infty} \frac{L^{2k}}{(2k)!} \|w\| + T|x_1 - x_2| \sum_{k=1}^{+\infty} \frac{L^{2k-1}}{(2k-1)!} \|w\|. \end{aligned} \tag{2.8}$$

In the same way, for $B_2[w]$ we have the estimate

$$\begin{aligned} &\| \int_0^{t_1} ds B_2[w](s, x_1) - \int_0^{t_2} ds B_2[w](s, x_2) \| \\ &\leqslant \int_{t_1}^{t_2} \|B_2[w](s, x_1)\|\, ds + \int_0^{t_1} \|B_2[w](s, x_1) - B_2[w](s, x_2)\| \\ &\leqslant |t_2 - t_1| \frac{1}{2} \sum_{k=0}^{+\infty} \frac{L^{2k+1}}{(2k+1)!} \|w\|^2 + T|x_1 - x_2| \frac{1}{2} \sum_{k=0}^{+\infty} \frac{L^{2k}}{(2k)!} \|w\|^2. \end{aligned} \tag{2.9}$$

This yields that the operator $\mathbb{F}$ is a mapping from $\mathbb{C}([0,T] \times [0,L])$ to $\mathbb{C}([0,T] \times [0,L])$. Now show that the operator is a mapping from $\mathbb{B}_R$ to $\mathbb{B}_R$. It is clear that

$$\begin{aligned} \|\mathbb{F}[w]\| &\leqslant \|u_0\| + \int_0^T ds \|B[w](s)\| \\ &\leqslant \|u_0\| + T \sum_{k=1}^{+\infty} \frac{L^{2k}}{(2k)!} \|w\| + \frac{T}{2} \sum_{k=0}^{+\infty} \frac{L^{2k+1}}{(2k+1)!} \|w\|^2. \end{aligned} \tag{2.10}$$

From (2.10) we get that for large enough $R > 0$,

$$\|u_0\| \leqslant \frac{R}{2},$$

and for small enough $T > 0$,

$$T \sum_{k=1}^{+\infty} \frac{L^{2k}}{(2k)!} R + \frac{T}{2} \sum_{k=0}^{+\infty} \frac{L^{2k+1}}{(2k+1)!} R^2 \leqslant \frac{R}{2}\,.$$

The result of Theorem 1 is true. Now let us demonstrate the contraction mapping $\mathbb{F}[w]$ on $\mathbb{B}_R$. It is not hard to show that the following chain of inequalities holds

$$\begin{aligned} \|\mathbb{F}[w_1]-\mathbb{F}[w_2]\| &\leqslant T\|B[w_1]-B[w_2]\| \\ &\leqslant T\Big[\sum_{k=1}^{+\infty}\frac{L^{2k}}{(2k)!}+\sum_{k=0}^{+\infty}\frac{L^{2k+1}}{(2k+1)!}R\Big]\|w_1-w_2\|, \end{aligned} \tag{2.11}$$

then from (2.11), for any

$$T\big[\sum_{k=1}^{+\infty}\frac{L^{2k}}{(2k)!}+\sum_{k=0}^{+\infty}\frac{L^{2k+1}}{(2k+1)!}R\big]\leqslant\frac{1}{2}$$

we have a contraction $\mathbb{F}[w]$ on $\mathbb{B}_R$. This means that the unique, local in time, solvability of the integral equation (2.7) is proved.

We claim that the solution w of (2.7) belongs to the class $\mathbb{C}([0,T_0)\times[0,L])$; moreover, either $T_0=+\infty$, or $T_0<+\infty$ and $\lim_{t\uparrow T_0}\|w\|=+\infty$. Assuming the converse: $T_0<+\infty$, then we have

$$\sup_{t\in[0,T_0]}\|w\|<+\infty.$$

From equation (2.7) we get the estimate

$$\|w'\|\leqslant c(T_0)<+\infty\quad\text{for all } t\in[0,T_0].$$

This implies that

$$|w(x)(t_1)-w(x)(t_2)|\leqslant\int_{t_1}^{t_2}|w'(x)(\tau)|\,d\tau\leqslant c(T_0)|t_2-t_1|,$$

where the constant $c(T_0)<+\infty$ does not depend on $x\in[0,L]$. Whence, for all $x\in[0,L]$ we can define $w(x)(T_0)$. Furthemore, if we take supremum of both sides of the last inequality, then we obtain

$$w(x)(T_0)\in\mathbb{B}[0,L],$$

where $\mathbb{B}[0,L]$ is the set of bounded functions on the line segment $[0,L]$. We shall see that

$$w(x)(T_0)\in\mathbb{C}[0,L].$$

Obviously, for all $\varepsilon>0$ there is $\delta(\varepsilon)>0$ such that for all $x_1,x_2\in[0,L]$,

$$|x_1-x_2|\leqslant\delta(\varepsilon)/2$$

and we can choose $t_1,t_2\in[0,T_0)$ such that

$$|T_0-t_1|\leqslant\delta(\varepsilon)/4,\quad |T_0-t_2|\leqslant\delta(\varepsilon)/4$$

imply

$$|t_1-t_2|\leqslant|T_0-t_1|+|T_0-t_2|\leqslant\delta(\varepsilon)/2, |t_1-t_2|+|x_1-x_2|\leqslant\delta(\varepsilon),$$

so the following chain of inequalities holds:

$$\begin{aligned} &|w(x_1)(T_0)-w(x_2)(T_0)| \\ &\leqslant|w(x_1)(T_0)-w(x_1)(t_1)|+|w(x_1)(t_1)-w(x_2)(t_2)|+|w(x_2)(T_0)-w(x_2)(t_2)| \\ &\leqslant c(T_0)|T_0-t_1|+|w(x_1)(t_1)-w(x_2)(t_2)|+c(T_0)|T_0-t_2| \\ &\leqslant c(T_0)(\frac{\varepsilon}{4}+\frac{\varepsilon}{2}+\frac{\varepsilon}{4})=c(T_0)\varepsilon. \end{aligned} \tag{2.12}$$

Therefore, we can define
$$w(x)(T_0) \in \mathbb{C}([0, L])$$
and can extend the classical solution over the time moment $T_0 > 0$, taking the following initial function $w(x)(t)$ for $t = T_0$. This contradiction concludes the result of existance of a maximal solution. Finally, using so called "bootstrap" method for the integral equation (2.7) under the additional condition $u_0(x) \in \mathbb{C}_0^{(2)}([0, L])$, we get that
$$w(x)(t) \in \mathbb{C}^{(1)}([0, T_0); \mathbb{C}_0^{(2)}([0, L])).$$
proof is complete. □

Now we shall obtain a blow-up result.

Theorem 2.2. *Suppose that for some positive $\lambda \geqslant 3$, the initial function satisfies the condition*
$$J(0) > \frac{m}{k}, \tag{2.13}$$
where
$$J(t) = \int_0^L (L-x)^{\lambda-3}[\lambda(\lambda-1) - (L-x)^2]((L-x)u - (\lambda-1))\, dx,$$
or
$$\begin{aligned} &\int_0^L (L-x)^{\lambda-2}[\lambda(\lambda-1) - (L-x)^2]u_0\, dx \\ &> (\lambda-1)L^{\lambda-1}\Big|\frac{\lambda(\lambda-1)^2}{\lambda-2} - 2\frac{1}{\lambda}(\lambda-1)L^2 + \frac{\lambda L^4}{\lambda(\lambda+2)}\Big|^{1/2} \\ &\quad + \frac{\lambda(\lambda-1)^2}{\lambda-2}L^{\lambda-2} - \frac{\lambda-1}{\lambda}L^{\lambda}, \end{aligned}$$
then the classical solution of problem (2.1)–(2.2) does not exist globally in time. Furthermore, the following estimate holds
$$J(t) \geqslant \frac{m}{k}\frac{(kJ(0)+m) + (kJ(0)-m)\exp(2mkt)}{(kJ(0)+m) - (kJ(0)-m)\exp(2mkt)}, \tag{2.14}$$
consequently,
$$\lim_{t\to T_b} J(t) = +\infty, \quad T_b \leqslant -\frac{1}{2mk}\ln(\frac{kJ(0)-m}{kJ(0)+m}), \tag{2.15}$$
$$m^2 = \frac{(\lambda-1)^2L^{\lambda}}{2}, \quad k^2 = \frac{L^{2-\lambda}}{2}\Big|\frac{\lambda(\lambda-1)^2}{\lambda-2} - 2\frac{1}{\lambda}(\lambda-1)L^2 + \frac{\lambda L^4}{\lambda(\lambda+2)}\Big|^{-1}.$$

Proof. To obtain sufficient conditions of the blow-up we use the method of nonlinear capacity. Let us take the test function
$$\varphi(x) = (L-x)^{\lambda}, \quad \lambda \geqslant 3$$
to remove boundary conditions on the right side ($x = L$). Multiplying both sides of (1.2) by $\varphi(x)$ and integrating by parts over $[0, L]$, we use the following equalities
$$\int_0^L (L-x)^{\lambda}u_{xx}(x,t)\, dx = \lambda(\lambda-1)\int_0^L (L-x)^{\lambda-2}u\, dx,$$
$$\int_0^L (L-x)^{\lambda}uu_x(x,t)\, dx = \frac{\lambda}{2}\int_0^L (L-x)^{\lambda-1}u^2\, dx,$$

and get the ordinary differential equation

$$\begin{aligned} &\frac{d}{dt}\int_0^L (L-x)^{\lambda-2}[\lambda(\lambda-1)-(L-x)^2]u\,dx \\ &= -\lambda(\lambda-1)\int_0^L (L-x)^{\lambda-2}u\,dx + \frac{\lambda}{2}\int_0^L u^2(L-x)^{\lambda-1}\,dx. \end{aligned} \tag{2.16}$$

Constructing the perfect square in the right side of (2.16) and adding in the left side the time-independent function, we obtain

$$\begin{aligned} &\frac{d}{dt}\int_0^L (L-x)^{\lambda-3}[\lambda(\lambda-1)-(L-x)^2]((L-x)u-(\lambda-1))\,dx \\ &= \frac{\lambda}{2}\int_0^L [u(L-x)-(\lambda-1)]^2(L-x)^{\lambda-3}\,dx - \frac{\lambda(\lambda-1)^2}{2}\int_0^L (L-x)^{\lambda-1}\,dx. \end{aligned} \tag{2.17}$$

By Holder's inequality

$$\Big|\int ab\Big| \leqslant \Big|\int a^2\Big|^{1/2}\Big|\int b^2\Big|^{1/2} \tag{2.18}$$

we can substitute

$$\begin{gathered} a^2 = [u(L-x)-(\lambda-1)]^2(L-x)^{\lambda-3}, \\ b^2 = [\lambda(\lambda-1)-(L-x)^2]^2(L-x)^{\lambda-3}, \end{gathered}$$

and integrate over the line segment $[0,L]$. Then (2.18) gives the following estimate

$$\int_0^L [u(L-x)-(\lambda-1)]^2(L-x)^{\lambda-3}\,dx \geqslant J^2\Big|\int_0^L b^2dx\Big|^{-1}.$$

From equation (2.17) we get the ODE

$$\frac{dJ(t)}{dt} \geqslant \frac{\lambda J^2}{2}\Big|\int_0^L b^2dx\Big|^{-1} - \frac{\lambda(\lambda-1)^2}{2}\int_0^L (L-x)^{\lambda-1}\,dx, \tag{2.19}$$

where

$$J(t) = \int_0^L (L-x)^{\lambda-3}[\lambda(\lambda-1)-(L-x)^2]((L-x)u-(\lambda-1))\,dx.$$

The final inequality (2.19) can be written as

$$\frac{dJ(t)}{dt} \geqslant k^2J^2 - m^2, \tag{2.20}$$

where

$$m^2 = \frac{\lambda(\lambda-1)^2}{2}\int_0^L (L-x)^{\lambda-1}\,dx = \frac{(\lambda-1)^2L^\lambda}{2},$$

$$\begin{aligned} k^2 &= \frac{\lambda}{2}\Big|\int_0^L b^2dx\Big|^{-1} \\ &= \frac{\lambda}{2}\Big|\int_0^L [\lambda^2(\lambda-1)^2 - 2\lambda(\lambda-1)(L-x)^2 + (L-x)^4](L-x)^{\lambda-3}\,dx\Big|^{-1} \\ &= \frac{L^{2-\lambda}}{2}\Big|\frac{\lambda(\lambda-1)^2}{\lambda-2} - 2\frac{1}{\lambda}(\lambda-1)L^2 + \frac{L^4}{\lambda(\lambda+2)}\Big|^{-1}. \end{aligned}$$

Integrating the inequality (2.20), we complete the proof. □

Note that the Theorem 2.2 is proved for any fixed $L \in (0, +\infty)$. But we should stress that the time moment $T_0 > 0$ is the function of $L > 0$ such that the following situation is possible:

$$T_0 \to +0 \quad \text{as } L \to +\infty.$$

Theorem 2.3. *Suppose that the initial data satisfies the inequalities*

$$J(0) > 1, \quad \lambda > 1, \tag{2.21}$$

where

$$J(t) = \int_0^{+\infty} (u - \lambda) e^{-\lambda x}\, dx,$$

or, which is the same,

$$\int_0^{+\infty} u_0(x) e^{-\lambda x}\, dx > 2\,.$$

Then the global in time solution of the problem (2.1), (2.2) *does not exist in the half-space ($L = \infty$). Furthermore,*

$$J(t) \geqslant \frac{1 + C \exp(\lambda^2 t/(\lambda^2 - 1))}{1 - C \exp(\lambda^2 t/(\lambda^2 - 1))}, \tag{2.22}$$

where

$$\lim_{t \to T_b} J(t) = +\infty, \quad T_b \leqslant -\frac{\lambda^2 - 1}{\lambda^2} \ln C, \quad C = \frac{J(0) - 1}{J(0) + 1}. \tag{2.23}$$

Proof. To remove conditions for $L = +\infty$ we use the test function $\varphi(x) = \exp(-\lambda x)$, $\lambda > 1$. Multiplying both sides of (1.2) by $\varphi(x)$, we integrate by parts over $[0, \infty)$. By the boundary conditions we have the following equalities

$$\begin{gathered}
\int_0^{+\infty} e^{-\lambda x} u_{xx}(x,t)\, dx = \lambda^2 \int_0^{+\infty} e^{-\lambda x} u(x,t)\, dx, \\
\int_0^{+\infty} e^{-\lambda x} u u_x(x,t)\, dx = \frac{\lambda}{2} \int_0^{+\infty} e^{-\lambda x} u^2(x,t)\, dx.
\end{gathered}$$

So we can rewrite the equation (2.1) as

$$\frac{d}{dt} \int_0^\infty e^{-\lambda x} [\lambda^2 - 1] u\, dx = -\lambda^2 \int_0^\infty e^{-\lambda x} u\, dx + \frac{\lambda}{2} \int_0^\infty e^{-\lambda x} u^2\, dx. \tag{2.24}$$

As above, we make the perfect square in the right side of (2.24) and add the time-independent function in the left side, we obtain

$$\frac{d}{dt} \int_0^{+\infty} e^{-\lambda x} [\lambda^2 - 1](u - \lambda)\, dx = \frac{\lambda}{2} \int_0^{+\infty} (u - \lambda)^2 e^{-\lambda x}\, dx - \frac{\lambda^3}{2} \int_0^{+\infty} e^{-\lambda x}\, dx. \tag{2.25}$$

It is not hard to prove the inequality

$$\begin{aligned}
\Big| \int_0^{+\infty} (u - \lambda) e^{-\lambda x}\, dx \Big|^2 &\leqslant \int_0^{+\infty} (u - \lambda)^2 e^{-\lambda x}\, dx \int_0^{+\infty} e^{-\lambda x}\, dx \\
&= \frac{1}{\lambda} \int_0^{+\infty} (u - \lambda)^2 e^{-\lambda x}\, dx.
\end{aligned} \tag{2.26}$$

Thus from (2.25) we get the ODE

$$\frac{dJ(t)}{dt} \geqslant \frac{\lambda^2}{2(\lambda^2 - 1)} J^2 - \frac{\lambda^2}{2(\lambda^2 - 1)}, \tag{2.27}$$

where
$$J(t)=\int_0^{+\infty}(u-\lambda)e^{-\lambda x}\,dx.$$
Integrating inequality (2.27), we obtain the stated result. □

3. Blow-up for the Rosenau-Burgers equation

Let us consider (1.3) with the initial-boundary conditions
$$\frac{\partial}{\partial t}(u_{xxxx}+u)+u_{xx}-uu_x=0,\quad x\in(0,L),\quad t>0, \tag{3.1}$$
$$u(0,t)=u_x(0,t)=u_{xx}(0,t)=u_{xxx}(0,t)=0,\quad u(x,0)=u_0(x), \tag{3.2}$$
where $L\in(0,+\infty)$. Assume that there is such $T>0$, that classical solution u of the problem (3.1)–(3.2) exists and satisfies
$$u(x)(t)\in\mathbb{C}^{(1)}([0,T];\mathbb{C}_0^{(4)}([0,L]))\,. \tag{3.3}$$
By definition, if $f\in\mathbb{C}_0^{(4)}$, then $f\in\mathbb{C}^{(4)}$ and $f(0)=f_x(0)=f_{xx}(0)=f_{xxx}(0)=0$. In this section we prove the following result.

Theorem 3.1. *For any initial data $u_0\in\mathbb{C}_0^{(4)}([0,L])$ there exists only one solution u of problem* (3.1), (3.2) *such that*
$$u(x)(t)\in\mathbb{C}^{(1)}([0,T_0);\mathbb{C}_0^{(4)}([0,L])),$$
for some $T_0>0$ such that either $T_0=+\infty$ or $T_0<+\infty$ and
$$\limsup_{t\uparrow T_0}\sup_{x\in[0,L]}|u(x)(t)|=+\infty. \tag{3.4}$$

The proof of the above theorem is the same as that for the Theorem 2.1. So we omit it. Let us prove now a blow-up result.

Theorem 3.2. *Suppose that there exists such $\lambda\geqslant 7$ that initial function satisfies the inequality*
$$J(0)>\frac{m}{k}, \tag{3.5}$$
where
$$J(t)=\int_0^L(L-x)^{\lambda-5}[\lambda(\lambda-1)(\lambda-2)(\lambda-3)+(L-x)^4]((L-x)u-(\lambda-1))\,dx.$$
then the the classical solution of (3.1) *does not exist globally in time. Moreover, for m and k from* (3.2) *the following estimate holds*
$$J(t)\geqslant\frac{m}{k}\frac{(kJ(0)+m)+(kJ(0)-m)\exp(2mkt)}{(kJ(0)+m)-(kJ(0)-m)\exp(2mkt)}, \tag{3.6}$$
and, consequently,
$$\lim_{t\to T_b}J(t)=+\infty, \tag{3.7}$$
where
$$T_b\leqslant-\frac{1}{2mk}\ln\Big(\frac{kJ(0)-m}{kJ(0)+m}\Big),\quad m^2=\frac{(\lambda-1)^2L^\lambda}{2},$$
$$k^2=\frac{L^{6-\lambda}}{2}\left|\frac{\lambda(\lambda-1)^2(\lambda-2)^2(\lambda-3)^2}{(\lambda-6)}+2(\lambda-1)(\lambda-3)L^4+\frac{L^8}{\lambda(\lambda+2)}\right|^{-1}$$

Proof. Multiplying both sides (3.1) by $\varphi(x) = (L-x)^\lambda$, we integrate by parts. By the boundary conditions, we have

$$\int_0^L (L-x)^\lambda u_{xxxx}(x,t)\,dx = \lambda(\lambda-1)(\lambda-2)(\lambda-3)\int_0^L (L-x)^{\lambda-4}u(x,t)\,dx;$$

By this equality we can rewrite (1.3) as the integro-differential expression

$$\begin{aligned}&\frac{d}{dt}\int_0^L (L-x)^{\lambda-4}[\lambda(\lambda-1)(\lambda-2)(\lambda-3)+(L-x)^4]u\,dx\\ &= -\lambda(\lambda-1)\int_0^L (L-x)^{\lambda-2}u\,dx + \frac{\lambda}{2}\int_0^L u^2(L-x)^{\lambda-1}\,dx.\end{aligned} \tag{3.8}$$

As before, in the right-hand side of (3.8) we complete the square and in the left-hand side add the time-independent function:

$$\begin{aligned}&\frac{d}{dt}\int_0^L (L-x)^{\lambda-5}[\lambda(\lambda-1)(\lambda-2)(\lambda-3)+(L-x)^4]((L-x)u-(\lambda-1))\,dx\\ &= \frac{\lambda}{2}\int_0^L [u(L-x)-(\lambda-1)]^2(L-x)^{\lambda-3}\,dx - \frac{\lambda(\lambda-1)^2}{2}\int_0^L (L-x)^{\lambda-1}\,dx.\end{aligned} \tag{3.9}$$

Substituting

$$\begin{gathered}a^2 = [u(L-x)-(\lambda-1)]^2\,(L-x)^{\lambda-3},\\ b^2 = [\lambda(\lambda-1)(\lambda-2)(\lambda-3)-(L-x)^4]^2(L-x)^{\lambda-7}\end{gathered}$$

in (2.18) and integrating over the segment $[0,L]$, we obtain

$$\begin{aligned}&\int_0^L [u(L-x)-(\lambda-1)]^2(L-x)^{\lambda-3}\,dx \leqslant J^2\Big|\int_0^L b^2dx\Big|^{-1}\\ &= J^2\Big|\frac{\lambda^2(\lambda-1)^2(\lambda-2)^2(\lambda-3)^2}{(\lambda-6)}L^{\lambda-6} + 2\lambda(\lambda-1)(\lambda-3)L^{\lambda-2} + \frac{L^{\lambda+2}}{\lambda+2}\Big|^{-1}.\end{aligned} \tag{3.10}$$

Finally, from (3.9) we obtain the ordinary differential inequality

$$\frac{dJ(t)}{dt} \geqslant k^2J^2 - m^2, \tag{3.11}$$

where $m^2 = \frac{(\lambda-1)^2L^\lambda}{2}$,

$$\begin{gathered}k^2 = \frac{L^{6-\lambda}}{2}\Big|\frac{\lambda(\lambda-1)^2(\lambda-2)^2(\lambda-3)^2}{(\lambda-6)} + 2(\lambda-1)(\lambda-3)L^4 + \frac{L^8}{\lambda(\lambda+2)}\Big|^{-1},\\ J(t) = \int_0^L (L-x)^{\lambda-5}[\lambda(\lambda-1)(\lambda-2)(\lambda-3)+(L-x)^4]((L-x)u-(\lambda-1))\,dx.\end{gathered}$$

Integrating (3.11), we complete the proof. □

As for equation (1.2), we can obtain results for the Rosenau-Burgers in the unbounded domain. For problem (3.1)-(3.2) on the half-line $[0,+\infty)$ we shall prove the following result.

Theorem 3.3. *Suppose that the initial data satisfies the inequality*

$$\int_0^{+\infty} u_0(x)e^{-\lambda x}\,dx > 2,$$

then the classical solution of the problem for Rosenau-Burgers on the half line does not exist globally in time. Furthermore, there exists the lower estimate

$$J(t) \geqslant \frac{1 + C\exp(\lambda^2 t/(\lambda^4+1))}{1 - C\exp(\lambda^2 t/(\lambda^4+1))}, \quad J(t) = \int_0^{+\infty} (u-\lambda)e^{-\lambda x}\,dx, \tag{3.12}$$

and, consequently,

$$\lim_{t\to T_b} J(t) = +\infty, \quad T_b \leqslant -\frac{\lambda^4+1}{\lambda^2}\ln C, \quad C = \frac{J(0)-1}{J(0)+1}. \tag{3.13}$$

Proof. The proof is similar to the Theorem 2.3; so we present on the main points. First let us multiply both sides of (1.3) by $\varphi(x) = \exp(-\lambda x)$, with $\lambda > 1$, and integrate by parts over $[0, L)$:

$$\frac{d}{dt}\int_0^\infty e^{-\lambda x}[\lambda^4+1]u\,dx = -\lambda^2\int_0^\infty e^{-\lambda x}u\,dx + \frac{\lambda}{2}\int_0^\infty e^{-\lambda x}u^2\,dx. \tag{3.14}$$

Then extracting perfect square in the side part (3.14) and adding a time-independed function to the left-hand side, we obtain the equality

$$\frac{d}{dt}\int_0^{+\infty} e^{-\lambda x}[\lambda^4+1](u-\lambda)\,dx = \frac{\lambda}{2}\int_0^{+\infty}(u-\lambda)^2 e^{-\lambda x}\,dx - \frac{\lambda^3}{2}\int_0^{+\infty} e^{-\lambda x}\,dx. \tag{3.15}$$

By (2.26) we can rewrite (3.15) as

$$\frac{dJ(t)}{dt} \geqslant \frac{\lambda^2}{2(\lambda^4+1)}J^2 - \frac{\lambda^2}{2(\lambda^4+1)}, \tag{3.16}$$

where

$$J(t) = \int_0^{+\infty}(u-\lambda)e^{-\lambda x}\,dx.$$

Integrating (3.16), we obtain the result of the theorem. □

4. Blow-up for equation (1.4)

It is readily seen that the method of nonlinear capacity is usable for Sobolev type equations also with additional terms like higher-order derivatives $u^{(n)}$. As an example, we consider the problem for the equation, which describes processes in cristalline semiconductors and has the third-order derivative:

$$(u_{xx} - u)_t + u_{xxx} - uu_x = 0, \quad t > 0,\ x \in (0, L),\ L > 0, \tag{4.1}$$

$$u(0,t) = u_x(0,t) = u_{xx}(0,t) = 0, \quad u(x,0) = u_0(x), \quad x \in [0, L], \quad t \geqslant 0. \tag{4.2}$$

The physical model for problem (4.1)–(4.2) is presented in [31]. Let us prove the solvability result.

Theorem 4.1. *For any function $\tilde{u}_0(x)$ such that*

$$\tilde{u}_0(x) = \begin{cases} u_0(x), & \text{for } x \geqslant 0; \\ 0, & \text{for } x < 0, \end{cases} \quad \tilde{u}_0(x) \in \mathbb{C}^{(4)}(-\infty, L],$$

there exists only one solution u of (4.1), (4.2) such that

$$u(x)(t) \in \mathbb{C}^{(1)}([0, T_0); \mathbb{C}_0^{(3)}([0, L])),$$

for some $T_0 > 0$. *By definition, if* $f \in \mathbb{C}_0^{(3)}$, *then* $f \in \mathbb{C}^{(3)}$ *and* $f(0) = f_x(0) = f_{xx}(0) = 0$. *Moreover, it can be proved that either* $T_0 = +\infty$ *or* $T_0 < +\infty$ *and the following limiting equality holds*

$$\sup_{t \uparrow T_0} \sup_{x \in [0,L]} |u(x)(t)| = +\infty.$$

Remark 4.2. By the boundary conditions (4.2) and natural matching condition of initial and boundary data it is easily shown that $\tilde{u}_0(x) \in \mathbb{C}_0^{(3)}(-\infty, L]$, however, larger smoothness is not indispensable. In addition, it is clear that necessary and sufficient conditions for $\tilde{u}_0(x) \in \mathbb{C}^{(4)}(-\infty, L]$ are the following

$$u_0(x) \in \mathbb{C}^{(4)}([0, L]) \quad \text{and} \quad (u_0)_{xxx}(0+0) = (u_0)_{xxxx}(0+0) = 0.$$

Proof of Theorem 4.1. As above for equation (1.2), taking into account the boundary conditions, integrating with respect to x and inverting the operator

$$(\mathbb{I} - G*), \quad \text{where} \quad G * f = \int_0^x dy \int_0^y f(z)\, dz,$$

we obtain the equation

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = \mathbb{Q}[u](x, t) \equiv \sum_{k=0}^{+\infty} [G*]^k \int_0^x u\, dy + \frac{1}{2} \sum_{k=0}^{+\infty} [G*]^k \int_0^x u^2\, dy, \tag{4.3}$$

where we use that $\int_0^x u_y\, dy = u(x)$, and, thus,

$$\sum_{k=1}^{+\infty} [G*]^k u_x = \sum_{k=0}^{+\infty} [G*]^k \int_0^x u\, dy.$$

From the definition of operator $\mathbb{Q}[u](x, t)$ it follows that $\mathbb{Q}[u](0, t) = 0$, moreover, $u(0, t) = 0$. Then we can continue the function $\mathbb{Q}[u](x, t)$ by zero for $x < 0$, furthermore, this function $\mathbb{Q}[u](x, t)$ is smooth function with respect to the first input. Corresponding to this continuation we define

$$f(x, t) \equiv \widetilde{\mathbb{Q}}[u](x, t) \quad \text{and} \quad \tilde{u}(x, t).$$

Now consider the differential problem

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} = f(x, t), \quad v(x, 0) = v_0(x), \quad v(x, t) \equiv \tilde{u}(x, t). \tag{4.4}$$

The solution of (4.3) we can be written as

$$u(x, t) = \tilde{u}_0(x - t) + \int_0^t f(x - t + \tau, \tau)\, d\tau, \quad x \in [0, L],\ t \in [0, T], \tag{4.5}$$

where $L > 0$ is fixed, and $T > 0$ is small enough.

To prove the local solvability of the integral equation (4.5) we use the contraction mapping method. In the notation of the previous section we can rewrite the integral equation (4.5) in the abstract form

$$u(x, t) = \mathbb{F}[u](x, t) \equiv \tilde{u}_0(x - t) + \int_0^t \widetilde{\mathbb{Q}}[u](x - t + \tau, \tau)\, d\tau. \tag{4.6}$$

As before, it can easily be checked that, if $u_0(x) \in \mathbb{C}_0^{(3)}([0, L])$, then

$$\mathbb{F}[\cdot] : \mathbb{C}([0, T] \times [0, L]) \to \mathbb{C}([0, T] \times [0, L]).$$

Let us prove now that $\mathbb{F}[\cdot]$ is an operator between $\mathbb{B}_R$ and $\mathbb{B}_R$ for large enough $R > 0$ and small $T > 0$. Indeed, we can fix $R > 0$ such large that

$$\|u_0\| \leqslant \frac{R}{2}.$$

On the other hand, we have the inequality

$$\|u\| \leqslant \|u_0\| + T \sum_{k=0}^{+\infty} \frac{(L^*)^{2k+1}}{(2k+1)!} \big[\|u\| + \frac{1}{2}\|u\|^2\big], \quad L^* = \max\{L, |L-T|\}. \tag{4.7}$$

From (4.7), for small enough $T > 0$,

$$T \sum_{k=0}^{+\infty} \frac{(L^*)^{2k+1}}{(2k+1)!} [R + \frac{R^2}{2}] \leqslant \frac{R}{2},$$

and we obtain our result. As in the second section it is easily shown that $\mathbb{F}$ is contraction mapping on a space $\mathbb{B}_R$, if we change L to L^*. Thus, we complete the proof of local in time solvability of the integral equation (4.5).

Now we can continue the solution in time. Assume that $u_0(x) \in \mathbb{C}_0^{(3)}([0, L])$, then from the necessary matching condition of initial and boundary data we obtain that $\tilde{u}_0(x) \in \mathbb{C}^{(4)}((-\infty, L])$. Moreover, it can easily be checked that

$$u(x)(t) \in \mathbb{C}_0^{(3)}([0, T] \times [0, L]).$$

Let us denote

$$w(x)(t) \equiv \tilde{u}_0(x - t) + \int_0^t \widetilde{\mathbb{Q}}[u](x - t + \tau, \tau)\, d\tau,$$

then under the condition $T_0 < +\infty$, the following inequality holds

$$\sup_{t \in [0, T_0]} \|u\| < +\infty.$$

By a standard way we obtain $w(x)(T_0) \in \mathbb{C}([0, L])$. At the same time we can write

$$w(x)(T_0) = \tilde{u}_0(x - T_0) + \int_0^{T_0} \widetilde{\mathbb{Q}}[u](x - T_0 + \tau, \tau)\, d\tau \in \mathbb{C}^{(1)}([0, L]).$$

Therefore, in (4.3), taking $u(x)(t)$ for $t = T_0$ as initial data, we can continue the solution over the time moment T_0. This contradiction concludes the proof of existence of the maximum solution.

Using the "bootstrap" method, it is possible to show that, if $\tilde{u}_0(x) \in \mathbb{C}^{(4)}(-\infty, L]$, then the solution of (4.5) belong to the class $\mathbb{C}^{(1)}([0, T_0); \mathbb{C}_0^{(3)}([0, L]))$. $\square$

As above, to obtain sufficient conditions of blow-up we use the method of nonlinear capacity for the power test function $\varphi(x) = (L - x)^5$. Suppose that the solution $u(x, t)$ of problem (1.4), (4.2) is classical:

$$u(x)(t) \in \mathbb{C}^{(1)}([0, T]; \mathbb{C}^{(3)}([0; L])) \quad \text{for some } T > 0.$$

Multiplying both sides of (1.4) by the test function $\varphi(x)$ and integrating by parts, we obtain the equality:

$$\frac{d}{dt} \int_0^L (L-x)^3[20-(L-x)^2]u\, dx = \frac{5}{2} \int_0^L u^2(L-x)^4\, dx + 60 \int_0^L u(L-x)^2\, dx. \tag{4.8}$$

As before, let us make in the right-hnad side (2.16) the perfect square and in the left add the time-independent function

$$\frac{d}{dt}\int_0^L (L-x)[20-(L-x)^2]((L-x)^2u+12)\,dx = \frac{5}{2}\int_0^L [(L-x)^2u+12]^2\,dx - 360L. \tag{4.9}$$

The following inequalities can be easily proved:

$$\begin{aligned} &\Big|\int_0^L (L-x)[20-(L-x)^2]((L-x)^2u+12)\,dx\Big|^2 \\ &\leqslant \Big|\int_0^L (L-x)^2[20-(L-x)^2]^2\,dx\Big|\Big|\int_0^L ((L-x)^2u+12)^2\,dx\Big| \\ &= \frac{5}{2k^2}\int_0^L ((L-x)^2u+12)^2\,dx, \end{aligned} \tag{4.10}$$

where

$$k^2 = \Big|\frac{160L^3}{3} - \frac{16L^5}{5} + \frac{2L^7}{35}\Big|^{-1}.$$

From equation (4.9) and estimate (4.10) we obtain the ordinary differential inequality

$$\frac{dJ(t)}{dt} \geqslant k^2J^2 - 360L, \tag{4.11}$$

where

$$J(t) = \int_0^L (L-x)[20-(L-x)^2]((L-x)^2u+12)\,dx.$$

Integrating (4.11), we obtain the following result.

Theorem 4.3. *Let the initial function satisfies the condition*

$$J(0) = \int_0^L (L-x)[20-(L-x)^2]((L-x)^2u_0(x)+12)\,dx > \sqrt{\frac{360L}{k^2}} \tag{4.12}$$

or

$$\int_0^L (L-x)^3[20-(L-x)^2]u_0(x)\,dx > 120L^2\sqrt{\frac{4}{3} - \frac{2L^2}{25} + \frac{L^4}{700}} - 120L^2 + 3L^4,$$

then there is no global in time classical solutions of (1.4). *Moreover, the following lower bound holds*

$$J(t) \geqslant \frac{6\sqrt{10L}}{k}\frac{1 + C\exp(12\sqrt{10L}kt)}{1 - C\exp(12\sqrt{10L}kt)}, \tag{4.13}$$

and, consequently, $\lim_{t\to T_b} J(t) = +\infty$,

$$T \leqslant T_b \leqslant -\frac{1}{12\sqrt{10L}k}\ln C,$$

$$C = \frac{kJ(0) - 6\sqrt{10L}}{kJ(0) + 6\sqrt{10L}}, \quad k^2 = \Big|\frac{160L^3}{3} - \frac{16L^5}{5} + \frac{2L^7}{35}\Big|^{-1}.$$

Using the nonlinearity capacity method with the test function $\varphi(x) = \exp(-\lambda x)$, $\lambda > 1$, we obtain the blow-up result on the halfline. Multiplying both sides of (1.4) equation by $\varphi(x)$ and integrating by parts, we obtain the equality:

$$\frac{d}{dt}\int_0^\infty e^{-\lambda x}[\lambda^2-1]u\,dx = \lambda^3\int_0^\infty e^{-\lambda x}u\,dx + \frac{\lambda}{2}\int_0^\infty e^{-\lambda x}u^2\,dx. \tag{4.14}$$

Similarly to the bounded domain, forming perfect square and adding time-independent function, we obtain that

$$\frac{d}{dt}\int_0^{+\infty} e^{-\lambda x}[\lambda^2-1](u+\lambda^2)\,dx = \frac{\lambda}{2}\int_0^{+\infty}(u+\lambda^2)^2 e^{-\lambda x}\,dx - \frac{\lambda^5}{2}\int_0^{+\infty} e^{-\lambda x}\,dx. \tag{4.15}$$

It is not hard to prove the following chain of inequalities:

$$\begin{aligned}\Big|\int_0^{+\infty}(u+\lambda^2)e^{-\lambda x}\,dx\Big|^2 &\leqslant \int_0^{+\infty}(u+\lambda^2)^2 e^{-\lambda x}\,dx\int_0^{+\infty} e^{-\lambda x}\,dx\\ &= \frac{1}{\lambda}\int_0^{+\infty}(u+\lambda^2)^2 e^{-\lambda x}\,dx.\end{aligned} \tag{4.16}$$

Thus, we can rewrite (4.15) as

$$\frac{dJ(t)}{dt} \geqslant \frac{\lambda^2}{2(\lambda^2-1)}J^2 - \frac{\lambda^4}{2(\lambda^2-1)}, \tag{4.17}$$

where

$$J(t) = \int_0^{+\infty}(u+\lambda^2)e^{-\lambda x}\,dx.$$

After integrating (4.17), we can formulate the following statement.

Theorem 4.4. *Suppose that the initial data satisfies the condition*

$$J(0) = \int_0^{+\infty}(u_0(x)+\lambda^2)e^{-\lambda x}\,dx > 1, \quad \lambda > 1\,. \tag{4.18}$$

Then the classical solution of (1.4), (4.2) *does not exist globally in time. Furthermore, the following inequality holds*

$$J(t) \geqslant \frac{1+C\exp(\lambda^3 t/(\lambda^2-1))}{1-C\exp(\lambda^3 t/(\lambda^2-1))}, \tag{4.19}$$

consequently,

$$\lim_{t\to T_b} J(t) = +\infty, \quad T_b \leqslant -\frac{\lambda^2-1}{\lambda^3}\ln C, \quad C = \frac{J(0)-\lambda}{J(0)+\lambda}. \tag{4.20}$$

Conclusion. This article shows that the nonlinear capacity method allow us to study not only standard Kortweg-de Vries equations, but also other problems of modern mathematical physics: problems for Benjamin-Bona-Mahony-Burgers, Rosenau-Burgers and Korteweg-de Vries-Benjamin-Bona-Mahony equations. Moreover, it is possible to unite the proof of blow-up in problems with nonclassical boundary conditions and the existence of correct blow-up solutions.

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