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ULAM-HYERS-RASSIAS STABILITY OF SEMILINEAR DIFFERENTIAL EQUATIONS WITH IMPULSES

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Abstract. In this article, we present Ulam-Hyers-Rassias and Ulam-Hyers stability results for semilinear differential equations with impulses on a compact interval. An example is also provided to illustrate our results.

1. Introduction

Many researchers paid attention to the stability properties of all kinds of equations since Ulam [23] raised the famous stability problem of functional equations (Ulam problem) in 1940. Such problems have been taken up by Hyers [8], Rassias [17] and other mathematicians. Recently, the study of this area has the grown to be one of the most important subjects in the mathematical analysis area. For the advanced contribution on Ulam problem, we refer the reader to András and Kolumbán [1], András and Mészáros [2], Burger et al [4], Cădariu [5], Cimpean and Popa [6], Hyers [9], Hegyi and Jung [7], Jung [10, 11], Lungu and Popa [12], Miura et al [13, 14], Obłoza [15, 16], Rassias [18, 19], Rus [20, 21], Takahasi et al [22] and Wang et al [24].

However, Ulam-Hyers-Rassias stability of semilinear differential equations with impulses have not been studied. Motivated by recent works [21, 24], we investigate Ulam-Hyers-Rassias stability of the following semilinear differential equations with impulses

$$\begin{gathered} x'(t) = \lambda x(t) + f(t, x(t)), \quad t \in J' := J \setminus \{t_1, \dots, t_m\},\ J := [0, T], \\ \Delta x(t_k) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \end{gathered} \tag{1.1}$$

where $0 < T < +\infty$, $\lambda > 0$, $f : J \times \mathbb{R} \to \mathbb{R}$ is continuous, $I_k : \mathbb{R} \to \mathbb{R}$ and t_k satisfy $0 = t_0 < t_1 < \dots < t_m < t_{m+1} = T$, $\Delta x(t_k) := x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{\epsilon \to 0^+} x(t_k + \epsilon)$ and $x(t_k^-) = \lim_{\epsilon \to 0^-} x(t_k + \epsilon)$ represent the right and left limits of $x(t)$ at $t = t_k$.

We will adopt the concepts in Wang et al [24] and introduce four types of Ulam stabilities (see Definitions 3.1–3.4): Ulam-Hyers stability, generalized Ulam-Hyers stability, Ulam-Hyers-Rassias stability and generalized Ulam-Hyers-Rassias stability for the equation (1.1). Next, we present the Ulam-Hyers-Rassias stability results

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for the equation (1.1) on a compact interval by virtue of an integral inequality of Gronwall type for piecewise continuous functions (see Lemma 2.2).

2. Preliminaries

In this section, we introduce some notation, and preliminary facts. Throughout this paper, let $C(J,\mathbb{R})$ be the Banach space of all continuous functions from J into $\mathbb{R}$ with the norm $\|x\|_C := \sup\{|x(t)| : t \in J\}$ for $x \in C(J,\mathbb{R})$. We also introduce the Banach space $PC(J,\mathbb{R}) := \{x : J \to \mathbb{R} : x \in C((t_k, t_{k+1}],\mathbb{R}), k = 0,\dots,m$ and there exist $x(t_k^-)$ and $x(t_k^+), k = 1,\dots,m,$ with $x(t_k^-) = x(t_k)\}$ with the norm $\|x\|_{PC} := \sup\{|x(t)| : t \in J\}$. Denote $PC^1(J,\mathbb{R}) := \{x \in PC(J,\mathbb{R}) : x' \in PC(J,\mathbb{R})\}$. Set $\|x\|_{PC^1} := \max\{\|x\|_{PC}, \|x'\|_{PC}\}$. It can be seen that endowed with the norm $\|\cdot\|_{PC^1}$, $PC^1(J,\mathbb{R})$ is also a Banach space.

Definition 2.1. By a PC^1-solution of the impulsive Cauchy problem

$$\begin{gathered} x'(t) = \lambda x(t) + f(t, x(t)), \quad t \in J', \\ \Delta x(t_k) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \\ x(0) = x_0, \quad x_0 \in \mathbb{R}, \end{gathered} \tag{2.1}$$

we mean the function $x \in PC^1(J,\mathbb{R})$ that satisfies

$$x(t) = e^{\lambda t}x_0 + \int_0^t e^{\lambda(t-s)} f(s, x(s))ds + \sum_{0<t_k<t} e^{\lambda(t-t_k)} I_k(x(t_k^-)), \quad t \in J.$$

Bainov and Hristova [3] studied the following integral inequality of Gronwall type for piecewise continuous functions which can be used in the sequel.

Lemma 2.2. *Let for $t \ge t_0 \ge 0$ the following inequality hold*

$$x(t) \le a(t) + \int_{t_0}^t g(t, s)x(s)ds + \sum_{t_0<t_k<t} \beta_k(t)x(t_k), \tag{2.2}$$

where $\beta_k(t)(k \in \mathbb{N})$ are nondecreasing functions for $t \ge t_0$, $a \in PC([t_0,\infty),\mathbb{R}_+)$, a is nondecreasing and $g(t, s)$ is a continuous nonnegative function for $t, s \ge t_0$ and nondecreasing with respect to t for any fixed $s \ge t_0$. Then, for $t \ge t_0$, the following inequality is valid:

$$x(t) \le a(t) \prod_{t_0<t_k<t} (1 + \beta_k(t)) \exp\Big(\int_{t_0}^t g(t, s)ds\Big).$$

3. Basic concepts and remarks

Here, we adopt the concepts in Wang et al [24] and introduce Ulam's type stability concepts for the equation (1.1).

Let $\epsilon > 0$, $\psi \ge 0$ and $\varphi \in PC(J,\mathbb{R}_+)$ is nondecreasing. We consider the set of inequalities

$$\begin{gathered} |y'(t) - \lambda y(t) - f(t, y(t))| \le \epsilon, \quad t \in J', \\ |\Delta y(t_k) - I_k(y(t_k^-))| \le \epsilon, \quad k = 1, 2, \dots, m; \end{gathered} \tag{3.1}$$

the set of inequalities

$$\begin{gathered} |y'(t) - \lambda y(t) - f(t, y(t))| \le \varphi(t), \quad t \in J', \\ |\Delta y(t_k) - I_k(y(t_k^-))| \le \psi, \quad k = 1, 2, \dots, m; \end{gathered} \tag{3.2}$$

and the set of inequalities

$$\begin{aligned} &|y'(t) - \lambda y(t) - f(t, y(t))| \le \epsilon\varphi(t), \quad t \in J', \\ &|\Delta y(t_k) - I_k(y(t_k^-))| \le \epsilon\psi, \quad k = 1, 2, \dots, m. \end{aligned} \tag{3.3}$$

Definition 3.1. Equation (1.1) is Ulam-Hyers stable if there exists a real number $c_{f,m} > 0$ such that for each $\epsilon > 0$ and for each solution $y \in PC^1(J, \mathbb{R})$ of the inequality (3.1) there exists a solution $x \in PC^1(J, R)$ of the equation (1.1) with

$$|y(t) - x(t)| \le c_{f,m}\epsilon, \quad t \in J.$$

Definition 3.2. Equation (1.1) is generalized Ulam-Hyers stable if there exists $\theta_{f,m} \in C(\mathbb{R}_+, \mathbb{R}_+)$, $\theta_{f,m}(0) = 0$ such that for each solution $y \in PC^1(J, \mathbb{R})$ of the inequality (3.1) there exists a solution $x \in PC^1(J, \mathbb{R})$ of the equation (1.1) with

$$|y(t) - x(t)| \le \theta_{f,m}(\epsilon), \quad t \in J.$$

Definition 3.3. Equation (1.1) is Ulam-Hyers-Rassias stable with respect to (φ, ψ) if there exists $c_{f,m,\varphi} > 0$ such that for each $\epsilon > 0$ and for each solution $y \in PC^1(J, \mathbb{R})$ of the inequality (3.3) there exists a solution $x \in PC^1(J, \mathbb{R})$ of the equation (1.1) with

$$|y(t) - x(t)| \le c_{f,m,\varphi}\epsilon(\varphi(t) + \psi), \quad t \in J.$$

Definition 3.4. The equation (1.1) is generalized Ulam-Hyers-Rassias stable with respect to (φ, ψ) if there exists $c_{f,m,\varphi} > 0$ such that for each solution $y \in PC^1(J, \mathbb{R})$ of the inequality (3.2) there exists a solution $x \in PC^1(J, \mathbb{R})$ of the equation (1.1) with

$$|y(t) - x(t)| \le c_{f,m,\varphi}(\varphi(t) + \psi), \ t \in J.$$

Remark 3.5. It is clear that: (i) Definition 3.1 implies Definition 3.2; (ii) Definition 3.3 implies Definition 3.4; (iii) Definition 3.3 for $\varphi(t) = \psi = 1$ implies Definition 3.1.

Remark 3.6. A function $y \in PC^1(J, \mathbb{R})$ is a solution of the inequality (3.1) if and only if there is $g \in PC(J, \mathbb{R})$ and a sequence g_k, $k = 1, 2, \dots, m$ (which depend on y) such that

(i) $|g(t)| \le \epsilon\varphi(t)$, $t \in J$ and $|g_k| \le \epsilon\psi$, $k = 1, 2, \dots, m$;
(ii) $y'(t) = f(t, y(t)) + g(t)$, $t \in J'$;
(iii) $\Delta y(t_k) = I_k(y(t_k^-)) + g_k$, $k = 1, 2, \dots, m$.

One can have similar remarks for inequalities (3.2) and (3.1).

Remark 3.7. If $y \in PC^1(J, \mathbb{R})$ is a solution of the inequality (3.3) then y is a solution of the integral inequality

$$\begin{aligned} &\Big|y(t) - e^{\lambda t}y(0) - \sum_{i=1}^{k} e^{\lambda(t-t_i)}I_i(y(t_i^-)) - \int_0^t e^{\lambda(t-s)}f(s, y(s))ds\Big| \\ &\le e^{\lambda t}m\epsilon\psi + \epsilon\int_0^t e^{\lambda(t-s)}\varphi(s)ds, \quad t \in J. \end{aligned} \tag{3.4}$$

In fact, by Remark 3.6 we have

$$\begin{aligned} y'(t) &= f(t, y(t)) + g(t), \quad t \in J', \\ \Delta y(t_k) &= I_k(y(t_k^-)) + g_k, \quad k = 1, 2, \dots, m. \end{aligned} \tag{3.5}$$

Clearly, the solution of (3.5) is given by

$$y(t) = e^{\lambda t}y(0) + \sum_{i=1}^{k} e^{\lambda(t-t_i)} I_i(y(t_i^-)) + \sum_{i=1}^{k} e^{\lambda(t-t_i)} g_i + \int_0^t e^{\lambda(t-s)} f(s, y(s))ds + \int_0^t e^{\lambda(t-s)} g(s)ds,\ t \in (t_k, t_{k+1}].$$

From this it follows that

$$\begin{aligned} &\Big|y(t) - e^{\lambda t}y(0) - \sum_{i=1}^{k} e^{\lambda(t-t_i)} I_i(y(t_i^-)) - \int_0^t e^{\lambda(t-s)} f(s, y(s))ds\Big| \\ &\le \sum_{i=1}^{m} e^{\lambda(t-t_i)}|g_i| + \int_0^t e^{\lambda(t-s)}|g(s)|ds \\ &\le e^{\lambda t} m\epsilon\psi + \epsilon \int_0^t e^{\lambda(t-s)}\varphi(s)ds. \end{aligned}$$

Clearly, one can give similar remarks for the solutions of the inequalities (3.2) and (3.1).

4. Ulam-Hyers-Rassias stability results

We use the following assumptions:

- (H1) $f \in C(J \times \mathbb{R}, \mathbb{R})$.
- (H2) There exists $L_f(\cdot) \in C(J, \mathbb{R}_+)$ such that
$$|f(t, u_1) - f(t, u_2)| \le L_f(t)|u_1 - u_2|, \quad \text{for each } t \in J \text{ and all } u_1, u_2 \in \mathbb{R}.$$
- (H3) There exists $\rho_k > 0$ such that $|I_k(u_1) - I_k(u_2)| \le \rho_k|u_1 - u_2|$ for each $u_1, u_2 \in \mathbb{R}$ and $k = 1, 2, \dots, m$.
- (H4) Let $\varphi \in C(J, \mathbb{R}_+)$ be a nondecreasing function. There exists $c_\varphi > 0$ such that
$$\int_0^t \varphi(s)ds \le c_\varphi \varphi(t), \quad \text{for each } t \in J.$$

Now, we are ready to state the following Ulam-Hyers-Rassias stable result.

Theorem 4.1. *Assume that* (H1)–(H4) *are satisfied. Then* (1.1) *is Ulam-Hyers-Rassias stable with respect to* (φ, ψ).

Proof. Let $y \in PC^1(J, \mathbb{R})$ be a solution of the inequality (3.3). Denote by x the unique solution of the impulsive Cauchy problem

$$\begin{gathered} x'(t) = \lambda x(t) + f(t, x(t)), \quad t \in J', \\ \Delta x(t_k) = I_k(x(t_k^-)), \quad k = 1, 2, \dots, m, \\ x(0) = y(0). \end{gathered} \tag{4.1}$$

Then we have

$$x(t) = \begin{cases} e^{\lambda t}y(0) + \int_0^t e^{\lambda(t-s)} f(s, x(s))ds, & \text{for } t \in [0, t_1], \\ e^{\lambda t}y(0) + e^{\lambda(t-t_1)} I_1(x(t_1^-)) + \int_0^t e^{\lambda(t-s)} f(s, x(s))ds, & \text{for } t \in (t_1, t_2], \\ \dots \\ e^{\lambda t}y(0) + \sum_{k=1}^{m} e^{\lambda(t-t_k)} I_k(x(t_k^-)) + \int_0^t e^{\lambda(t-s)} f(s, x(s))ds, & \text{for } t \in (t_m, T]. \end{cases}$$

By (3.4), for each $t \in (t_k, t_{k+1}]$, we have

$$\begin{aligned}
&\Big|y(t) - e^{\lambda t}y(0) - \sum_{i=1}^{k} e^{\lambda(t-t_i)} I_i(y(t_i^-)) - \int_0^t e^{\lambda(t-s)} f(s, y(s))ds\Big| \\
&\le e^{\lambda t} m\epsilon\psi + \epsilon \int_0^t e^{\lambda(t-s)}\varphi(s)ds \\
&\le \varepsilon e^{\lambda T}(m + c_\varphi)[\psi + \varphi(t)].
\end{aligned}$$

Hence for each $t \in (t_k, t_{k+1}]$, it follows that

$$\begin{aligned}
&|y(t) - x(t)| \\
&\le \Big|y(t) - e^{\lambda t}y(0) - \sum_{i=1}^{k} e^{\lambda(t-t_i)} I_i(y(t_i^-)) - \int_0^t e^{\lambda(t-s)} f(s, y(s))ds\Big| \\
&\quad + \sum_{i=1}^{k} e^{\lambda(t-t_i)} |I_i(y(t_i^-)) - I_i(x(t_i^-))| + \int_0^t e^{\lambda(t-s)} |f(s, y(s)) - f(s, x(s))|ds \\
&\le \varepsilon e^{\lambda T}(m + c_\varphi)[\psi + \varphi(t)] + \int_0^t e^{\lambda(t-s)} L_f(s)|y(s) - x(s)|ds \\
&\quad + e^{\lambda T} \sum_{i=1}^{k} \rho_i |y(t_i^-) - x(t_i^-)|.
\end{aligned}$$

By Lemma 2.2, we obtain

$$\begin{aligned}
|y(t) - x(t)| &\le \varepsilon e^{\lambda T}(m + c_\varphi)[\psi + \varphi(t)]\Big(\prod_{0<t_k<t} (1 + \rho_k e^{\lambda t}) e^{\int_0^t e^{\lambda(t-s)} L_f(s)ds}\Big) \\
&\le c_{f,m,\varphi}\varepsilon(\varphi(t) + \psi), \quad t \in J,
\end{aligned}$$

where

$$c_{f,m,\varphi} := e^{\lambda T}(m + c_\varphi) \prod_{k=1}^{m} (1 + \rho_k e^{\lambda T}) e^{e^{\lambda T} \int_0^T L_f(s)ds} > 0.$$

Thus, (1.1) is Ulam-Hyers-Rassias stable with respect to (φ, ψ). The proof is complete. □

Next, we replace (H3) by

(H3*) There exist nondecreasing functions $\rho_k \in C(\mathbb{R}_+, \mathbb{R}_+)$, with $\rho_k(0) = 0$ such that
$$|I_k(u_1) - I_k(u_2)| \le \rho_k(|u_1 - u_2|),$$
for each $u_1, u_2 \in \mathbb{R}$ and $k = 1, 2, \dots, m$.

Next, we present the following Ulam-Hyers stable result.

Theorem 4.2. *Assume that* (H1), (H2) *and* (H3*) *are satisfied. Then* (1.1) *is generalized Ulam-Hyers stable.*

Proof. From the proof in Theorem 4.1, we are led to the inequality

$$\begin{aligned}
|v(t)| &\le \varepsilon e^{\lambda T}(m + T) + e^{\lambda T} \int_0^t L_f(s)|v(s)|ds \\
&\quad + e^{\lambda T} \sum_{i=1}^{k} \rho_i(|v(t_i^-)|), \quad t \in (t_k, t_{k+1}],
\end{aligned} \tag{4.2}$$

where $v(t) := y(t) - x(t)$.

Let $M_k := \sup_{t\in[t_k,t_{k+1}]}\{|v(t)|\}$ for $k = 0,\dots,m$. Then the inequality (4.2) implies

$$|v(t)| \le (m+T)e^{\lambda T}\epsilon + e^{\lambda T}\int_0^t L_f(s)\,|v(s)|\,ds + e^{\lambda T}\sum_{i=1}^{k}\rho_i(M_{i-1})$$

for $t \in (t_k, t_{k+1}]$. Using the standard Gronwall inequality we obtain

$$M_k \le e^{\lambda T}\Big((m+T)\epsilon + \sum_{i=1}^{k}\rho_i(M_{i-1})\Big)e^{e^{\lambda T}\int_0^T L_f(s)ds}. \tag{4.3}$$

Setting

$$\theta_0(\epsilon) = (m+T)\epsilon e^{e^{\lambda T}\int_0^T L_f(s)ds},$$
$$\theta_k(\epsilon) = \Big((m+T)\epsilon + \sum_{i=1}^{k}\rho_i(e^{\lambda T}\theta_{i-1}(\epsilon))\Big)e^{e^{\lambda T}\int_0^T L_f(s)ds}, \quad k = 1,\dots,m.$$

Obviously, the inequality (4.3) implies

$$M_k \le e^{\lambda T}\theta_k(\epsilon), \quad k = 0,\dots,m.$$

Let $\theta_{f,m}(\epsilon) = \max\{e^{\lambda T}\theta_k(\epsilon) : k = 0,\dots,m\}$. Hence

$$|v(t)| \le \theta_{f,m}(\epsilon).$$

Clearly $\theta_{f,m} \in C(\mathbb{R}_+,\mathbb{R}_+)$ and $\theta_{f,m}(0) = 0$. Thus, the equation (1.1) is generalized Ulam-Hyers stable. The proof is complete. □

5. Example

Let $\lambda = 1$, $\varphi(t) = t$, $\psi = 1$. We consider the linear impulsive ordinary differential equation

$$\begin{gathered} x'(t) = x(t), \quad t \in [0,2]\setminus\{1\}, \\ \Delta x(1) = \frac{|x(1^-)|}{1+|x(1^-)|}, \end{gathered} \tag{5.1}$$

and the inequalities

$$\begin{gathered} |y'(t) - y(t)| \le \epsilon t, \quad t \in ([0,2]\setminus\{1\}, \\ \Big|\Delta y(1) - \frac{|y(1^-)|}{1+|y(1^-)|}\Big| \le \epsilon, \quad \epsilon > 0. \end{gathered} \tag{5.2}$$

Let $y \in PC^1([0,2],\mathbb{R})$ be a solution of inequality (5.2). Then there exist $g \in PC^1([0,2],\mathbb{R})$ and $g_1 \in \mathbb{R}$ such that:

$$|g(t)| \le \epsilon t, \quad t \in [0,2],\ |g_1| \le \epsilon, \tag{5.3}$$
$$y'(t) = y(t) + g(t), \quad t \in [0,2]\setminus\{1\}, \tag{5.4}$$
$$\Delta y(1) = \frac{|y(1^-)|}{1+|y(1^-)|} + g_1. \tag{5.5}$$

Integrating (5.4) from 0 to t via (5.5), we have

$$y(t) = e^t y(0) + \chi_{(1,2]}(t)e^{t-1}\Big(\frac{|y(1^-)|}{1+|y(1^-)|} + g_1\Big) + \int_0^t e^{t-s}g(s)ds,$$

for the characteristic function $\chi_{(1,2]}(t)$ on $(1,2]$.

Let us consider the solution x of (5.1) given by

$$x(t) = e^t y(0) + \chi_{(1,2]}(t) e^{t-1} \frac{|x(1^-)|}{1+|x(1^-)|}.$$

We have

$$\begin{aligned}
|y(t)-x(t)| &= \Big|\chi_{(1,2]}(t)e^{t-1}\Big(\frac{|y(1^-)|}{1+|y(1^-)|} - \frac{|x(1^-)|}{1+|x(1^-)|} + g_1\Big) + \int_0^t e^{t-s} g(s)ds\Big| \\
&\le e^t|y(1^-)-x(1^-)| + e^t|g_1| + e^t\int_0^t |g(s)|ds \\
&\le e^t|y(1^-)-x(1^-)| + e^t\epsilon + \epsilon e^t\int_0^t sds \\
&\le e^t|y(1^-)-x(1^-)| + e^t\epsilon + \epsilon e^t\frac{1}{2}t^2 \\
&\le e^t|y(1^-)-x(1^-)| + e^t\epsilon(1+t),\ t\in[0,2],
\end{aligned}$$

which gives

$$|y(t)-x(t)| \le e^2(1+e^2)\epsilon(t+1), \quad t\in[0,2].$$

Thus, (5.1) is Ulam-Hyers-Rassias stable with respect to $(t,1)$.

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