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EXISTENCE OF SOLUTIONS FOR CRITICAL ELLIPTIC SYSTEMS WITH BOUNDARY SINGULARITIES

JIANFU YANG, LINLI WU

Abstract. This article concerns the existence of positive solutions to the nonlinear elliptic system involving critical Hardy-Sobolev exponent

$$\begin{gathered}
-\Delta u = \frac{2\lambda\alpha}{\alpha+\beta}\frac{u^{\alpha-1}v^{\beta}}{|\pi(x)|^s} - u^p, \quad \text{in } \Omega, \\
-\Delta v = \frac{2\lambda\beta}{\alpha+\beta}\frac{u^{\alpha}v^{\beta-1}}{|\pi(x)|^s} - v^p, \quad \text{in } \Omega, \\
u > 0, \quad v > 0, \quad \text{in } \Omega, \\
u = v = 0, \quad \text{on } \partial\Omega,
\end{gathered}$$

where $N \geq 4$ and Ω is a C^1 bounded domain in $\mathbb{R}^N$, $0 < s < 2$, $\alpha + \beta = 2^*(s) = \frac{2(N-s)}{N-2}$, $\alpha, \beta > 1$, $\lambda > 0$ and $1 \leq p < \frac{N}{N-2}$.

Let $\mathcal{P}$ be a linear subspace of $\mathbb{R}^N$ such that $k = \dim_{\mathbb{R}} \mathcal{P} \geq 2$, and π be the orthogonal projection on $\mathcal{P}$ with respect to the Euclidean structure. We consider mainly the case when $\mathcal{P}^\perp \cap \Omega = \emptyset$ and $\mathcal{P}^\perp \cap \partial\Omega \neq \emptyset$. We show that there exists $\lambda^* > 0$ such that the system above possesses at least one positive solution for $0 < \lambda < \lambda^*$ provided that at each point $x \in \mathcal{P}^\perp \cap \partial\Omega$ the principal curvatures of $\partial\Omega$ at x are non-positive, but not all vanish.

1. Introduction

Let $\mathcal{P}$ be a linear subspace of $\mathbb{R}^N$ such that $k = \dim_{\mathbb{R}} \mathcal{P} \geq 2$, and π be the orthogonal projection on $\mathcal{P}$ with respect to the Euclidean structure. In this paper, we are concerned with the existence of positive solutions of the following nonlinear elliptic system involving critical Hardy-Sobolev exponent

$$\begin{gathered}
-\Delta u = \frac{2\lambda\alpha}{\alpha+\beta}\frac{u^{\alpha-1}v^{\beta}}{|\pi(x)|^s} - u^p, \quad \text{in } \Omega, \\
-\Delta v = \frac{2\lambda\beta}{\alpha+\beta}\frac{u^{\alpha}v^{\beta-1}}{|\pi(x)|^s} - v^p, \quad \text{in } \Omega, \\
u > 0, v > 0, \quad \text{in } \Omega, \\
u = v = 0, \quad \text{on } \partial\Omega,
\end{gathered} \tag{1.1}$$

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where $N \geq 4$ and Ω is a C^1 bounded domain in $\mathbb{R}^N$. We assume in this paper that $0 < s < 2$, $\alpha + \beta = 2^*(s) = \frac{2(N-s)}{N-2}$, $\alpha, \beta > 1$, $\lambda > 0$ and $1 < p < \frac{N}{N-2}$.

For the one equation case, if $k = N$, the problem is related to the Caffarelli-Kohn-Nirenberg inequalities. It was discussed in [5] the existence of minimizer of the best constant of the Caffarelli-Kohn-Nirenberg inequalities and related subjects. In particular, it shows that if $0 \in \Omega$, the best Hardy-Sobolev constant

$$\mu_{2^*(s),s}(\Omega) = \inf_{u \in H_0^1(\Omega)\setminus\{0\}} \frac{\int_\Omega |\nabla u|^2 dx}{\left(\int_\Omega \frac{u^{2^*(s)}}{|x|^s} dx\right)^{2/2^*(s)}} \tag{1.2}$$

is never attained unless $\Omega = \mathbb{R}^N$ and $\mu_{2^*(s),s}(\Omega) = \mu_{2^*(s),s}(\mathbb{R}^N)$. If $s = 0$, the quantity $\mu_{2^*(s),s}(\Omega)$ is the best Sobolev constant

$$S = S(\Omega) = \inf_{u \in H_0^1(\Omega)\setminus\{0\}} \frac{\int_\Omega |\nabla u|^2 dx}{\left(\int_\Omega |u|^{2^*} dx\right)^{2/2^*}},$$

where $2^* = \frac{2N}{N-2}$ is the critical Sobolev exponent and S is achieved if and only if $\Omega = \mathbb{R}^N$, see [19]. Related results can also be found in [8] and [18].

In contrast with the case $0 \in \Omega$, if $0 \in \partial\Omega$, the problem is closely related to the properties of the curvature of $\partial\Omega$ at 0. Ghoussoub and Kang showed in [9] that there exists a solution of the problem

$$-\Delta u = \frac{u^{2^*(s)-1}}{|x|^s} + \lambda u^p, \quad u > 0 \quad \text{in } \Omega; \quad u = 0, \quad \text{on } \partial\Omega,$$

where $\lambda > 0$, $1 < p < \frac{N+2}{N-2}$, $0 \in \partial\Omega$ and the mean curvature of $\partial\Omega$ at 0 is negative. Such a result was proved by the global compactness method. Moreover, Ghoussoub and Robert in [10] have proved that $\mu_{2^*(s),s}(\Omega)$ is achieved if $0 \in \partial\Omega$ and the mean curvature of $\partial\Omega$ at 0 is negative. For the elliptic equation with two critical exponents

$$-\Delta u = \frac{u^{2^*(s)-1}}{|x|^s} + \lambda u^{\frac{N+2}{N-2}}, \quad u > 0 \quad \text{in } \Omega; \quad u = 0, \quad \text{on } \partial\Omega, \tag{1.3}$$

using the blow-up method, Hsai et al [13] prove that problem (1.3) possesses at least one positive solution.

In [17], the Hardy-Sobolev inequality

$$\mu_{2^*(s),\mathcal{P}}\Big(\int_{\mathbb{R}^N} \frac{|u|^{2^*(s)}}{|\pi(x)|^s} dx\Big)^{\frac{2^*(s)}{2}} \leq \int_{\mathbb{R}^N} |\nabla u|^2 dx, \ \forall u \in D^{1,2}(\mathbb{R}^N) \tag{1.4}$$

was established for all $u \in D^{1,2}(\mathbb{R}^N)$. Then, it was shown in [7] that $\mu_{2^*(s),\mathcal{P}}(\Omega) \geq \mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N) > 0$ for all smooth domain $\Omega \subset \mathbb{R}^N$. The attainability of $\mu_{2^*(s),\mathcal{P}}(\Omega)$ depends on the position between $\mathcal{P}$ and Ω, this was discussed in [12].

In this article, we study the existence of positive solutions of (1.1). In [14], positive solutions of problem (1.1) were found in non-contractible domains if $\lambda = 0, k = N$ and $s = 0$. In [20], the existence of sign-changing solutions was obtained for (1.1) with $k = N$ and $s = 0$. For further results for the system we refer the references in [14] and [20].

Equation (1.1) involves the Hardy type potential, that is $s \neq 0$ and possibly, $k \leq N$, and the lower order terms are negative, which will push the energy up.

We will prove that (1.1) possesses at least one positive solution by the blow up argument. The limiting problem after blowing up is as follows.

$$
\begin{gathered}
-\Delta u=\frac{2\alpha}{\alpha+\beta}\frac{u^{\alpha-1}v^{\beta}}{|\pi(x)|^{s}},\quad \text{in } \mathbb{R}^N_+,\\
-\Delta v=\frac{2\beta}{\alpha+\beta}\frac{u^{\alpha}v^{\beta-1}}{|\pi(x)|^{s}},\quad \text{in } \mathbb{R}^N_+,\\
u>0,\quad v>0,\quad \text{in } \mathbb{R}^N_+,\\
u=v=0,\quad \text{on } \partial\mathbb{R}^N_+.
\end{gathered}
\tag{1.5}
$$

Denote

$$
\mu_{\alpha,\beta,\mathcal{P}}(\Omega)=\inf_{(u,v)\in(H_0^1(\Omega))^2\setminus\{0\}}\frac{\int_\Omega(|\nabla u|^2+|\nabla v|^2)dx}{\big(\int_\Omega\frac{u^\alpha v^\beta}{|\pi(x)|^s}dx\big)^{\frac{2}{2^*(s)}}}
\tag{1.6}
$$

for a domain $\Omega\subset\mathbb{R}^N$. The solution of (1.4) will be obtained by showing that $\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)$ is achieved. The minimizer of $\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)$ is the least energy solution of (1.4) up to a multiplicative constant. It was observed in [2] that $\mu_{\alpha,\beta,\mathcal{P}}(\Omega)$ and $\mu_{\alpha+\beta,\mathcal{P}}(\Omega)$ are closely related. Precisely, we have

$$
\mu_{\alpha,\beta,\mathcal{P}}(\Omega)=\big[(\frac{\alpha}{\beta})^{\frac{\beta}{\alpha+\beta}}+(\frac{\alpha}{\beta})^{\frac{-\alpha}{\alpha+\beta}}\big]\mu_{\alpha+\beta,\mathcal{P}}(\Omega)
\tag{1.7}
$$

for $\alpha+\beta\le 2^*$. Moreover, if w_0 realizes $\mu_{\alpha+\beta,s}(\Omega)$, then $u_0=Aw_0$ and $v_0=Bw_0$ realizes $\mu_{\alpha,\beta,\mathcal{P}}(\Omega)$ for any real constants A and B such that $\frac{A}{B}=\sqrt{\frac{\alpha}{\beta}}$.

In the case $\Omega=\mathbb{R}^N_+$, it was proved in [12] that $\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N_+)$ is achieved by a function $u\in H_0^1(\mathbb{R}^N_+)$ provided that $\mathcal{P}^\perp\subset\partial\mathbb{R}^N_+$. This implies that $\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)$ is achieved if $\alpha+\beta=2^*(s)$ and $\mathcal{P}^\perp\subset\partial\mathbb{R}^N_+$. Hence, there exists a least energy entire solution of (1.4) in this case.

To deal with (1.1), we consider a related subcritical problem, and obtain a sequence of solutions of the subcritical problems. Then, we analyse the blow up behavior of the approximating sequence. Since the coefficient of lower order terms are negative, the energy of the corresponding functional becomes larger, it makes difficult to find the upper compact bound. Our main result is as follows.

Theorem 1.1. *Let Ω be a smooth bounded domain of $\mathbb{R}^N$, $N\ge 3$, and let $\mathcal{P}$ be a linear subspace of $\mathbb{R}^N$ such that $k=\dim_{\mathbb{R}}\mathcal{P}\ge 2$. Suppose $s\in(0,2)$, then we have*

- (i) *If $\mathcal{P}^\perp\cap\Omega\ne\emptyset$, problem* (1.1) *possesses at least one positive solution provided that $s=1$.*
- (ii) *If $\mathcal{P}^\perp\cap\bar\Omega=\emptyset$, problem* (1.1) *possesses at least one positive solution.*
- (iii) *If $\mathcal{P}^\perp\cap\Omega=\emptyset$ and $\mathcal{P}^\perp\cap\partial\Omega\ne\emptyset$, there exists $\lambda^*>0$ such that for $0<\lambda<\lambda^*$ problem* (1.1) *possesses at least one positive solution provided that at each point $x\in\mathcal{P}^\perp\cap\partial\Omega$ the principle curvatures of $\partial\Omega$ at x are non-positive, but not all vanish.*

In section 2, we find a suitable upper bound for the mountain pass level and prove (i) and (ii) of Theorem 1.1, then using this bound and the blow-up argument, we prove (iii) of Theorem 1.1 in section 3.

2. Preliminaries

We recall that

$$\mu_{2^*(s),\mathcal{P}}(\Omega)=\inf_{u\in H_0^1(\Omega)\setminus\{0\}}\frac{\int_\Omega|\nabla u|^2\,dx}{\left(\int_\Omega\frac{u^{2^*(s)}}{|\pi(x)|^s}\,dx\right)^{\frac{2}{2^*(s)}}},\tag{2.1}$$

where $2^*(s)=\frac{2(N-2)}{N-2}$, $s\in(0,2)$ and π is the orthogonal projection on $\mathcal{P}$ with respect to the Euclidean structure. The attainability of $\mu_{2^*(s),\mathcal{P}}(\Omega)$ depends on the position between Ω and $\mathcal{P}$. Actually, it was proved in [12] that if $\mathcal{P}^\perp\cap\Omega\neq\emptyset$, $\mu_{2^*(s),\mathcal{P}}(\Omega)=\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N)$. Therefore, $\mu_{2^*(s),\mathcal{P}}(\Omega)$ is not achieved. If $\mathcal{P}^\perp\cap\bar\Omega=\emptyset$, the problem becomes subcritical without singularities, thus $\mu_{2^*(s),\mathcal{P}}(\Omega)$ is attained. Finally, if $\mathcal{P}^\perp\cap\Omega=\emptyset$ and $\mathcal{P}^\perp\cap\partial\Omega\neq\emptyset$, $\mu_{2^*(s),\mathcal{P}}(\Omega)$ is achieved provided that the principle curvatures of $\partial\Omega$ at $x\in\mathcal{P}^\perp\cap\partial\Omega$ are non-positive, and do not all vanish. Furthermore, the following lemma was also shown in [12].

Lemma 2.1. *There exists a minimizer $u\in C^1(\bar{\mathbb{R}}^N_+)\cap H_0^1(\mathbb{R}^N_+)$ of $\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N_+)$ such that*

$$\begin{gathered}-\Delta u=\frac{u^{2^*(s)-1}}{|\pi(x)|^s}\quad\text{in } \mathbb{R}^N_+,\\ u>0\quad\text{in } \mathbb{R}^N_+,\quad u=0\quad\text{on } \partial\mathbb{R}^N_+\end{gathered}\tag{2.2}$$

in $\mathcal{D}'(\mathbb{R}^N_+)$ satisfying $\int_{\mathbb{R}^N_+}|\nabla u|^2\,dx=\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}}$, provided that $2\le k\le N-1$ and $\mathcal{P}^\perp\subset\partial\mathbb{R}^N_+$.

Let $u\in C^1(\bar{\mathbb{R}}^N_+)\cap H_0^1(\mathbb{R}^N_+)$ be the minimizer of $\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N_+)$. We have the following estimates.

Lemma 2.2. *There exists $C>0$ such that*

$$|u(x)|\le C(1+|x|)^{1-N},\quad|\nabla u(x)|\le C(1+|x|)^{-N}\tag{2.3}$$

for $x\in\mathbb{R}^N_+$.

Proof. Let

$$u^*(x)=|x|^{-(N-2)}u(\frac{x}{|x|^2}),\quad x\in\mathbb{R}^N_+,$$

be the Kelvin transformation of u. Since $u\in D_0^{1,2}(\mathbb{R}^N_+)$, we may verify that the u^* also satisfies equation (2.2), and both $\int_{\mathbb{R}^N_+}|\nabla u^*|^2\,dx$ and $\int_{\mathbb{R}^N_+}\frac{|u^*|^{2^*(s)}}{|\pi(x)|^s}\,dx$ are finite.

Next, by a regularity result in [12], $u^*\in C^1(\bar{\mathbb{R}}^N_+)$. It implies in a standard way that (2.3) holds. The proof is complete. □

By (1.7), we see that $\mu_{\alpha,\beta,\mathcal{P}}(\Omega)$ and $\mu_{2^*(s),\mathcal{P}}(\Omega)$ are closely related if $\alpha+\beta=2^*(s)$, which and Lemma 2.2 allow us to state the following result.

Proposition 2.3. *Suppose $\alpha+\beta=2^*(s)$. Then*

(i) *$\mu_{\alpha,\beta,\mathcal{P}}(\Omega)=\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N)$ if $\mathcal{P}^\perp\cap\Omega\neq\emptyset$, and $\mu_{\alpha,\beta,\mathcal{P}}(\Omega)$ is not achieved.*
(ii) *If $\mathcal{P}^\perp\cap\bar\Omega=\emptyset$, $\mu_{\alpha,\beta,\mathcal{P}}(\Omega)$ is attained.*
(iii) *If $\mathcal{P}^\perp\cap\Omega=\emptyset$ and $\mathcal{P}^\perp\cap\partial\Omega\neq\emptyset$, $\mu_{\alpha,\beta,\mathcal{P}}(\Omega)$ is achieved provided that the principle curvatures of $\partial\Omega$ at $x\in\mathcal{P}^\perp\cap\partial\Omega$ are non-positive, and do not all vanish.*

Moreover, all components of the minimizer of $\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)$ satisfy the decaying law in (2.3).

Proof of (i) and (ii) of Theorem 1.1. In the case (i), problem (1.1) is a critical problem with singularities in Ω. The existence of positive solution of the problem can be proved by the mountain pass theorem as [1], [4]. In the case (ii), problem (1.1) is a subcritical problem without singularities, the result is readily obtained. □

In the rest of the paper, we only consider the case (iii); that is, we assume that $\mathcal{P}^\perp \cap \Omega = \emptyset$ and $\mathcal{P}^\perp \cap \partial\Omega \neq \emptyset$.

In the following, we establish the upper bound for the mountain pass level. We recall that by [10], $\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N_+)$ is achieved by a function $u \in H^1_0(\mathbb{R}^N_+)$ if $\mathcal{P}^\perp \subset \partial\mathbb{R}^N_+$. This implies that $\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)$ is achieved if $\alpha+\beta = 2^*(s)$. Hence, there exists a least energy entire solution of system (1.4).

The energy functional for (1.1) is

$$I_\lambda(u,v) = \int_\Omega \Big(\frac{1}{2}|\nabla u|^2 + \frac{1}{2}|\nabla v|^2 - \frac{2\lambda}{2^*(s)}\frac{u^\alpha v^\beta}{|\pi(x)|^s} + \frac{1}{p+1}u^{p+1} + \frac{1}{p+1}v^{p+1}\Big)\,dx\,,$$

which is well defined on $H^1_0(\Omega)$. It is well known that to find positive solutions of problem (1.1) is equivalent to find nonzero critical points of functional I_λ in $H^1_0(\Omega) \times H^1_0(\Omega)$. Now, we bound the mountain pass level for the functional I_λ.

Lemma 2.4. *Suppose that Ω is a C^1 bounded domain in $\mathbb{R}^N$ with $\mathcal{P}^\perp \cap \Omega = \emptyset$ and $\mathcal{P}^\perp \cap \partial\Omega \neq \emptyset$. There exist $\lambda^* > 0$ and nonnegative functions u_0 and v_0 in $H^1_0(\Omega) \setminus \{0\}$ such that for $0 < \lambda < \lambda^*$ and $1 \le p < \frac{N}{N-2}$, we have $I_\lambda(u_0, v_0) < 0$ and*

$$\max_{0\le t\le 1} I_\lambda(tu_0, tv_0) < (2\lambda)^{\frac{-2}{2^*(s)-2}}\Big(\frac{1}{2} - \frac{1}{2^*(s)}\Big)\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}}.$$

provided that the principle curvatures of $\partial\Omega$ at $x \in \mathcal{P}^\perp \cap \partial\Omega$ are non-positive, and do not all vanish.

Proof. Let (u,v) be the minimizer of $\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)$, such that

$$\int_{\mathbb{R}^N_+} |\nabla u|^2\,dx + \int_{\mathbb{R}^N_+} |\nabla v|^2\,dx = \mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+), \qquad \int_{\mathbb{R}^N_+} \frac{u^\alpha v^\beta}{|\pi(x)|^s}\,dx = 1.$$

Then, there exist A, $B \in \mathbb{R}$ such that $u = Aw$, $v = Bw$ with $\frac{A}{B} = \sqrt{\frac{\alpha}{\beta}}$, where w is a minimizer of $\mu_{2^*(s),\mathcal{P}}(\mathbb{R}^N_+)$. Since

$$|w(x)| \le C(1+|x|)^{1-N}, \qquad |\nabla w(x)| \le C(1+|x|)^{-N},$$

we obtain

$$|u(x)| \le C(1+|x|)^{1-N}, \quad |\nabla u(x)| \le C(1+|x|)^{-N}, \tag{2.4}$$

$$|v(x)| \le C(1+|x|)^{1-N}, \quad |\nabla v(x)| \le C(1+|x|)^{-N}. \tag{2.5}$$

Moreover, (u,v) satisfies

$$-\Delta u = \frac{\alpha}{\alpha+\beta}\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)\frac{u^{\alpha-1}v^\beta}{|\pi(x)|^s}, \quad -\Delta v = \frac{\beta}{\alpha+\beta}\mu_{\alpha,\beta,\mathcal{P}}(\mathbb{R}^N_+)\frac{u^\alpha v^{\beta-1}}{|\pi(x)|^s}, \quad \text{in } \mathbb{R}^N_+. \tag{2.6}$$

Let $x_0 \in \mathcal{P}^\perp \cap \partial\Omega$. Since $\mathcal{P}^\perp \cap \Omega = \emptyset$, we have $\mathcal{P}^\perp \subset T_{x_0}\partial\Omega$, where $T_{x_0}\partial\Omega$ is the tangent space of the smooth manifold $\partial\Omega$ at x_0. Thus, $(T_{x_0}\partial\Omega)^\perp \subset \mathcal{P}$.

Denote $k = \dim_{\mathbb{R}} \mathcal{P}$, we choose a direct orthonormal basis $(e_1, \dots, e_N)$ of $\mathbb{R}^N$ such that $e_1 = n_{x_0}$ is the outward normal of $\partial\Omega$ at x_0, $\operatorname{span}\{e_1, \dots, e_k\} = \mathcal{P}$ and $\operatorname{span}\{e_{k+1}, \dots, e_N\} = \mathcal{P}^\perp$. For any $x \in \mathbb{R}^N$, we denote $x = (x_1, y, z)$, where $x_1 \in \mathbb{R}$, $y \in \operatorname{span}\{e_2, \dots, e_k\}$ and $z \in \mathcal{P}^\perp$.

Since $\partial\Omega$ is smooth, there exist open sets U, V of $\mathbb{R}^N$ such that $0 \in U$ and $x_0 \in V$, and there exist $\varphi \in C^\infty(U, V)$ and $\varphi_0 \in C^\infty(U')$ with $U' = \{(y, z) :$ there exists $x_1 \in \mathbb{R}$ such that $(x_1, y, z) \in U\}$ such that

(i) $\varphi : U \to V$ is a C^∞ diffeomorphism, $\varphi(0) = x_0$;
(ii) $\varphi(U \cap \{x_1 > 0\}) = \varphi(U) \cap \Omega$ and $\varphi(U \cap \{x_1 = 0\}) = \varphi(U) \cap \partial\Omega$;
(iii) $\varphi_0(0) = 0$ and $\nabla\varphi_0(0) = 0$;
(iv) $\varphi(x_1, y, z) = (x_1 - \varphi_0(y, z), y, z) + x_0$ for all $(x_1, y, z) \in U$.

Denote $\psi = \varphi^{-1}$. We choose a small positive number r_0 so that there exist neighborhoods V and $\tilde{V}$ of x_0, such that $\psi(V) = B_{r_0}(0)$, $\psi(V \cap \Omega) = B_{r_0}^+(0)$, $\psi(\tilde{V}) = B_{\frac{r_0}{2}}(0)$, $\psi(\tilde{V} \cap \Omega) = B_{\frac{r_0}{2}}^+(0)$. For $\varepsilon > 0$, we define

$$\tilde{u}_\varepsilon(x) = \varepsilon^{-\frac{N-2}{2}} \eta(x) u\big(\frac{\psi(x)}{\varepsilon}\big) := \eta(x) u_\varepsilon, \quad \tilde{v}_\varepsilon(x) = \varepsilon^{-\frac{N-2}{2}} \eta(x) v\big(\frac{\psi(x)}{\varepsilon}\big) := \eta(x) v_\varepsilon,$$

where $\eta \in C_0^\infty(V)$ is a positive cut-off function with $\eta \equiv 1$ in $\tilde{V}$. In what follows, we estimate each term in $I_\lambda(t\tilde{u}_\varepsilon, t\tilde{v}_\varepsilon)$. We have

$$\int_\Omega |\nabla \tilde{u}_\varepsilon|^2 \, dx = \int_\Omega (|\nabla\eta|^2 u_\varepsilon^2 + \eta^2 |\nabla u_\varepsilon|^2 + 2\nabla\eta \nabla u_\varepsilon \eta u_\varepsilon) \, dx.$$

Since

$$\int_\Omega \eta u_\varepsilon \nabla\eta \nabla u_\varepsilon \, dx = -\int_\Omega |\nabla\eta|^2 u_\varepsilon^2 \, dx - \int_\Omega \nabla\eta\eta\nabla u_\varepsilon u_\varepsilon \, dx - \int_\Omega \eta(\Delta\eta)|u_\varepsilon|^2 \, dx,$$

we obtain

$$\int_\Omega |\nabla \tilde{u}_\varepsilon|^2 \, dx = \int_{\Omega\cap U} \eta^2 |\nabla u_\varepsilon|^2 \, dx - \int_{\Omega\cap U} \eta(\Delta\eta)|u_\varepsilon|^2 \, dx.$$

By the change of the variable $X = \frac{\psi(x)}{\varepsilon} \in B_{r_0/\varepsilon}^+(0)$ and (2.4), we obtain

$$\begin{aligned}
\Big| \int_{\Omega\cap U} \eta(\Delta\eta) u_\varepsilon^2 \, dx \Big| &\le C\varepsilon^2 \int_{B_{r_0/\varepsilon}^+(0) \setminus B_{\frac{r_0}{2\varepsilon}}^+(0)} \eta(\varphi(\varepsilon X)) \big|\Delta\eta(\varphi(\varepsilon X))\big| u^2(X) \, dX \\
&= O(\varepsilon^2)
\end{aligned}$$

and since $\nabla_x u_\varepsilon(x) = \varepsilon^{-\frac{N}{2}} \nabla_X u(\frac{\psi(x)}{\varepsilon}) \nabla_x \psi(x)$, we deduce for $X' = (X_2, \dots, X_N)$ and $\nabla' = (\partial_{X_2}, \dots, \partial_{X_N})$ that

$$\begin{aligned}
&\int_{\Omega\cap U} \eta^2 |\nabla u_\varepsilon|^2 \, dx \\
&\le \int_{\mathbb{R}^N_+} |\nabla u|^2 \, dX - 2\int_{B_{r_0/\varepsilon}^+} \eta^2(\varphi(\varepsilon X)) \partial_1 u(X) \nabla' u(X) (\nabla'\varphi_0)(\varepsilon X') \, dX \\
&\quad + \int_{B_{r_0/\varepsilon}^+} \eta^2(\varphi(\varepsilon X)) |\nabla' u(X)|^2 |(\nabla'\varphi_0)(\varepsilon X')|^2 \, dX = I_1 + I_2 + I_3.
\end{aligned}$$

Using that

$$|\nabla'\varphi_0(X')| = O(|X'|), \quad \varphi_0(X') = \sum_{i=2}^N \alpha_i X_i^2 + o(1)(|X'|^2)$$

and (2.4), we have

$$I_3 \le C \int_{\mathbb{R}^N} (1+|X|)^{-2N} |\varepsilon X|^2 \, dX = O(\varepsilon^2).$$

Integrating by parts, we infer that

$$\begin{aligned} I_2 &= \frac{4}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \eta(\varphi(\varepsilon X)) \nabla' \eta(\varphi(\varepsilon X)) \partial_1 u(X) \nabla' u(X) \varphi_0(\varepsilon X') \, dX \\ &\quad + \frac{2}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \eta^2(\varphi(\varepsilon X)) \nabla' \partial_1 u(X) \nabla' u(X) \varphi_0(\varepsilon X') \, dX \\ &\quad + \frac{2}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \eta^2(\varphi(\varepsilon X)) \partial_1 u(X) \sum_{i=2}^N \partial_{ii} u(X) \varphi_0(\varepsilon X') \, dX = I_{21} + I_{22} + I_{23}. \end{aligned}$$

By (2.4),

$$|I_{21}| \le C\varepsilon^2 \int_{B^+_{r_0/\varepsilon}(0) \setminus B_{\frac{r_0}{2\varepsilon}}(0)} (1+|X|)^{-2N} |X|^2 \, dX \le C\varepsilon^N.$$

In the same way, $I_{22} = O(\varepsilon^N)$. By equation (2.6),

$$\sum_{i=2}^N \partial_{ii} u(X) = \Delta u - \partial_{11} u(X) = -\frac{\alpha\lambda}{\alpha+\beta} \mu_{\alpha,\beta,s}(\mathbb{R}^N_+) \frac{u^{\alpha-1} v^\beta}{|\pi(X)|^s} - \partial_{11} u(X).$$

Therefore,

$$I_{23} = -\frac{2}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \eta^2(\varphi(\varepsilon X)) \partial_1 u(X) \frac{\alpha\lambda}{\alpha+\beta} \mu_{\alpha,\beta,s}(\mathbb{R}^N_+) \frac{u^{\alpha-1} v^\beta}{|\pi(X)|^s} \varphi_0(\varepsilon X') \, dX \tag{2.7}$$

$$\quad - \frac{2}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \eta^2(\varphi(\varepsilon X)) \partial_1 u(X) \partial_{11} u(X) \varphi_0(\varepsilon X') \, dX := F_1 + F_2. \tag{2.8}$$

Since $u = Aw$,

$$F_1 = -\frac{C_0}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \eta^2(\varphi(\varepsilon X)) \frac{\partial_1 w(X)^{2^*(s)}}{|\pi(X)|^s} \varphi_0(\varepsilon X') \, dX,$$

where $C_0 = \frac{2\alpha\lambda}{(2^*(s))^2} \mu_{\alpha,\beta,s}(\mathbb{R}^N_+) A^\alpha B^\beta$. Integrating by parts, we obtain

$$\begin{aligned} F_1 &= C_0 \int_{B^+_{r_0/\varepsilon}} \frac{2\eta(\varphi(\varepsilon X)) \partial_1 \eta(\varphi(\varepsilon X)) \varphi_0(\varepsilon X')}{|\pi(X)|^s} w^{2^*(s)} \, dX \\ &\quad - \frac{C_0 s}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \frac{\eta^2(\varphi(\varepsilon X)) \varphi_0(\varepsilon X') X_1}{|\pi(X)|^{s+2}} w^{2^*(s)} \, dX \\ &= F_{11} + F_{12}. \end{aligned}$$

We may verify as above that $F_{11} = O(\varepsilon^{\frac{N^2-N-Ns+2}{N-2}})$.

Now, we estimate F_2. Integrating by parts, we deduce

$$\begin{aligned} F_2 &= \frac{1}{\varepsilon} \int_{B^+_{r_0/\varepsilon}} \partial_1 [\eta^2(\varphi(\varepsilon X)) \varphi_0(\varepsilon X')] (\partial_1 u)^2 \, dX \\ &\quad + \frac{1}{\varepsilon} \int_{B^+_{r_0/\varepsilon} \cap \{X_1 = 0\}} \eta^2(\varphi(\varepsilon X)) \varphi_0(\varepsilon X') (\partial_1 u)^2 \nu^N \, dS_X \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\varepsilon}\int_{B^+_{r_0/\varepsilon}} 2\eta(\varphi(\varepsilon X))\partial_1[\eta(\varphi(\varepsilon X))]\varphi_0(\varepsilon X')(\partial_1 u)^2\,dX\\
&\quad+\frac{1}{\varepsilon}\int_{B^+_{r_0/\varepsilon}\cap\partial\mathbb{R}^N_+}\eta^2(\varphi(\varepsilon X))\varphi_0(\varepsilon X')(\partial_1 u)^2\,dS_X\\
&= F_{21}+F_{22}.
\end{aligned}
$$

It can be shown that $F_{21}=O(\varepsilon^{N-1})$. Hence,

$$
I_2 = F_{12}+F_{22}+O(\varepsilon^{N-1}).
$$

Since $\eta(\varphi(\varepsilon X))\equiv 1$ in $B^+_{\frac{r_0}{2\varepsilon}}$, we have

$$
\begin{aligned}
F_{12} &= -\frac{C_0 s}{\varepsilon}\int_{B^+_{r_0/\varepsilon}\setminus B^+_{\frac{r_0}{2\varepsilon}}}\frac{\eta^2(\varphi(\varepsilon X))\varphi_0(\varepsilon X')X_1}{|\pi(X)|^{s+2}}w^{2^*(s)}\,dX\\
&\quad-\frac{C_0 s}{\varepsilon}\int_{B^+_{\frac{r_0}{2\varepsilon}}}\frac{\varphi_0(\varepsilon X')X_1}{|\pi(X)|^{s+2}}w^{2^*(s)}\,dX = J_1+J_2.
\end{aligned}
$$

We have

$$
\begin{aligned}
J_1 &\le C\varepsilon\int_{B^+_{r_0/\varepsilon}\setminus B^+_{\frac{r_0}{2\varepsilon}}}\frac{|\pi(X)|^3(1+|X|)^{(1-N)2^*(s)}}{|\pi(X)|^{s+2}}\,dX\\
&\le C\varepsilon\Big(\int_{(B^+_{r_0/\varepsilon}\setminus B^+_{\frac{r_0}{2\varepsilon}})\cap\mathbb{R}^{N-k}}\frac{1}{|x|^{\frac{2^*(s)(N-1)}{2}}}\,dx\Big)\Big(\int_{(B^+_{r_0/\varepsilon}\setminus B^+_{\frac{r_0}{2\varepsilon}})\cap\mathbb{R}^{k}}\frac{|x|^{1-s}}{|x|^{\frac{2^*(s)(N-1)}{2}}}\,dx\Big)\\
&\le C\varepsilon^{\frac{N(N-s)}{N-2}}.
\end{aligned}
$$

In the same way,

$$
\begin{aligned}
J_2 &= -\frac{C_0 s}{\varepsilon}\int_{\mathbb{R}^N_+}\frac{\varphi_0(\varepsilon X')X_1}{|\pi(X)|^{s+2}}w^{2^*(s)}\,dX-\frac{C_0 s}{\varepsilon}\int_{\mathbb{R}^N_+\setminus B^+_{r_0/\varepsilon}}\frac{\varphi_0(\varepsilon X')X_1}{|\pi(X)|^{s+2}}w(X)^{2^*(s)}\,dX\\
&= -\frac{C_0 s}{\varepsilon}\int_{\mathbb{R}^N_+}\frac{\varphi_0(\varepsilon X')X_1}{|\pi(X)|^{s+2}}w^{2^*(s)}\,dX+O(\varepsilon^{\frac{N(N-s)}{N-2}})\\
&= -\varepsilon C_0 s\sum_{i=2}^N\alpha_i\int_{\mathbb{R}^N_+}\frac{X_i^2X_1w(y)^{2^*(s)}}{|\pi(X)|^{s+2}}dX(1+o(1))+O(\varepsilon^{\frac{N(N-s)}{N-2}})\\
&= -\frac{s\varepsilon c_1}{N-1}\int_{\mathbb{R}^N_+}\frac{|X'|^2X_1w(X)^{2^*(s)}}{|\pi(X)|^{s+2}}dX\sum_{i=2}^N\alpha_i(1+o(1))+O(\varepsilon^{\frac{N(N-s)}{N-2}})\\
&= -C_0K_1H(0)(1+o(1))\varepsilon+O(\varepsilon^{\frac{N(N-s)}{N-2}}),
\end{aligned}
$$

where

$$
H(0)=\frac{1}{N-1}\sum_{i=2}^N\alpha_i,\quad K_1=s\int_{\mathbb{R}^N_+}\frac{|X'|^2X_1w^{2^*(s)}}{|\pi(X)|^{s+2}}\,dX.
$$

Similarly,

$$
F_{22}=\frac{1}{\varepsilon}\int_{(B^+_{r_0/\varepsilon}\setminus B^+_{\frac{r_0}{2\varepsilon}})\cap\{X_1=0\}}\eta^2(\varphi(\varepsilon X))\varphi_0(\varepsilon X')(\partial_1 u(X))^2\,dS_X
$$

$$+\frac{1}{\varepsilon}\int_{B^+_{\frac{r_0}{2\varepsilon}}\cap\{X_1=0\}}\varphi_0(\varepsilon X')(\partial_1 u(X))^2\,dS_X = L_1+L_2.$$

Also

$$\begin{aligned}L_1&\le\frac{C}{\varepsilon}\int_{\{\frac{r_0}{2}<|\varepsilon X'|\le r_0\}}|(\partial_1 u)(0,X')|^2|\varphi_0(\varepsilon X')|\,dX'\\&\le C\varepsilon\int_{\{\frac{r_0}{2}<|\varepsilon X'|\le r_0\}}|X'|^{-2N+2}\,dX'=O(\varepsilon^N).\end{aligned}$$

Using that

$$\int_{\mathbb{R}^{N-1}\setminus(B^+_{\frac{r_0}{2\varepsilon}}\cap\{X_1=0\})}\varphi_0(\varepsilon X')(\partial_N u(X))^2\,dS_X=O(\varepsilon^N),$$

one finds

$$\begin{aligned}L_2&=\frac{1}{\varepsilon}\int_{\mathbb{R}^{N-1}}\varphi_0(\varepsilon X')(\partial_1 u(X))^2\,dSX+O(\varepsilon^{N-1})\\&=\varepsilon\sum_{i=2}^N\alpha_i\int_{\mathbb{R}^{N-1}}[(\partial_1 u)(0,X')]^2X_i^2\,dX'(1+o(1))+O(\varepsilon^{N-1})\\&=K_2H(0)(1+o(1))\varepsilon+O(\varepsilon^{N-1}),\end{aligned}$$

where $K_2=\int_{\mathbb{R}^{N-1}}|(\partial_N u)(0,X')|^2|X'|^2\,dX'$. Consequently,

$$\int_\Omega|\nabla\tilde u_\varepsilon|^2\,dx=\int_{\mathbb{R}^N_+}|\nabla u|^2\,dX-(C_0K_1-K_2)H(0)(1+o(1))\varepsilon+O(\varepsilon^2),$$

and similarly,

$$\int_\Omega|\nabla\tilde v_\varepsilon|^2\,dx=\int_{\mathbb{R}^N_+}|\nabla v|^2\,dX-(C_1K_1-K_2)H(0)(1+o(1))\varepsilon+O(\varepsilon^2).$$

where $C_1=\frac{2\beta\lambda}{(2^*(s))^2}\mu_{\alpha,\beta,s}(\mathbb{R}^N_+)A^\alpha B^\beta$.

Next, let $X=\frac{\psi(x)}{\varepsilon}$. We estimate

$$\int_\Omega\frac{\tilde u_\varepsilon^\alpha\tilde v_\varepsilon^\beta}{|\pi(x)|^s}\,dx\ge\int_{\Omega\cap\tilde V}\frac{\tilde u_\varepsilon^\alpha\tilde v_\varepsilon^\beta}{|\pi(x)|^s}\,dx=\int_{\Omega\cap\tilde V}\frac{u_\varepsilon^\alpha v_\varepsilon^\beta}{|\pi(x)|^s}\,dx=\int_{B^+_{\frac{r_0}{2\varepsilon}}}\frac{u^\alpha(X)v^\beta(X)}{|\frac{\pi(\varphi(\varepsilon X))}{\varepsilon}|^s}\,dX$$

since $\eta\equiv1$ in $\Omega\cap\tilde V$. We recall that $x_0\in\mathcal{P}^\perp\cap\partial\Omega$, then we may write $\pi(\varphi(\varepsilon X))=(\varepsilon x_1+\varphi_0(\varepsilon y,\varepsilon z),\varepsilon y,0)$ and

$$|\pi(\varphi(\varepsilon X))|^2=\varepsilon^2|\pi(X)|^2\Big(1+\frac{2X_1\varphi_0(\varepsilon X')}{\varepsilon|\pi(X)|^2}+\frac{\varphi_0^2(\varepsilon X')}{\varepsilon^2|\pi(X)|^2}\Big).$$

Therefore,

$$\begin{aligned}\frac{1}{|\frac{\varphi(\varepsilon X)}{\varepsilon}|^s}&=\frac{1}{|\pi(X)|^s}\Big(1-\frac{sX_1\varphi_0(\varepsilon X')}{\varepsilon|\pi(X)|^2}-\frac{s\varphi_0^2(\varepsilon X')}{2\varepsilon^2|\pi(X)|^2}\Big)\\&\quad+\frac{1}{|\pi(X)|^s}O\Big(\frac{2X_1\varphi_0(\varepsilon X')}{\varepsilon|\pi(X)|^2}+\frac{\varphi_0^2(\varepsilon X')}{\varepsilon^2|\pi(X)|^2}\Big).\end{aligned}$$

This and

$$\int_{\mathbb{R}^N_+\setminus B^+_{\frac{r_0}{2\varepsilon}}} \frac{u^\alpha v^\beta}{|\pi(X)|^s}\,dX = O(\varepsilon^{\frac{N(N-s)}{N-2}})$$

enable us to show that

$$\begin{aligned}
\int_{\Omega\cap\tilde U} \frac{\tilde u_\varepsilon^\alpha \tilde v_\varepsilon^\beta}{|\pi(x)|^s}\,dx &= \int_{B^+_{\frac{r_0}{2\varepsilon}}} \frac{u^\alpha v^\beta}{|\pi(X)|^s}\,dX - \frac{s}{\varepsilon}\int_{B^+_{\frac{r_0}{2\varepsilon}}} \frac{X_1\varphi(\varepsilon X')u^\alpha(X)v^\beta(X)}{|\pi(X)|^{s+2}}\,dX + O(\varepsilon^2)\\
&= \int_{\mathbb{R}^N_+} \frac{u^\alpha v^\beta}{|\pi(X)|^s}\,dX - \frac{s}{\varepsilon}\int_{B^+_{\frac{r_0}{2\varepsilon}}} \frac{X_1\varphi(\varepsilon X')u^\alpha v^\beta}{|\pi(X)|^{s+2}}\,dX + O(\varepsilon^2).
\end{aligned}$$

Moreover,

$$\begin{aligned}
&-\frac{s}{\varepsilon}\int_{B^+_{\frac{r_0}{2\varepsilon}}} \frac{X_1\varphi(\varepsilon y')u^\alpha v^\beta}{|\pi(X)|^{s+2}}\,dX\\
&= -\frac{s}{\varepsilon}A^\alpha B^\beta\int_{B^+_{\frac{r_0}{2\varepsilon}}} \frac{X_1\varphi(\varepsilon X')w^{2^*(s)}}{|\pi(X)|^{s+2}}\,dX\\
&= -s\varepsilon\sum_{i=2}^N \alpha_i A^\alpha B^\beta\int_{\mathbb{R}^N_+} \frac{X_1X_i^2w^{2^*(s)}}{|\pi(X)|^{s+2}}\,dX(1+o(1)) + O(\varepsilon^{\frac{N(N-s)}{N-2}})\\
&= -\frac{s\varepsilon}{N-1}A^\alpha B^\beta\int_{\mathbb{R}^N_+} \frac{X_1|X'|^2w^{2^*(s)}}{|\pi(X)|^{s+2}}\,dX\sum_{i=2}^N\alpha_i(1+o(1)) + O(\varepsilon^{\frac{N(N-s)}{N-2}}).
\end{aligned}$$

Hence,

$$\int_{\Omega\cap\tilde U} \frac{\tilde u_\varepsilon^\alpha \tilde v_\varepsilon^\beta}{|\pi(x)|^s}\,dx = \int_{\mathbb{R}^N_+} \frac{u^\alpha v^\beta}{|\pi(X)|^s}\,dX - K_3H(0)(1+o(1))\varepsilon + O(\varepsilon^2),$$

where $K_3 = sA^\alpha B^\beta\int_{\mathbb{R}^N_+} \frac{X_1|X'|^2w^{2^*(s)}}{|\pi(X)|^{s+2}}\,dX = A^\alpha B^\beta K_1$.

Finally, let $X = \frac{\psi(x)}{\varepsilon} \in B^+_{r_0/\varepsilon}(0)$. We estimate

$$\begin{aligned}
\int_\Omega \tilde u_\varepsilon^{p+1}\,dx &= \varepsilon^{\frac{(2-N)(p+1)}{2}}\int_{\Omega\cap U}\eta^2(x)[u(\frac{\psi(x)}{\varepsilon})]^{p+1}\,dx\\
&= \varepsilon^{\frac{(2-N)(p+1)}{2}+N}\int_{B^+_{r_0/\varepsilon}} u^{p+1}\,dX\\
&= \varepsilon^{\frac{N+2}{2}-\frac{(N-2)p}{2}}\int_{\mathbb{R}^N_+} u^{p+1}\,dX + O(\varepsilon^{\frac{N(p+1)}{2}}).
\end{aligned}$$

Similarly,

$$\int_\Omega \tilde v_\varepsilon^{p+1}\,dx = \varepsilon^{\frac{N+2}{2}-\frac{(N-2)p}{2}}\int_{\mathbb{R}^N_+} v^{p+1}\,dX + O(\varepsilon^{\frac{N(p+1)}{2}}).$$

Since $q < \frac{N}{N-2}$, $\frac{N+2}{2} - \frac{(N-2)p}{2} > 1$. For $t \ge 0$, we have

$$\begin{aligned}
&I_\lambda(t\tilde u_\varepsilon, t\tilde v_\varepsilon)\\
&= \frac{t^2}{2}\Big(\int_{\mathbb{R}^N_+}|\nabla u|^2\,dX + \int_{\mathbb{R}^N_+}|\nabla v|^2\,dX\Big) - \frac{2t^{2^*(s)}\lambda}{2^*(s)}\int_{\mathbb{R}^N_+}\frac{u^\alpha v^\beta}{|\pi(X)|^s}\,dX
\end{aligned}$$

$$+\frac{H(0)}{2}[(2K_2 - C_0K_1 - C_1K_1)t^2 + \frac{4}{2^*(s)}(\lambda K_3 + o(1))t^{2^*(s)}]\varepsilon + O(\varepsilon^2)$$
$$= f_1(t) + \frac{H(0)}{2}\varepsilon f_2(t) + O(\varepsilon^2),$$

where
$$f_1(t) = \frac{t^2}{2}\mu_{\alpha,\beta,s}(\mathbb{R}^N_+) - \frac{2\lambda t^{2^*(s)}}{2^*(s)}.$$

It can be verified that
$$\max_{0\le t\le 1} f_1(t) = f_1(t_0) = (2\lambda)^{\frac{-2}{2^*(s)-2}}(\frac{1}{2} - \frac{1}{2^*(s)})\mu_{\alpha,\beta,s}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}},$$

with $t_0 = (\frac{1}{2\lambda}\mu_{\alpha,\beta,s}(\mathbb{R}^N_+))^{\frac{1}{2^*(s)-2}}$. Since $K_1 > 0$,
$$\begin{aligned} f_2(t_0) &= (2K_2 - C_0K_1 - C_1K_1)t_0^2 + \frac{4\lambda}{2^*(s)}K_3 t_0^{2^*(s)} \\ &= (2K_2 - \frac{2\lambda}{2^*(s)}A^\alpha B^\beta K_1)t_0^2 + \frac{4\lambda}{2^*(s)}A^\alpha B^\beta K_1 t_0^{2^*(s)} \\ &= 2K_2t_0^2 + \frac{2\lambda}{2^*(s)}A^\alpha B^\beta K_1(\frac{\mu_{\alpha,\beta,s}(\mathbb{R}^N_+)}{\lambda} - 1)t_0^2. \end{aligned}$$

Hence, $f_2(t_0) > 0$ if $\lambda > 0$ and small.

Since $H(0) < 0$, by choosing T large enough, we have $I_\lambda(T\tilde{u}_\varepsilon, T\tilde{v}_\varepsilon) < 0$ for $t \ge T$ and $\varepsilon \ge 0$ small. Let $u_0 = T\tilde{u}_\varepsilon$, $v_0 = T\tilde{v}_\varepsilon$. We obtain
$$\max_{0\le t\le 1} I_\lambda(tu_0, tv_0) < (2\lambda)^{\frac{-2}{2^*(s)-2}}(\frac{1}{2} - \frac{1}{2^*(s)})\mu_{\alpha,\beta,s}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}}$$

and
$$I_\lambda(u_0, v_0) < 0.$$

This completes the proof of Lemma 2.1. □

3. Existence of positive solution in Ω

Now we will use the blow up argument to prove (iii) of Theorem 1.1. For any $\varepsilon > 0$, by the mountain pass theorem, we have a positive solution pair $(u_\varepsilon, v_\varepsilon)$ of the subcritical system
$$\begin{gathered} -\Delta u_\varepsilon = \frac{2\alpha\lambda}{\alpha+\beta-\varepsilon}\frac{u_\varepsilon^{\alpha-1}v_\varepsilon^{\beta-\varepsilon}}{|\pi(x)|^s} - u_\varepsilon^{p-\varepsilon}, \quad \text{in } \Omega, \\ -\Delta v_\varepsilon = \frac{2\beta\lambda}{\alpha+\beta-\varepsilon}\frac{u_\varepsilon^{\alpha}v_\varepsilon^{\beta-1-\varepsilon}}{|\pi(x)|^s} - v_\varepsilon^{p-\varepsilon}, \quad \text{in } \Omega, \\ u_\varepsilon > 0, v_\varepsilon > 0, \quad \text{in } \Omega, \\ u_\varepsilon = v_\varepsilon = 0, \quad \text{on } \partial\Omega. \end{gathered} \tag{3.1}$$

Using Lemma 2.4, we see that the mountain pass level c_ε of (3.1) satisfies
$$c_\varepsilon = I^\varepsilon_\lambda(u_\varepsilon, v_\varepsilon) < (2\lambda)^{\frac{-2}{2^*(s)-2}}(\frac{1}{2} - \frac{1}{2^*(s)})\mu_{\alpha,\beta,s}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}} \tag{3.2}$$

if $0 < \lambda < \lambda^*$, where
$$I_\varepsilon(u_\varepsilon, v_\varepsilon) = \int_\Omega (\frac{1}{2}|\nabla u_\varepsilon|^2 + \frac{1}{2}|\nabla v_\varepsilon|^2 - \frac{2\lambda}{2^*(s)-\varepsilon}\frac{u_\varepsilon^\alpha v_\varepsilon^{\beta-\varepsilon}}{|\pi(x)|^s})\, dx$$

$$+\int_\Omega \Big(\frac{1}{p+1-\varepsilon}u_\varepsilon^{p+1-\varepsilon}+\frac{1}{p+1-\varepsilon}v_\varepsilon^{p+1-\varepsilon}\Big)\,dx.$$

It can be easily shown that both $\|u_\varepsilon\|_{H_0^1(\Omega)}$ and $\|v_\varepsilon\|_{H_0^1(\Omega)}$ are uniformly bounded for $\varepsilon>0$ small. Thus, there is a subsequence $\{(u_j,v_j)\}$ of $\{(u_\varepsilon,v_\varepsilon)\}$ such that

$$\begin{gathered} u_j \rightharpoonup u,\quad v_j \rightharpoonup v,\quad \text{in } H_0^1(\Omega),\\ u_j \to u,\quad v_j \to v,\quad \text{in } L^{p+1}(\Omega),\\ u_j \rightharpoonup u,\quad v_j \rightharpoonup v,\quad \text{in } L^{2^*(s)}(\Omega,|\pi(x)|^{-s}dx), \end{gathered}\tag{3.3}$$

with $u,v\ge 0$ and (u,v) is a solution of system (1.1). If (u,v) is a nontrivial solution, by the strong maximum principle, $u,v>0$, then we are done.

Now, we prove (u,v) is nontrivial. This will be shown by the blowing up argument. Suppose on the contrary that $u=v=0$ in Ω. By the regularity result, see for instance [12, Proposition 3.2], $u_\varepsilon,v_\epsilon\in C^1(\bar\Omega)$. Let $x_j,y_j\in\Omega$ be such that

$$M_j=u_j(x_j)=\max_{\bar\Omega}u_j(x),\quad N_j=v_j(y_j)=\max_{\bar\Omega}v_j(x).\tag{3.4}$$

Then, we have either $m_j\to\infty$ or $n_j\to\infty$ as $j\to\infty$. Indeed, on the contrary we would have $m_j\le C$ and $n_j\le C$ for a positive constant C. By the Sobolev embedding,

$$\int_\Omega \frac{u_j^\alpha v_j^{\beta-\varepsilon_j}}{|\pi(x)|^s}\,dx\le C\int_\Omega\frac{u_j^\alpha}{|\pi(x)|^s}\,dx\to 0$$

as $j\to\infty$. This implies

$$\int_\Omega(|\nabla u_j|^2+|\nabla v_j|^2)\,dx=2\int_\Omega\frac{u_j^\alpha v_j^{\beta-\varepsilon_j}}{|\pi(x)|^s}\,dx-\lambda\int_\Omega u_j^{p+1-\varepsilon_j}\,dx-\lambda\int_\Omega v_j^{p+1-\varepsilon_j}\,dx\to 0;$$

that is, $u_j\to 0$, $v_j\to 0$ strongly in $H_0^1(\Omega)$. It yields

$$0=\lim_{j\to\infty}\frac12\int_\Omega(|\nabla u_j|^2+|\nabla v_j|^2)\,dx=c>0$$

a contradiction.

Suppose $N_j\le M_j\to\infty$. Denote

$$\tilde u_j(x)=M_j^{-1}u_j(k_jx+x_j),\quad \tilde v_j(x)=M_j^{-1}v_j(k_jx+x_j),\quad \text{for } x\in\Omega_j,$$

where $k_j=M_j^{-\frac{2^*(s)-2-\varepsilon_j}{2-s}}$ and $\Omega_j=\{x\in\mathbb{R}^N \mid x_j+k_jx\in\Omega\}$. Obviously, $(\tilde u_j,\tilde v_j)$ satisfies

$$\begin{gathered} -\Delta\tilde u_j=\frac{2\alpha\lambda}{\alpha+\beta-\varepsilon_j}\Big(\frac{k_j}{|\pi(x_j)|}\Big)^s\frac{\tilde u_j^{\alpha-1}\tilde v_j^{\beta-\varepsilon_j}}{|\pi(\frac{x_j}{|\pi(x_j)|}+\frac{k_j}{|\pi(x_j)|}x)|^s}-k_j^2M_j^{p-1-\varepsilon_j}\tilde u_j^{p-\varepsilon_j},\quad \text{in } \Omega_j,\\ -\Delta\tilde v_j=\frac{2(\beta-\varepsilon_j)\lambda}{\alpha+\beta-\varepsilon_j}\Big(\frac{k_j}{|\pi(x_j)|}\Big)^s\frac{\tilde u_j^{\alpha}\tilde v_j^{\beta-1-\varepsilon_j}}{|\pi(\frac{x_j}{|\pi(x_j)|}+\frac{k_j}{|\pi(x_j)|}x)|^s}-k_j^2M_j^{p-1-\varepsilon_j}\tilde v_j^{p-\varepsilon_j},\quad \text{in } \Omega_j,\\ 0\le\tilde u_j,\tilde v_j\le 1,\quad \text{in } \Omega_j,\\ \tilde u_j=\tilde v_j=0,\quad \text{on } \partial\Omega_j. \end{gathered}\tag{3.5}$$

Since $M_j\to\infty$, $k_j\to 0$ as $j\to\infty$. Furthermore, we have

$$k_j^2M_j^{p-1-\varepsilon_j}=k_j^{2-\frac{(2-s)(p-\varepsilon_j-1)}{2^*(s)-2-\varepsilon_j}}\to 0\quad \text{as } j\to\infty$$

as the facts that $k_j \to 0$ and $2 - \frac{(2-s)(p-\varepsilon_j-1)}{2^*(s)-2-\varepsilon_j} > 0$; i.e, $p < \frac{N+2}{N-2}$.

We will show that $M_j = O(1)N_j$. First, we claim that $|\pi(x_j)| = O(k_j)$ as $j \to \infty$. Suppose on the contrary that $\limsup_{j\to\infty} \frac{|\pi(x_j)|}{k_j} = \infty$.

Because $(\tilde{u}_j, \tilde{v}_j)$ is uniformly bounded in $C^1_{\rm loc}$, we may assume that $\tilde{u}_j \to u, \tilde{v}_j \to v$ in $C^0_{\rm loc}$. Suppose now $x_j \to x_0 \in \bar{\Omega}$. There are two cases:

(i) $x_0 \in \Omega$ or $x_0 \in \partial\Omega$ and $\frac{\operatorname{dist}(x_j,\partial\Omega)}{k_j} \to \infty$; and

(ii) $x_0 \in \partial\Omega$ and $\frac{\operatorname{dist}(x_j,\partial\Omega)}{k_j} \to \sigma \geq 0$.

In the case (i), we have $\Omega_j \to \mathbb{R}^N$ as $j \to \infty$ and (u, v) satisfies

$$
\begin{gathered}
\Delta u = 0, \quad \Delta v = 0 \quad \text{in } \mathbb{R}^N, \\
0 \leq u, v \leq 1, \quad u(0) = 1.
\end{gathered}
$$

Furthermore,

$$
\int_{\Omega_j} \tilde{u}_j^{\frac{2N}{N-2}}\, dy = k_j^{\frac{N\varepsilon_j}{2^*(s)-2-\varepsilon_j}} \int_\Omega u_j^{\frac{2N}{N-2}}\, dx \leq C, \quad \text{and} \quad \int_{\Omega_j} \tilde{v}_j^{\frac{2N}{N-2}}\, dy \leq C,
$$

which yields

$$
\int_{\mathbb{R}^N} u^{\frac{2N}{N-2}}\, dy < \infty, \quad \int_{\mathbb{R}^N} v^{\frac{2N}{N-2}}\, dy < \infty.
$$

However, by the Liouville theorem, $u \equiv v \equiv 1$ for $x \in \mathbb{R}^N$. This is a contradiction.

In case (ii), after an orthogonal transformation, we have $\Omega_j \to \mathbb{R}^N_+ = \{x = (x_1, \dots, x_N) \mid x_1 > 0\}$ as $j \to \infty$ and $\tilde{u}_j$, $\tilde{v}_j$ converge to some u, v uniformly in every compact subset of $\mathbb{R}^N_+$. Now, $u(0) = 1$ and $0 \leq v(0) \leq 1$. Hence, (u, v) satisfies

$$
\begin{gathered}
\Delta u = 0, \quad \Delta v = 0 \quad \text{in } \mathbb{R}^N_+, \\
0 \leq u, v \leq 1 \quad \text{in } \mathbb{R}^N_+, \\
u = v = 0 \quad \text{on } \partial\mathbb{R}^N_+.
\end{gathered}
$$

By the boundary condition and the maximum principle, $u \equiv v \equiv 0$ for $x \in \mathbb{R}^N_+$ which violate to $u(0) = 1$. Consequently, $\limsup_{j\to\infty} \frac{|\pi(x_j)|}{k_j} < \infty$. Since $k_j \to 0$, we have $\pi(x_j) \to 0$ as $j \to \infty$.

Next, we show that $\liminf_{j\to\infty} \frac{|\pi(x_j)|}{k_j} > 0$. Were it not the case, we would have, up to a subsequence, that $\lim_{j\to\infty} \frac{|\pi(x_j)|}{k_j} = 0$. Then $(\tilde{u}_j, \tilde{v}_j)$ satisfies

$$
\begin{gathered}
-\Delta \tilde{u}_j = \frac{2\alpha\lambda}{\alpha+\beta-\varepsilon_j} \frac{\tilde{u}_j^{\alpha-1}\tilde{v}_j^{\beta-\varepsilon_j}}{|\frac{\pi(x_j)}{k_j} + \pi(x)|^s} - k_j^2 M_j^{p-1-\varepsilon_j} \tilde{u_j}^{p-\varepsilon_j}, \quad \text{in } \Omega_j, \\
-\Delta \tilde{v}_j = \frac{2(\beta-\varepsilon_j)\lambda}{\alpha+\beta-\varepsilon_j} \frac{\tilde{u}_j^{\alpha}\tilde{v}_j^{\beta-1-\varepsilon_j}}{|\frac{\pi(x_j)}{k_j} + \pi(x)|^s} - k_j^2 M_j^{p-1-\varepsilon_j} \tilde{v_j}^{p-\varepsilon_j}, \quad \text{in } \Omega_j, \\
0 \leq \tilde{u}_j, \tilde{v}_j \leq 1, \quad \text{in } \Omega_j, \\
\tilde{u}_j = \tilde{v}_j = 0, \quad \text{on } \partial\Omega_j,
\end{gathered} \tag{3.6}
$$

Up to a rotation, we have $\Omega_j \to \mathbb{R}^N_+$ and $\tilde{u}_j$, $\tilde{v}_j$ converge to some u, v uniformly in compact subsets of $\mathbb{R}^N_+$ respectively, where (u,v) satisfies

$$
\begin{gathered}
-\Delta u = \frac{2\alpha\lambda}{\alpha+\beta}\frac{u^{\alpha-1}v^{\beta}}{|\pi(x)|^s}, \quad -\Delta v = \frac{2\beta\lambda}{\alpha+\beta}\frac{u^{\alpha}v^{\beta-1}}{|\pi(x)|^s} \quad \text{in } \mathbb{R}^N_+,\\
0 \le u, v \le 1 \quad \text{in } \mathbb{R}^N_+, \quad u = v = 0 \quad \text{on } \partial\mathbb{R}^N_+.
\end{gathered}
$$

The boundary condition violates to $u(0) = 1$. Hence, $\liminf_{j\to\infty} \frac{|\pi(x_j)|}{k_j} > 0$.

Now, we complete the proof of Theorem 1.1 by showing that problem (1.1) has a nontrivial solution.

First, we remark that $\operatorname{dist}(x_j, \partial\Omega) = O(k_j)$. Indeed, since $\mathcal{P}^\perp \cap \Omega = \emptyset$, we have $x_j - \pi(x_j) \in \mathcal{P}^\perp \subset \mathbb{R}^N \setminus \Omega$. Because $x_j \in \Omega$, there exists $t_j \in (0,1)$ such that $t_j x_j + (1-t_j)(x_j - \pi(x_j)) \in \partial\Omega$. Therefore,

$$
d(x_j, \partial\Omega) \le |x_j - (t_j x_j + (1-t_j)(x_j - \pi(x_j)))| = (1-t_j)|\pi(x_j)| \le |\pi(x_j)| = O(k_j).
$$

Hence, we may assume $\frac{\operatorname{dist}(x_j,\partial\Omega)}{k_j} \to \sigma \ge 0$. By an affine transformation, we find $(\tilde{u}_j, \tilde{v}_j)$ converges to (u,v) uniformly in any compact subset of $\mathbb{R}^N_+$ and (u,v) satisfies

$$
\begin{gathered}
-\Delta u = \frac{2\alpha\lambda}{\alpha+\beta}\frac{u^{\alpha-1}v^{\beta}}{|\pi(x)|^s}, \quad -\Delta v = \frac{2\beta\lambda}{\alpha+\beta}\frac{u^{\alpha}v^{\beta-1}}{|\pi(x)|^s} \quad \text{in } \mathbb{R}^N_+,\\
u, v > 0 \quad \text{in } \mathbb{R}^N_+; \quad u = v = 0 \quad \text{on } \partial\mathbb{R}^N_+
\end{gathered} \tag{3.7}
$$

with $u(0, \dots, \sigma) = 1$. By the definition of $\mu_{\alpha,\beta,s}(\Omega)$, we have

$$
\mu_{\alpha,\beta,s}(\Omega_j) \le \frac{\int_\Omega (|\nabla \tilde{u}_j|^2 + |\nabla \tilde{v}_j|^2)\,dx}{\big(\int_\Omega \frac{\tilde{u}_j^\alpha \tilde{v}_j^{\beta-\varepsilon}}{|x|^s}\,dx\big)^{\frac{2}{2^*(s)}}},
$$

and then

$$
\mu_{\alpha,\beta,s}(\mathbb{R}^N_+) \le \frac{\int_{\mathbb{R}^N_+} (|\nabla u|^2 + |\nabla v|^2)\,dy}{\big(\int_{\mathbb{R}^N_+} \frac{u^\alpha v^\beta}{|\pi(x)|^s}\,dx\big)^{\frac{2}{2^*(s)}}} = 2\lambda\Big(\int_{\mathbb{R}^N_+} \frac{u^\alpha v^\beta}{|\pi(x)|^s}\,dx\Big)^{\frac{2^*(s)-2}{2^*(s)}};
$$

that is,

$$
\int_{\mathbb{R}^N_+} (|\nabla u|^2 + |\nabla v|^2)\,dx = 2\lambda \int_{\mathbb{R}^N_+} \frac{u^\alpha v^\beta}{|\pi(x)|^s}\,dx \ge (2\lambda)^{\frac{-2}{2^*(s)-2}} \mu_{\alpha,\beta,s}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}}. \tag{3.8}
$$

Furthermore, noting that

$$
\begin{aligned}
\lim_{j\to\infty} \int_\Omega (|\nabla u_j|^2 + |\nabla v_j|^2)\,dx &= \lim_{j\to\infty} k_j^{-\frac{(N-2)\varepsilon_j}{2^*(s)-2-\varepsilon_j}} \int_{\Omega_j} (|\nabla \tilde{u}_j|^2 + |\nabla \tilde{v}_j|^2)\,dx\\
&\ge \lim_{j\to\infty} \int_{\Omega_j} (|\nabla \tilde{u}_j|^2 + |\nabla \tilde{v}_j|^2)\,dx\\
&\ge \int_{\mathbb{R}^N_+} (|\nabla u|^2 + |\nabla v|^2)\,dx,
\end{aligned} \tag{3.9}
$$

we derive from (3.2), (3.8), (3.9) that

$$
\begin{aligned}
c &= (\frac{1}{2} - \frac{1}{2^*(s)}) \lim_{j\to\infty} \int_\Omega (|\nabla u_j|^2 + |\nabla v_j|^2)\,dx\\
&\ge (\frac{1}{2} - \frac{1}{2^*(s)})(2\lambda)^{\frac{-2}{2^*(s)-2}} \mu_{\alpha,\beta,s}(\mathbb{R}^N_+)^{\frac{2^*(s)}{2^*(s)-2}},
\end{aligned}
$$

which yields a contradiction to (3.2). Thus, (u, v) is a nontrivial solution of (1.1) if $N_j \le M_j$.

Now we show $M_j = O(N_j)$. Indeed, since u is nontrivial, so is v. Otherwise, we would have

$$\begin{gathered} \Delta u = 0 \quad \text{in } \mathbb{R}^N_+, \\ 0 \le u \le 1, u(0, \dots, \sigma) = 1 \quad \text{in } \mathbb{R}^N_+, \\ u = 0 \quad \text{on } \partial\mathbb{R}^N_+. \end{gathered}$$

By the strong maximum principle, u would be a constant because it attains its maximum value inside $\mathbb{R}^N_+$. This yields a contradiction between $u(0, \dots, \sigma) = 1$ and the boundary condition. Therefore, there exists $y_0 \in \mathbb{R}^N_+$ such that $v(y_0) \ne 0$. Hence,

$$\tilde{v}_j(y_0) = m_j^{-1} v_j(x_j + k_j y_0) \to v(y_0) > 0$$

implying

$$1 \ge \frac{n_j}{m_j} \ge \frac{v_j(x_j + k_j y_0)}{m_j} \ge v(y_0) - \varepsilon > 0$$

for $\varepsilon > 0$ small and j large, namely, $N_j = O(1)M_j$ as $j \to \infty$. Replacing M_j by N_j in above blow up process, we may also derive a contradiction if we assume $u = v = 0$. Consequently, (1.1) has a positive nontrivial solution. The proof of Theorem 1.1 is complete.

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