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EXISTENCE OF BOUNDED POSITIVE SOLUTIONS OF A NONLINEAR DIFFERENTIAL SYSTEM

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ABSTRACT. In this article, we study the existence and nonexistence of solutions for the system

$$\begin{gathered}\frac{1}{A}(Au')' = pu^\alpha v^s \quad \text{on } (0,\infty),\\ \frac{1}{B}(Bu')' = qu^r v^\beta \quad \text{on } (0,\infty),\\ Au'(0)=0, \quad u(\infty)=a>0,\\ Bv'(0)=0, \quad v(\infty)=b>0,\end{gathered}$$

where $\alpha,\beta \geq 1$, $s,r \geq 0$, p,q are two nonnegative functions on $(0,\infty)$ and A, B satisfy appropriate conditions. Using potential theory tools, we show the existence of a positive continuous solution. This allows us to prove the existence of entire positive radial solutions for some elliptic systems.

1. Introduction

Existence and nonexistence of solutions of the elliptic system

$$\begin{gathered}\Delta u = p(|x|)f(v), \quad x\in\mathbb{R}^n\\ \Delta v = q(|x|)g(u), \quad x\in\mathbb{R}^n\end{gathered} \tag{1.1}$$

have been intensively studied in the previous years; see for example [2, 3, 4, 5, 6, 9, 10] and the references therein.

Lair and Wood [6] considered the existence of entire positive radial solutions to the system (1.1) when $f(v)=v^s$ and $g(u)=u^r$. More precisely, for the sublinear case where $r,s\in(0,1)$, they proved that if p and q satisfy the decay conditions

$$\int_0^\infty tp(t)dt<\infty, \quad \int_0^\infty tq(t)dt<\infty \tag{1.2}$$

then (1.1) has bounded solutions, and if

$$\int_0^\infty tp(t)dt=\infty, \quad \int_0^\infty tq(t)dt=\infty$$

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then (1.1) has large solutions. For the superlinear case, where $r, s \in (1,\infty)$, the authors proved the existence of an entire large positive solution of (1.1), provided that p and q satisfy (1.2).

Later, their results were extended by Cîrstea and Radulescu [2] which considered (1.1) under the following conditions on f and g:

$$\lim_{t\to\infty} \frac{f(cg(t))}{t} = 0, \quad \text{for all } c > 0.$$

To study (1.1), Ghanmi et al in [4] considered the system

$$\begin{gathered}
\frac{1}{A}(Au')' = p(t)g(v) \quad t \in (0,\infty),\\
\frac{1}{B}(Bv')' = q(t)f(u) \quad t \in (0,\infty),\\
u(0) = \alpha > 0, \quad v(0) = \beta > 0,\\
Au'(0) = 0, \quad Bv'(0) = 0,
\end{gathered}$$

where A, B are continuous functions on $(0,\infty)$, and p, q, f and g are nonnegative and continuous functions on $[0,\infty)$. They proved that if f and g are lipschitz continuous functions on each interval $[\epsilon,\infty)$, $\epsilon > 0$, system (1.1) has a unique bounded positive solution (u, v) satisfying $u, v \in C([0,\infty)) \cap C^1((0,\infty))$.

In this article, we are interested in the study of positive radial solutions to the semilinear elliptic system

$$\begin{gathered}
\Delta u = p(|x|)u^\alpha v^s, \quad x \in \mathbb{R}^n\\
\Delta v = q(|x|)u^r v^\beta, \quad x \in \mathbb{R}^n
\end{gathered} \tag{1.3}$$

where $\alpha, \beta \geq 1$, $r, s \geq 0$ and $p, q : (0,\infty) \to [0,\infty)$ satisfying (1.2). To this end, we undertake a study of the system of semilinear differential equations

$$\begin{gathered}
\frac{1}{A}(Au')' = pu^\alpha v^s \quad \text{on } (0,\infty),\\
\frac{1}{B}(Bv')' = qu^r v^\beta \quad \text{on } (0,\infty),\\
Au'(0) = 0, \quad u(\infty) = a,\\
Bv'(0) = 0, \quad v(\infty) = b,
\end{gathered} \tag{1.4}$$

where $a, b > 0$ and the functions A and B satisfy condition (H0) below. In this paper, we denote $u(\infty) := \lim_{x\to\infty} u(x)$ and $Au'(0) := \lim_{x\to 0} A(x)u'(x)$.

To simplify our statement, we denote by $B^+((0,\infty))$ the set of nonnegative measurable functions on $(0,\infty)$. Also we refer to $C([0,\infty])$ the collection of all continuous functions u in $[0,\infty)$ such that $\lim_{x\to\infty} u(x)$ exists and $C_0([0,\infty))$ the subclass of $C([0,\infty])$ consisting of functions which vanish continuously at ∞.

Before presenting our main result, we would like to make some assumptions and recall some properties of the operator $Lu = \frac{1}{A}(Au')'$, while referring the reader to [7, 8] for furthers details. Throughout this paper, we say that a function A satisfies condition (H0) if

(H0) A is a continuous function on $[0,\infty)$, differentiable and positive on $(0,\infty)$ such that

$$\int_1^\infty \frac{dt}{A(t)} < \infty \quad \text{and} \quad \int_0^1 \frac{1}{A(t)}\Big(\int_0^t A(s)ds\Big)dt < \infty.$$

For a function A satisfying (H0), we denote by G the Green's function of the operator $Lu = \frac{1}{A}(Au')'$ on $(0,\infty)$ with Dirichlet conditions $Au'(0) = 0$, $u(\infty) = 0$; that is,

$$G(x,t) = A(t)\int_{x\vee t}^{\infty} \frac{dr}{A(r)}, \quad \text{for } (x,t) \in ((0,\infty))^2,$$

where $x \vee t := \max(x,t)$ and we refer to the potential of a function f in $B^+((0,\infty))$ by

$$Vf(x) = \int_0^{\infty} G(x,t)f(t)dt.$$

We point out that for each $f \in B^+((0,\infty))$ such that $Vf(0) < \infty$, the function $Vf \in C_0([0,\infty)) \cap C^1((0,\infty))$ and satisfies

$$\begin{gathered} L(Vf) = -f \quad \text{a.e. on } (0,\infty), \\ A(Vf)'(0) = 0, \quad Vf(\infty) = 0. \end{gathered}$$

Let us introduce the conditions imposed to the functions p and q:

(H1) $p, q : (0,\infty) \to [0,\infty)$ are two measurable functions such that

$$Vp(0) < \infty \quad \text{and} \quad Wq(0) < \infty.$$

Here for $f \in B^+((0,\infty))$, we denote

$$Wf(x) = \int_0^{\infty} H(x,t)f(t)dt,$$

where

$$H(x,t) = B(t)\int_{x\vee t}^{\infty} \frac{dr}{B(r)}.$$

Using a fixed point argument, we prove our main result.

Theorem 1.1. *Let A and B be two functions satisfying* (H0) *and let p, q be two functions satisfying* (H1). *Then for each $a, b > 0$, system* (1.4) *has a positive solution (u,v) satisfying $u, v \in C([0,\infty]) \cap C^1((0,\infty))$. Moreover, there exist $c_1, c_2 > 0$ such that for each $x \in [0,\infty)$, we have*

$$\begin{gathered} a\exp(-c_1 Vp(0)) \le u(x) \le a, \\ b\exp(-c_2 Wq(0)) \le v(x) \le b. \end{gathered}$$

Remark 1.2. If $A(t) = B(t) = t^{n-1}$, the condition (H1) is equivalent to (1.2). It follows by Theorem 1.1 that if p, q satisfy (1.2) then for each $a, b > 0$, the elliptic system (1.3) has a positive radial solution (u,v) continuous in $\mathbb{R}^n$ such that $\lim_{|x|\to\infty} u(x) = a$ and $\lim_{|x|\to\infty} v(x) = b$.

The outline of this article is as follows. In Section 2, we lay out some properties pertaining with potential theory and we give some useful results related to the operator $Lu = \frac{1}{A}(Au')'$. In particular, we establish an existence and a uniqueness result to the problem

$$\begin{gathered} Lu = p(x)u^{\alpha}, \quad x \in (0,\infty) \\ Au'(0) = 0, \quad u(\infty) = a > 0, \end{gathered} \tag{1.5}$$

where $\alpha \ge 1$ and $p \in B^+((0,\infty))$ such that $Vp(0) < \infty$. This allows us to prove Theorem 1.1 in Section 3 by using a technical method that requires a potential theory approach.

2. Preliminary results

Let A be a function satisfying (H0). The objective of this section is to give some technical results concerning the operator $Lu = \frac{1}{A}(Au')'$ and to recall some potential theory tools which are crucial to prove our main result.

Proposition 2.1. *Let $q \in B^+((0,\infty))$ such that $Vq(0) < \infty$. Then the family of functions*

$$F_q = \{x \to Vf(x) = \int_0^\infty G(x,t)f(t)dt; |f| \le q\}$$

is uniformly bounded and equicontinuous in $[0,\infty]$. Consequently F_q is relatively compact in $C_0([0,\infty))$.

Proof. By writing

$$Vf(x) = \int_x^\infty \frac{1}{A(t)}(\int_0^t A(r)f(r)dr)dt,$$

we deduce that for $x, x' \in [0,\infty)$, we have

$$|Vf(x) - Vf(x')| \le \int_x^{x'} \frac{1}{A(t)}\Big(\int_0^t A(r)q(r)dr\Big)dt.$$

Since $Vq(0) = \int_0^\infty \frac{1}{A(t)}(\int_0^t A(r)q(r)dr)dt < \infty$, it follows by the dominated convergence theorem the equicontinuity of F_q in $[0,\infty)$. Moreover, since

$$|Vf(x)| \le \int_x^\infty \frac{1}{A(t)}\Big(\int_0^t A(r)q(r)dr\Big)dt,$$

we deduce that $\lim_{x\to\infty} Vf(x) = 0$, uniformly in f. Which proves that F_q is uniformly bounded in $[0,\infty]$. Then by Ascoli's theorem, we deduce that F_q is relatively compact in $C_0([0,\infty))$. □

In what follows, we need the following lemma and we refer to [7, 8] for more details.

Lemma 2.2. *Let $q \in B^+((0,\infty))$ such that $Vq(0) < \infty$. Then the problem*

$$\begin{gathered} \frac{1}{A}(Au')' - qu = 0 \quad a.e. \text{ on } (0,\infty), \\ Au'(0) = 0, \quad u(0) = 1, \end{gathered} \tag{2.1}$$

has a unique solution $\psi \in C([0,\infty)) \cap C^1((0,\infty))$ satisfying for each $t \in [0,\infty)$,

$$1 \le \psi(t) \le \exp\Big(\int_0^t \frac{1}{A(s)}\Big(\int_0^s A(r)q(r)dr\Big)ds\Big).$$

Proof. Let K be the operator defined on $C([0,\infty))$ by

$$Kf(t) = \int_0^t \frac{1}{A(s)}\Big(\int_0^s A(r)q(r)f(r)dr\Big)ds, \quad t \in [0,\infty).$$

One can see that

$$0 \le K^n 1(t) \le \frac{(K1(t))^n}{n!}, \quad \text{for } t \in [0,\infty) \text{ and } n \in \mathbb{N}.$$

Then, the series $\sum_{n\geq 0} K^n 1$ converges uniformly to a function $\psi \in C([0,\infty))$ satisfying

$$\psi(t) = 1 + \int_0^t \frac{1}{A(s)}\Big(\int_0^s A(r)q(r)\psi(r)dr\Big)ds, \quad \text{for } t \in [0,\infty).$$

This implies that $\psi \in C^1((0,\infty))$ is a solution of problem (2.1). Moreover, we have

$$1 \leq \psi(t) \leq \sum_{n\geq 0} \frac{(K1(t))^n}{n!} = \exp(K1(t)), \quad \text{for } t \in [0,\infty).$$

Now, let u, v be two solutions in $C([0,\infty)) \cap C^1((0,\infty))$ of (2.1) and $\omega = |u-v|$, then

$$0 \leq \omega(t) \leq K\omega(t), \quad \text{for } t \in [0,\infty).$$

It follows that for $t \in [0,\infty)$ and $n \in \mathbb{N}$

$$0 \leq \omega(t) \leq K^n\omega(t) \leq \|\omega\|_\infty K^n 1(t) \leq \|\omega\|_\infty \frac{(K1(t))^n}{n!}.$$

By letting $n \to \infty$, we deduce that $\omega(t) = 0$, for $t \in [0,\infty)$ and so $u = v$ on $[0,\infty)$. □

We denote by G_q the Green's function of the operator

$$u \mapsto \frac{1}{A}(Au')' - qu$$

on $(0,\infty)$ with Dirichlet conditions $Au'(0) = 0$, $u(\infty) = 0$. Then

$$G_q(x,t) = A(t)\psi(x)\psi(t)\int_{x\vee t}^\infty \frac{dr}{A(r)\psi^2(r)}, \quad \text{for } x, t \in (0,\infty).$$

So we define the potential kernel V_q in $B^+((0,\infty))$ by

$$V_q f(x) = \int_0^\infty G_q(x,t)f(t)dt.$$

Note that V_q is the unique kernel which satisfies the resolvent equation

$$V = V_q + V_q(qV) = V_q + V(qV_q). \tag{2.2}$$

So if $u \in B^+((0,\infty))$ such that $V(qu)(0) < \infty$, we have

$$(I - V_q(q.))(I + V(q.))u = (I + V(q.))(I - V_q(q.))u = u. \tag{2.3}$$

To study problem (1.5), we recall an existence result given in [1] for the nonlinear problem

$$\begin{gathered} Lu = \frac{1}{A}(Au')' = u\varphi(.,u) \quad \text{in } (0,\infty), \\ Au'(0) = 0, \quad u(\infty) = a > 0. \end{gathered} \tag{2.4}$$

Here the nonlinear term φ satisfies the following hypotheses:

- (A1) φ is nonnegative measurable function in $[0,\infty) \times (0,\infty)$.
- (A2) For each $c > 0$, there exists $q_c \in B^+((0,\infty))$ such that $Vq_c(0) < \infty$ and for each $x \in (0,\infty)$, the function $t \to t(q_c(x) - \varphi(x,t))$ is continuous and nondecreasing on $[0,c]$.

Proposition 2.3 (see [1]). *For each $a > 0$, problem* (2.4) *has a positive bounded solution $u \in C([0,\infty]) \cap C^1((0,\infty))$ satisfying for each $x \in [0,\infty)$,*

$$e^{-Vq(0)}a \le u(x) \le a,$$

where $q := q_a$ is the function given in (A2).

Lemma 2.4. *Let $a > 0$ and φ be a function satisfying* (A1), (A2). *Let u be a positive function in $C([0,\infty]) \cap C^1((0,\infty))$. Then u is a solution of* (2.4) *if and only if u satisfies*

$$u + V(u\varphi(.,u)) = a \quad \text{on } [0,\infty). \tag{2.5}$$

Proof. Let u be a positive function in $C([0,\infty]) \cap C^1((0,\infty))$ satisfying (2.5), then $u \le a$. Let $q := q_a$ be the function given by (A2), then we have

$$u\varphi(.,u) \le qu \le aq.$$

Since $Vq(0) < \infty$, it follows by Proposition 2.1 that the function $v := V(u\varphi(.,u))$ is in $C_0([0,\infty))$ and so v satisfies

$$\begin{gathered} Lv = -u\varphi(.,u) \quad \text{a.e. on } (0,\infty), \\ Av'(0) = 0, \quad v(\infty) = 0. \end{gathered} \tag{2.6}$$

This together with (2.5) proves that u is a solution of (2.4).

Now, let u be a positive function in $C([0,\infty]) \cap C^1((0,\infty))$ satisfying (2.4). Since $Au'(0) = 0$, then $Au'(x) \ge 0$ for $x \in (0,\infty)$. It follows by $u(\infty) = a$ that $u \le a$. So, by hypothesis (A2), we have

$$u\varphi(.,u) \le aq.$$

Then using again Proposition 2.1, the function $v := V(u\varphi(.,u))$ satisfies (2.6). Put $w = u + V(u\varphi(.,u))$. Hence the function w is a solution of

$$\begin{gathered} Lw = 0 \quad \text{a.e. on } (0,\infty), \\ Aw'(0) = 0, \quad w(\infty) = a. \end{gathered}$$

It follows that $w = a$ and so u satisfies (2.5). □

Proposition 2.5. *Let $\alpha > 1$ and $p \in B^+((0,\infty))$ such that $Vp(0) < \infty$. Then for each $a > 0$, problem* (1.5) *has a unique solution $u \in C([0,\infty]) \cap C^1((0,\infty))$ satisfying*

$$a\exp(-\alpha a^{\alpha-1}Vp(0)) \le u(x) \le a. \tag{2.7}$$

Proof. Let $\varphi(x,t) = p(x)t^{\alpha-1}$, then it is obvious to see that φ satisfies (A1) and (A2) where q_a is explicitly given by $q_a(x) = \alpha a^{\alpha-1}p(x)$ for $x \in (0,\infty)$. So using Proposition 2.3, problem (1.5) has a solution u in $C([0,\infty]) \cap C^1((0,\infty))$ satisfying (2.7).

Let us prove uniqueness. Let u, $v \in C([0,\infty]) \cap C^1((0,\infty))$ be two solutions of (1.5) and put $w = u - v$. Then using Lemma 2.4, the function w satisfies

$$w + V(hw) = 0 \text{ on } (0,\infty), \tag{2.8}$$

where the function $h \in B^+((0,\infty))$ is defined by

$$h(x) := \begin{cases} p(x)\frac{u^\alpha(x) - v^\alpha(x)}{u(x)-v(x)} & \text{if } u(x) \ne v(x), \\ 0 & \text{if } u(x) = v(x). \end{cases}$$

Now, since $Vh(0) \leq \alpha a^{\alpha-1}Vp(0) < \infty$, we apply the operator $(I - V_h(h.))$ on both sides of (2.8), we obtain by (2.3) that $w = 0$ on $(0, \infty)$. So the uniqueness is proved. $\square$

3. Proof of Theorem 1.1

Let $E = C([0,\infty]) \times C([0,\infty])$ endowed with the norm $\|(u,v)\| = \|u\|_\infty + \|v\|_\infty$. Then $(E, \|.\|)$ is a Banach space. Now let $a, b > 0$, to apply a fixed-point argument, we consider the set

$$\Lambda = \big\{(u,v) \in E : ae^{-V\tilde{p}(0)} \leq u \leq a \text{ and } be^{-W\tilde{q}(0)} \leq v \leq b\big\},$$

where $\tilde{p} := \alpha a^{\alpha-1}b^s p$ and $\tilde{q} := \beta b^{\beta-1}a^r q$. Then Λ is a convex closed subset of E.

We define the operator T on Λ by $T(u,v) = (y,z)$ where (y,z) is the unique solution of the problem

$$\begin{gathered}
\frac{1}{A}(Ay')'(x) = p(x)v^s(x)y^\alpha(x), \quad x \in (0,\infty),\\
\frac{1}{B}(Bz')'(x) = q(x)u^r(x)z^\beta(x), \quad x \in (0,\infty),\\
Ay'(0) = 0, \quad y(\infty) = a,\\
Bz'(0) = 0, \quad z(\infty) = b.
\end{gathered}$$

Note that if $T(u,v) = (u,v)$ then (u,v) is a solution of (1.4). So we will use the Schauder's fixed point theorem to prove that T has a fixed point in Λ.

First, we point out that T is well defined and $T\Lambda \subset \Lambda$. Indeed, if $v \leq b$ then using Proposition 2.5, the problem

$$\begin{gathered}
\frac{1}{A}(Ay')'(x) = p(x)v^s(x)y^\alpha(x), \quad x \in (0,\infty),\\
Ay'(0) = 0, \quad y(\infty) = a,
\end{gathered}$$

has a unique solution y in $C([0,\infty])$ satisfying

$$a\exp(-V\tilde{p}(0)) \leq y \leq a.$$

A similar result holds for the problem

$$\begin{gathered}
\frac{1}{B}(Bz')'(x) = q(x)u^r(x)z^\beta(x), \quad x \in (0,\infty),\\
Bz'(0) = 0, \quad z(\infty) = b,
\end{gathered}$$

if the function u satisfies $u \leq a$.

Next, we prove that $T\Lambda$ is relatively compact in $C([0,\infty] \times [0,\infty])$. Let $(u,v) \in \Lambda$ and put $(y,z) = T(u,v)$. Using Lemma 2.4, the functions y and z satisfy

$$y + V(pv^sy^\alpha) = a \quad \text{on } [0,\infty), \tag{3.1}$$
$$z + W(qu^rz^\beta) = b \quad \text{on } [0,\infty). \tag{3.2}$$

Then for (x,t), $(x',t') \in ([0,\infty])^2$, we have

$$\begin{aligned}
&\|T(u,v)(x,t) - T(u,v)(x',t')\|\\
&= |y(x) - y(x')| + |z(t) - z(t')|\\
&= |V(pv^sy^\alpha)(x) - V(pv^sy^\alpha)(x')| + |W(qu^rz^\beta)(t) - W(qu^rz^\beta)(t')|.
\end{aligned}$$

Now, using that (u,v) and (y,z) are in Λ, it follows that $V(pv^sy^\alpha) \in F_{\frac{a}{\alpha}\tilde{p}}$ and $W(qu^rz^\beta) \in F_{\frac{b}{\beta}\tilde{q}}$. This implies, by Proposition 2.1, that $T\Lambda$ is equicontinuous in $[0,\infty] \times [0,\infty]$. Now, since $\{T(u,v)(x,t);\ (u,v) \in \Lambda\}$ is uniformly bounded in $[0,\infty] \times [0,\infty]$, we deduce by Ascoli's Theorem that $T\Lambda$ is relatively compact in $C([0,\infty] \times [0,\infty])$.

Let us prove the continuity of T in Λ. Let (u_n,v_n) be a sequence in Λ converging to $(u,v) \in \Lambda$ with respect to $\|.\|$. Put $(y_n,z_n) = T(u_n,v_n)$ and $(y,z) = T(u,v)$. Then

$$|T(u_n,v_n)(x,t) - T(u,v)(x,t)| = |y_n(x) - y(x)| + |z_n(t) - z(t)|.$$

We denote by $Y_n = y_n - y$ and $Z_n = z_n - z$. We start by evaluating Y_n. By (3.1), we have for $x \in [0,\infty]$

$$\begin{aligned} Y_n(x) &= V(pv^sy^\alpha)(x) - V(pv_n^sy_n^\alpha)(x) \\ &= V(py^\alpha(v^s - v_n^s))(x) - V(hY_n)(x), \end{aligned}$$

where $h \in B^+((0,\infty))$ and defined by

$$h(x) := \begin{cases} p(x)v_n^s(x)\frac{y_n^\alpha(x)-y^\alpha(x)}{y_n(x)-y(x)} & \text{if } y_n(x) \neq y(x), \\ 0 & \text{if } y_n(x) = y(x). \end{cases}$$

Since $Vh(0) < \infty$, applying the operator $(I - V_h(h.))$ on both side of

$$Y_n + V(hY_n) = V(py^\alpha(v^s - v_n^s)),$$

we obtain by (2.2) and (2.3) that

$$Y_n = V_h(py^\alpha(v^s - v_n^s)).$$

So,

$$|Y_n| \leq V(py^\alpha|v^s - v_n^s|).$$

Now, since $py^\alpha|v^s - v_n^s| \leq 2a^\alpha b^s p$ and $Vp(0) < \infty$, we deduce by the dominated convergence theorem, that

$$V(y^\alpha(v^s - v_n^s)p)(x) \to 0 \quad \text{as } n \to \infty.$$

It follows that $Y_n(x)$ converge to 0 as $n \to \infty$.

Analogously, we have $Z_n(x)$ converge to 0 as $n \to \infty$. This proves that for each $(x,t) \in [0,\infty) \times [0,\infty)$,

$$T(u_n,v_n)(x,t) \to T(u,v)(x,t) \quad \text{as } n \to \infty.$$

Now, since $T\Lambda$ is relatively compact in $C([0,\infty] \times [0,\infty])$, we deduce that

$$\|T(u_n,v_n) - T(u,v)\| \to 0 \quad \text{as } n \to \infty.$$

Hence, T is a compact mapping from Λ to itself. Then by the Schauder's fixed point theorem there exists $(u,v) \in \Lambda$ such that $T(u,v) = (u,v)$. So (u,v) is the desired solution. This completes the proof.

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