Electronic Journal of Differential Equations, Vol. 2012 (2012), No. 198, pp. 1–8.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

NONLINEAR FIRST-ORDER PERIODIC BOUNDARY-VALUE PROBLEMS OF IMPULSIVE DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. By using the fixed point theorem in cones, in this paper, existence criteria for single and multiple positive solutions to a class of nonlinear first-order periodic boundary value problems of impulsive dynamic equations on time scales are obtained. An example is given to illustrate the main results in this article.

1. Introduction

Let $\mathbb{T}$ be a time scale; i.e., is a nonempty closed subset of $\mathbb{R}$. Let 0, T be points in $\mathbb{T}$, an interval $(0,T)_{\mathbb{T}}$ denoting time scales interval, that is, $(0,T)_{\mathbb{T}} := (0,T) \cap \mathbb{T}$. Other types of intervals are defined similarly.

The theory of impulsive differential equations is emerging as an important area of investigation, since it is a lot richer than the corresponding theory of differential equations without impulse effects. Moreover, such equations may exhibit several real world phenomena in physics, biology, engineering, etc. (see [3, 17]). At the same time, the boundary value problems for impulsive differential equations and impulsive difference equations have received much attention [6, 11, 12, 18, 20, 24]. On the other hand, recently, the theory of dynamic equations on time scales has become a new important branch (See, for example, [4, 5, 10]). Naturally, some authors have focused their attention on the boundary value problems of impulsive dynamic equations on time scales [1, 2, 7, 9, 13, 14, 15, 22, 23]. However, to the best of our knowledge, few papers concerning PBVPs of impulsive dynamic equations on time scales with semi-position condition [22, 23].

In this paper, we are concerned with the existence of positive solutions for the following PBVPs of impulsive dynamic equations on time scales with semi-position condition

$$\begin{gathered} x^{\Delta}(t) + f(t, x(\sigma(t))) = 0, \quad t \in J := [0,T]_{\mathbb{T}},\ t \neq t_k,\ k = 1,2,\dots,m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1,2,\dots,m, \\ x(0) = x(\sigma(T)), \end{gathered} \tag{1.1}$$

2000 *Mathematics Subject Classification.* 39A10, 34B15.
Key words and phrases. Periodic boundary value problem; positive solution; fixed point; time scale; impulsive dynamic equation.

Submitted August 20, 2012. Published November 10, 2012.

where $\mathbb{T}$ is a time scale, $T > 0$ is fixed, $0, T \in \mathbb{T}$, $f \in C(J \times [0,\infty), (-\infty,\infty))$, $I_k \in C([0,\infty), (-\infty,\infty))$, $t_k \in (0,T)_{\mathbb{T}}$, $0 < t_1 < \cdots < t_m < T$, and for each $k = 1,2,\dots,m$, $x(t_k^+) = \lim_{h\to 0^+} x(t_k+h)$ and $x(t_k^-) = \lim_{h\to 0^-} x(t_k+h)$ represent the right and left limits of $x(t)$ at $t = t_k$.

Using fixed point theorems, Wang [22, 23] considered the existence of one or two positive solution to (1.1) when the following hypothesis holds (semi-position condition):

(A) There exists a positive number M such that
$$Mx - f(t,x) \geq 0 \text{ for } x \in [0,\infty), \quad t \in [0,T]_{\mathbb{T}}.$$

Motivated by the results mentioned above, in this paper, we shall obtain existence criteria for single and multiple positive solutions to (1.1) by means of a fixed point theorem in cones. It is worth noticing that: (i) Our hypotheses on nonlinearity f in this paper are weaker than condition (A) of [22, 23]; (ii) For the case $\mathbb{T} = \mathbb{R}$ and $I_k(x) \equiv 0, k = 1,2,\dots,m$, problem (1.1) reduces to the problem studied in [16] and for the case $I_k(x) \equiv 0, k = 1,2,\dots,m$, problem (1.1) reduces to the problem (in the one-dimension case) studied by [19]. The ideas in this article come from [21].

Theorem 1.1 ([8]). *Let X be a Banach space and K is a cone in X. Assume Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$. Let*
$$\Phi : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$$
be a continuous and completely continuous operator such that

(i) *$\|\Phi x\| \leq \|x\|$ for $x \in K \cap \partial\Omega_1$;*
(ii) *there exists $e \in K\backslash\{0\}$ such that $x \neq \Phi x + \lambda e$ for $x \in K \cap \partial\Omega_2$ and $\lambda > 0$.*

Then Φ has a fixed point in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

Remark 1.2. In Theorem 1.1, if (i) and (ii) are replaced by

(i) $\|\Phi x\| \leq \|x\|$ for $x \in K \cap \partial\Omega_2$;
(ii) there exists $e \in K\backslash\{0\}$ such that $x \neq \Phi x + \lambda e$ for $x \in K \cap \partial\Omega_1$ and $\lambda > 0$, then Φ has also a fixed point in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

2. Preliminaries

Throughout the rest of this paper, we assume that the points of impulse t_k are right-dense for each $k = 1,2,\dots,m$. We define
$$\begin{aligned} PC = \Big\{ & x \in [0,\sigma(T)]_{\mathbb{T}} \to \mathbb{R} : x_k \in C(J_k, R),\ k = 0,1,2,\dots,m \text{ and} \\ & \text{there exist } x(t_k^+) \text{ and } x(t_k^-) \text{ with } x(t_k^-) = x(t_k),\ k = 1,2,\dots,m \Big\}, \end{aligned}$$
where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]_{\mathbb{T}} \subset (0,\sigma(T)]_{\mathbb{T}}$, $k = 1,2,\dots,m$ and $J_0 = [0,t_1]_{\mathbb{T}}$, $t_{m+1} = \sigma(T)$. Let
$$X = \{x : x \in PC, \quad x(0) = x(\sigma(T))\}$$
with the norm $\|x\| = \sup_{t\in[0,\sigma(T)]_{\mathbb{T}}} |x(t)|$, then X is a Banach space.

Lemma 2.1 ([22, 23]). *Suppose $M > 0$ and $h : [0,T]_{\mathbb{T}} \to \mathbb{R}$ is rd-continuous, then x is a solution of*
$$x(t) = \int_0^{\sigma(T)} G(t,s)h(s)\triangle s + \sum_{k=1}^m G(t,t_k)I_k(x(t_k)), \quad t \in [0,\sigma(T)]_{\mathbb{T}},$$

where

$$G(t,s)=\begin{cases}\frac{e_M(s,t)e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}, & 0\le s\le t\le\sigma(T),\\ \frac{e_M(s,t)}{e_M(\sigma(T),0)-1}, & 0\le t<s\le\sigma(T),\end{cases}$$

if and only if x is a solution of the boundary-value problem

$$\begin{gathered}x^{\Delta}(t)+Mx(\sigma(t))=h(t),\quad t\in J:=[0,T]_{\mathbb{T}},\quad t\neq t_k,\quad k=1,2,\dots,m,\\ x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)),\quad k=1,2,\dots,m,\\ x(0)=x(\sigma(T)).\end{gathered}$$

Lemma 2.2. *Let $G(t,s)$ be defined as in Lemma 2.1. Then*

$$\frac{1}{e_M(\sigma(T),0)-1}\le G(t,s)\le\frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}$$

for all $t,s\in[0,\sigma(T)]_{\mathbb{T}}$.

Remark 2.3. Let $G(t,s)$ be defined as in Lemma 2.1, then $\int_0^{\sigma(T)}G(t,s)\triangle s=1/M$.

Let

$$K=\{x\in X:x(t)\ge\delta\|x\|,\ t\in[0,\sigma(T)]_{\mathbb{T}}\},$$

where $\delta=\frac{1}{e_M(\sigma(T),\quad 0)}\in(0,1)$. It is not difficult to verify that K is a cone in X.

For $u\in K$, we consider the problem

$$\begin{gathered}x^{\Delta}(t)+Mx(\sigma(t))=Mu(\sigma(t))-f(t,u(\sigma(t))),\\ t\in[0,T]_{\mathbb{T}},\ t\neq t_k,\ k=1,2,\dots,m,\\ x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)),\quad k=1,2,\dots,m,\\ x(0)=x(\sigma(T)).\end{gathered}\tag{2.1}$$

It follows from Lemma 2.1 that (2.1) has a unique solution,

$$x(t)=\int_0^{\sigma(T)}G(t,s)h_u(s)\triangle s+\sum_{k=1}^m G(t,t_k)I_k(x(t_k)),\quad t\in[0,\sigma(T)]_{\mathbb{T}},$$

where $h_u(s)=Mu(\sigma(s))-f(s,u(\sigma(s)))$, $s\in[0,T]_{\mathbb{T}}$.

We define an operator $\Phi:K\to X$ by

$$\Phi_x(t)=\int_0^{\sigma(T)}G(t,s)h_x(s)\triangle s+\sum_{k=1}^m G(t,t_k)I_k(x(t_k)),\quad t\in[0,\sigma(T)]_{\mathbb{T}}.$$

It is obvious that fixed points of Φ are solutions of (1.1).

Lemma 2.4. *The operator $\Phi:K\to X$ is completely continuous.*

The proof similar to that in [22, 23], so we omit it here.

3. Main results

In this section, by defining an appropriate cones, we impose the conditions on f which allow us to apply the fixed point theorem in cones to establish the existence criteria for single and multiple positive solutions of the problem (1.1).

Theorem 3.1. *Suppose that there exist a positive number $M > 0$ and $0 < \alpha < \beta$ such that*
$$Mx - f(t, \quad x) \geq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \beta].$$
Then (1.1) has at least one positive solution if one of the following two conditions holds: (i)
$$\begin{gathered} f(t,x) \leq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \alpha];\ \forall k, I_k(x) \geq 0,\ x \in [\delta\alpha, \alpha], \\ f(t,x) \geq 0 \text{ for } t \in [0,T]_{\mathbb{T}}, \quad x \in [\delta\beta, \beta];\ \forall k, I_k(x) \leq 0,\ x \in [\delta\beta, \beta], \end{gathered}$$
(ii)
$$\begin{gathered} f(t,x) \geq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \alpha];\ \forall k,\ I_k(x) \leq 0,\ x \in [\delta\alpha, \alpha], \\ f(t,x) \leq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta, \beta];\ \forall k,\ I_k(x) \geq 0,\ x \in [\delta\beta, \beta]. \end{gathered}$$

Proof. Define the open sets
$$\Omega_1 = \{x \in X : \|x\| < \alpha\}, \quad \Omega_2 = \{x \in X : \|x\| < \beta\}.$$
Firstly, we claim that $\Phi : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$. In fact, for any $x \in K \cap (\overline{\Omega}_2 \setminus \Omega_1)$, we have $\delta\alpha \leq x \leq \beta$, by Lemma 2.2
$$\|\Phi x\| \leq \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0) - 1}\Big[\int_0^{\sigma(T)} (Mx(\sigma(s)) - f(s, x(\sigma(s))))\triangle s + \sum_{k=1}^{m} I_k(x(t_k))\Big]$$
and
$$\begin{aligned} (\Phi x)(t) &= \int_0^{\sigma(T)} G(t,s)h_x(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(x(t_k)) \\ &\geq \frac{1}{e_M(\sigma(T),0) - 1}\Big[\int_0^{\sigma(T)} (Mx(\sigma(s)) - f(s, x(\sigma(s))))\triangle s + \sum_{k=1}^{m} I_k(x(t_k))\Big]. \end{aligned}$$
So
$$(\Phi x)(t) \geq \frac{1}{e_M(\sigma(T),0)}\|\Phi x\| = \delta\|\Phi x\|; \quad \text{i.e., } \Phi x \in K.$$
Therefore, $\Phi : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$.

Secondly, we prove the result provided conditions (i) holds. By the first inequality of (i), we have
$$Mx - f(t, \quad x) \geq Mx, \quad t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \alpha].$$
Let $e \equiv 1$, then $e \in K$. We assert that
$$x \neq \Phi x + \lambda e \quad \text{for } x \in K \cap \partial\Omega_1 \text{ and } \lambda > 0. \tag{3.1}$$
If not, there would exist $x_0 \in K \cap \partial\Omega_1$ and $\lambda_0 > 0$ such that $x_0 = \Phi x_0 + \lambda_0 e$.

Since $x_0 \in K \cap \partial\Omega_1$, it follows that $\delta\alpha = \delta\|x_0\| \leq x_0(t) \leq \alpha$. Let $\mu = \min_{t \in [0,\sigma(T)]_{\mathbb{T}}} x_0(t)$, then for any $t \in [0, \sigma(T)]_{\mathbb{T}}$, we have
$$\begin{aligned} x_0(t) &= (\Phi x_0)(t) + \lambda_0 \\ &= \int_0^{\sigma(T)} G(t,s)[Mx_0(\sigma(s)) - f(s, x_0(\sigma(s)))]\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(x_0(t_k)) + \lambda_0 \\ &\geq \int_0^{\sigma(T)} G(t,s)Mx_0(\sigma(s))\triangle s + \lambda_0 \end{aligned}$$

$$\geq \mu \int_0^{\sigma(T)} G(t,s)M\triangle s + \lambda_0 = \mu + \lambda_0.$$

This implies that $\mu \geq \mu + \lambda_0$, and this is a contradiction. Therefore (3.1) holds.

On the other hand, by using the second inequality of (i), we have

$$Mx - f(t, \quad x) \leq Mx, \quad t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta, \beta].$$

We assert that

$$\|\Phi x\| \leq \|x\| \text{ for } x \in K \cap \partial\Omega_2. \tag{3.2}$$

In fact, if $x \in K \cap \partial\Omega_2$, then $\delta\beta = \delta\|x\| \leq x(t) \leq \beta$; we have

$$\begin{aligned}
(\Phi x)(t) &= \int_0^{\sigma(T)} G(t,s)[Mx(\sigma(s)) - f(s, \quad x(\sigma(s)))]\triangle s + \sum_{k=1}^m G(t,t_k)I_k(x(t_k)) \\
&\leq \int_0^{\sigma(T)} G(t,s)Mx(\sigma(s))\triangle s \\
&\leq \int_0^{\sigma(T)} G(t,s)M\triangle s\|x\| = \|x\|.
\end{aligned}$$

Therefore, $\|\Phi x\| \leq \|x\|$.

It follows from Remark 1.2, (3.1) and (3.2) that Φ has a fixed point $x \in K \cap (\overline{\Omega}_2 \setminus \Omega_1)$. In a similar way, we can prove the result by Theorem 1.1 if condition (ii) holds. □

Theorem 3.2. *Suppose that there exist a positive number $M > 0$ and $0 < \alpha < \rho < \beta$ such that*

$$Mx - f(t, \quad x) \geq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \beta].$$

Then (1.1) *has at least two positive solutions if one of the following two conditions holds (i)*

$$\begin{gathered}
f(t,x) \leq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \alpha];\ \forall k,\ I_k(x) \geq 0,\ x \in [\delta\alpha, \alpha], \\
f(t,x) > 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\rho, \rho];\ \forall k,\ I_k(x) < 0,\ x \in [\delta\rho, \rho], \\
f(t,x) \leq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta, \beta];\ \forall k,\ I_k(x) \geq 0,\ x \in [\delta\beta, \beta],
\end{gathered}$$

(ii)

$$\begin{gathered}
f(t,x) \geq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha, \alpha];\ \forall k,\ I_k(x) \leq 0,\ x \in [\delta\alpha, \alpha], \\
f(t,x) < 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\rho, \rho];\ \forall k,\ I_k(x) > 0,\ x \in [\delta\rho, \rho], \\
f(t,x) \geq 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta, \beta];\ \forall k,\ I_k(x) \leq 0,\ x \in [\delta\beta, \beta],
\end{gathered}$$

Proof. We prove only the result when condition (i) holds. In a similar way we can obtain the result if condition (ii) holds. Define Ω_1, Ω_2 as in Theorem 3.1 and define

$$\Omega_3 = \{x \in X : \|x\| < \rho\}.$$

Similar to the proof of Theorem 3.1, we can prove that

$$x \neq \Phi x + \lambda e \text{ for } x \in K \cap \partial\Omega_1 \text{ and } \lambda > 0, \tag{3.3}$$

$$x \neq \Phi x + \lambda e \text{ for } x \in K \cap \partial\Omega_2 \text{ and } \lambda > 0, \tag{3.4}$$

where $e \equiv 1 \in K$, and

$$\|\Phi x\| < \|x\| \quad \text{for } x \in K \cap \partial\Omega_3. \tag{3.5}$$

Thus we can obtain the existence of two positive solutions x_1 and x_2 by using Theorem 1.1 and Remark 1.2, respectively. It is easy to see that $\alpha \le \|x_1\| < \rho < \|x_2\| \le \beta$. □

Theorem 3.3. *Suppose that there exist a positive number $M > 0$ and $0 < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \dots < \alpha_n < \beta_n$ such that*

$$Mx - f(t, \quad x) \ge 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha_1, \beta_n].$$

Then (1.1) has at least n multiple positive solutions x_i ($1 \le i \le n$) satisfying $\alpha_i \le \|x_i\| \le \beta_i$, $1 \le i \le n$, if one of the following two conditions holds (i)

$$f(t,x) \le 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha_i, \alpha_i];\ \forall k,\ I_k(x) \ge 0,\ x \in [\delta\alpha_i, \alpha_i],\ 1 \le i \le n,$$
$$f(t,x) \ge 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta_i, \beta_i];\ \forall k, I_k(x) \le 0,\ x \in [\delta\beta_i, \beta_i],\ 1 \le i \le n,$$

(ii)

$$f(t,x) \ge 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha_i, \alpha_i];\ \forall k, I_k(x) \le 0,\ x \in [\delta\alpha_i, \alpha_i],\ 1 \le i \le n,$$
$$f(t,x) \le 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta_i, \beta_i];\ \forall k, I_k(x) \ge 0,\ x \in [\delta\beta_i, \beta_i],\ 1 \le i \le n.$$

Remark 3.4. In theorem 3.3, if (i) and (ii) are replaced by (iii)

$$f(t,x) < 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha_i, \alpha_i];\ \forall k, I_k(x) > 0,\ x \in [\delta\alpha_i, \alpha_i],\ 1 \le i \le n,$$
$$f(t,x) > 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta_i, \beta_i];\ \forall k, I_k(x) < 0,\ x \in [\delta\beta_i, \beta_i],\ 1 \le i \le n;$$

(iv)

$$f(t,x) > 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\alpha_i, \alpha_i];\ \forall k, I_k(x) < 0,\ x \in [\delta\alpha_i, \alpha_i],\ 1 \le i \le n,$$
$$f(t,x) < 0 \quad \text{for } t \in [0,T]_{\mathbb{T}},\ x \in [\delta\beta_i, \beta_i];\ \forall k, I_k(x) > 0,\ x \in [\delta\beta_i, \beta_i],\ 1 \le i \le n.$$

Then (1.1) has at least $2n - 1$ multiple positive solutions.

4. Examples

Example 4.1. Let $\mathbb{T} = [0,1] \cup [2,3]$. We consider the following problem on $\mathbb{T}$:

$$\begin{gathered} x^{\Delta}(t) + f(t, x(\sigma(t))) = 0, \quad t \in [0,3]_{\mathbb{T}},\ t \ne \frac{1}{2}, \\ x(\frac{1}{2}^+) - x(\frac{1}{2}^-) = I(x(\frac{1}{2})), \\ x(0) = x(3), \end{gathered} \tag{4.1}$$

where $T = 3$, $f(t,x) = x - x^{1/2} + \frac{7}{64}$, and $I(x) = x^{1/2} - x$.

Let $M = 1$, $\alpha = e^2/32$, $\beta = 4e^2$. Then $e_M(\sigma(T), 0) = 2e^2$, $\delta = 1/(2e^2)$, it is easy to see that

$$Mx - f(t,x) = x^{1/2} - \frac{7}{64} \ge \frac{1}{8} - \frac{7}{64} = \frac{1}{64} > 0, \quad \text{for } x \in [\frac{1}{64}, 4e^2] = [\delta\alpha, \beta],$$

and

$$f(t,x) = x - x^{1/2} + \frac{7}{64} \le \frac{1}{64} - \frac{1}{8} + \frac{7}{64} = 0, \quad \text{for } x \in [\frac{1}{64}, \frac{e^2}{32}] = [\delta\alpha, \alpha];$$
$$f(t,x) = x - x^{1/2} + \frac{7}{64} > 0, \quad \text{for } x \in [2, 4e^2] = [\delta\beta, \beta];$$
$$I(x) = x^{1/2} - x \ge \frac{1}{8} - \frac{1}{64} > 0, \quad \text{for } x \in [\frac{1}{64}, \frac{e^2}{32}] = [\delta\alpha, \alpha];$$
$$I(x) = x^{1/2} - x \le 2^{1/2} - 2 < 0, \quad \text{for } x \in [2, 4e^2] = [\delta\beta, \beta].$$

Therefore, by Theorem 3.1, it follows that (4.1) has at least one positive solution.

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