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POSITIVE PERIODIC SOLUTIONS OF NONLINEAR FIRST-ORDER FUNCTIONAL DIFFERENCE EQUATIONS WITH A PARAMETER

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Abstract. We obtain the existence and multiplicity of positive T-periodic solutions for the difference equations

$$\Delta x(n) = a(n, x(n)) - \lambda b(n)f(x(n - \tau(n)))$$

and

$$\Delta x(n) + a(n, x(n)) = \lambda b(n)f(x(n - \tau(n))),$$

where $f(\cdot)$ may be singular at $x = 0$. Using a fixed point theorem in cones, we extend recent results in the literature.

1. Introduction

In recent years, there has been considerable interest in the existence of periodic solutions of the equation

$$x'(t) = \tilde{a}(t, x(t)) - \lambda\tilde{b}(t)\tilde{f}(x(t - \tau(t))), \tag{1.1}$$

where $\lambda > 0$ is a positive parameter, $\tilde{a}$ is continuous in x and T-periodic in t, $\tilde{b} \in C(\mathbb{R}, [0,\infty))$ and $\tau \in C(\mathbb{R}, \mathbb{R})$ are T-periodic functions, $\int_0^T \tilde{b}(t)dt > 0$, $f \in C([0,\infty), [0,\infty))$. (1.1) has been proposed as a model for a variety of physiological processes and conditions including production of blood cells, respiration, and cardiac arrhythmias. See, for example, [1, 2, 4, 6, 7, 13, 11, 14, 15, 16] and the references therein.

In this article, we study the existence of positive T-periodic solutions of a discrete analogues to (1.1) of the form

$$\Delta x(n) = a(n, x(n)) - \lambda b(n)f(x(n - \tau(n))), \quad n \in \mathbb{Z} \tag{1.2}$$

and

$$\Delta x(n) + a(n, x(n)) = \lambda b(n)f(x(n - \tau(n))), \quad n \in \mathbb{Z}, \tag{1.3}$$

where $\mathbb{Z}$ is the set of integer numbers, $T \in \mathbb{N}$ is a fixed integer, $a : \mathbb{Z}\times[0,\infty) \to [0,\infty)$ is continuous in x and T-periodic in n, $b : \mathbb{Z} \to [0,+\infty)$, $\tau : \mathbb{Z} \to \mathbb{Z}$ are T-periodic

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and $\sum_{n=0}^{T-1} b(n) > 0$, $f \in C((0,+\infty),(0,+\infty))$ and may have a repulsive singularity near $x = 0$, $\lambda > 0$ is a parameter.

So far, relatively little is known about the existence of positive periodic solutions of (1.2) and (1.3). To our best knowledge, Ma [10] dealt with the special equations of (1.2) and (1.3) of the form

$$\Delta x(n) = a(n)g(x(n))x(n) - \lambda b(n)f(x(n-\tau(n))) \tag{1.4}$$

and

$$\Delta x(n) + a(n)g(x(n))x(n) = \lambda b(n)f(x(n-\tau(n))), \tag{1.5}$$

with certain values of λ, for which there exist positive T-periodic solutions of (1.4) and (1.5), respectively. If $g(x(n)) \equiv 1$, this special case see [8, 9, 12]. All these authors [8, 9, 10, 12] focus their attention on the fact that the number of positive T-periodic solutions can be determined by the behaviors of the quotient of $f(x)/x$ at $\{0,+\infty\}$. However, our main results show the number of positive T-periodic solutions can be determined by the behaviors of the quotient of $f(x)/x$ at $[0,\infty]$.

It is the purpose of this paper to study more general equations (1.2) and (1.3) and generalize the main results of Ma [10]. We also establish some existence and multiplicity for (1.2) and (1.3), respectively. The main tool we will use is the fixed point index theory [3, 5]. Throughout this paper, we denote the product of $x(n)$ from $n = a$ to $n = b$ by $\prod_{n=a}^{b} x(n)$ with the understanding that $\prod_{n=a}^{b} x(n) = 1$ for all $a > b$.

The rest of the paper is arranged as follows: In Section 2, we give some preliminary results. In Section 3 we state and prove some existence results of positive periodic solutions for (1.2) and (1.3). Finally, Section 4 is devoted to improving some results of Ma [10]. For related results on the associated differential equations, see Weng and Sun [14].

2. Preliminaries

In this article, we make the following assumptions:

(H1) There exist functions $a_1, a_2 : \mathbb{Z} \to [0,+\infty)$ are $T-$periodic functions such that $\sum_{n=0}^{T-1} a_1(n) > 0$, $\sum_{n=0}^{T-1} a_2(n) > 0$ and $a_1(n)x(n) \le a(n,x(n)) \le a_2(n)x(n)$ for $n \in \mathbb{Z}$ and $x > 0$. In addition, $\lim_{x\to 0} \frac{a(n,x)}{x}$ exists for $n \in \mathbb{Z}$.

(H2) $a(n,x)$ is continuous in x and T-periodic in n, $b : \mathbb{Z} \to [0,+\infty)$, $\tau : \mathbb{Z} \to \mathbb{Z}$ are T-periodic and $B := \sum_{n=0}^{T-1} b(n) > 0$; $f \in C((0,+\infty),(0,+\infty))$ and may have a repulsive singularity near $x = 0$.

Denote

$$\sigma_i = \prod_{s=0}^{T-1} (1 + a_i(s))^{-1}, \quad i = 1, 2, \quad m = \frac{\sigma_2}{1-\sigma_2}, \quad M = \frac{1}{1-\sigma_1}.$$

From (H1), it is clear that $0 < \frac{m}{M} < 1$. Let

$$E := \{x : \mathbb{Z} \to \mathbb{R} : x(n+T) = x(n)\}$$

be the Banach space with the norm $\|x\| = \max_{n\in\mathbb{Z}} |x(n)|$. Define the cone

$$P := \{x \in E : x(n) \ge 0,\ x(n) \ge \frac{m}{M}\|x\|\},$$

and the operator $A_\lambda : P \to E$ by

$$(A_\lambda x)(n) = \lambda \sum_{s=n}^{n+T-1} G_x(n,s)b(s)f(x(s-\tau(s))), \quad n \in \mathbb{Z},$$

where

$$G_x(n,s) = \frac{\prod_{k=n}^{s}(1+\frac{a(k,x(k))}{x(k)})^{-1}}{1-\prod_{k=1}^{T}(1+\frac{a(k,x(k))}{x(k)})^{-1}}, \quad n \le s \le n+T.$$

It follows from (H1) that

$$m \le G_x(n,s) \le M.$$

If (H1) and (H2) hold and $x \in P$, then

$$\lambda m \sum_{s=n}^{n+T-1} b(s)f(x(s-\tau(s))) \le \|A_\lambda x\| \le \lambda M \sum_{s=n}^{n+T-1} b(s)f(x(s-\tau(s))). \tag{2.1}$$

The construction of $G_x(n,s)$ is due to Ma [10]. Following the approach in [10], we can easily prove the following two Lemmas. Similar arguments have been also employed in [12]. We remark that the process of proofs are similar and are omitted.

Lemma 2.1. *Assume that* (H1), (H2) *hold. Then $A_\lambda(P) \subset P$ and $A_\lambda : P \to P$ is compact and continuous.*

Lemma 2.2. *Assume that* (H1), (H2) *hold. Then $x \in P$ is a solution of* (1.2) *if and only if x is a fixed point of A_λ in P.*

The following well-known result of the fixed point index is crucial in our arguments.

Lemma 2.3 ([3, 5])**.** *Let E be a Banach space and K be a cone in E. For $r > 0$, define $K_r = \{u \in K : \|u\| < r\}$. Assume that $T : \overline{K}_r \to K$ is completely continuous such that $Tu \ne u$ for $u \in \partial K_r = \{u \in K : \|u\| = r\}$.*

(i) *If $\|Tu\| \ge \|u\|$ for $u \in \partial K_r$, then $i(T, K_r, K) = 0$.*
(ii) *If $\|Tu\| \le \|u\|$ for $u \in \partial K_r$, then $i(T, K_r, K) = 1$.*

3. Existence of positive periodic solutions for (1.2) and (1.3)

In this section, we shall provide two explicit intervals of λ such that (1.2) and (1.3) have at least one positive T-periodic solution.

Theorem 3.1. *Assume that* (H1), (H2) *hold and there exist R, r such that $R > r > 0$ and*

$$m^2 \min_{x\in[\frac{m}{M}r,r]} \frac{f(x)}{x} > M^2 \max_{x\in[R,\frac{M}{m}R]} \frac{f(x)}{x}. \tag{3.1}$$

Then, for each λ satisfying

$$\frac{M}{m^2 B \min_{x\in[\frac{m}{M}r,r]} \frac{f(x)}{x}} < \lambda \le \frac{1}{MB \max_{x\in[R,\frac{M}{m}R]} \frac{f(x)}{x}}, \tag{3.2}$$

equation (1.2) *has a positive T-periodic solution x satisfying $r < x \le \frac{M}{m}R$.*

Proof. According to (3.1), the set $\{\lambda : \lambda \text{ satisfies } (3.2)\}$ is nonempty. It follows from (3.2) that

$$\frac{f(x)}{x} > \frac{M}{\lambda m^2 B}, \quad \forall x \in [\frac{m}{M}r, r] \quad \text{and} \quad \frac{f(x)}{x} \le \frac{1}{\lambda MB}, \quad x \in [R, \frac{M}{m}R].$$

Define the open sets

$$\Omega_1 := \{x \in E : \|x\| < r\}, \quad \Omega_2 := \{x \in E : \|x\| < \frac{M}{m}R\}.$$

If $x \in \partial\Omega_1 \cap P$, then $\|x\| = r$ and $\frac{m}{M}r \le x \le r$. According to (2.1), it follows that

$$\begin{aligned} A_\lambda x(n) &\ge \lambda m \sum_{s=n}^{n+T-1} b(s) f(x(s-\tau(s))) \\ &> \lambda m \sum_{s=n}^{n+T-1} b(s) \frac{M}{\lambda m^2 B} x(s-\tau(s))) \\ &\ge \frac{M}{mB} \sum_{s=0}^{T-1} b(s) \frac{m}{M} r = r = \|x\|. \end{aligned}$$

Hence $\|A_\lambda x\| > \|x\|$, $x \in \partial\Omega_1 \cap P$. From Lemma 2.3, we have that

$$i(A_\lambda, \Omega_1 \cap P, P) = 0.$$

If $x \in \partial\Omega_2 \cap P$, then $\|x\| = \frac{M}{m}R$ and $R \le x \le \frac{M}{m}R$. According to (2.1), it follows that

$$\begin{aligned} \|A_\lambda x\| &\le \lambda M \sum_{s=n}^{n+T-1} b(s) f(x(s-\tau(s))) \\ &\le \lambda M \sum_{s=n}^{n+T-1} b(s) \frac{1}{\lambda MB} x(s-\tau(s))) \\ &\le \frac{1}{B} \sum_{s=0}^{T-1} b(s) \frac{M}{m} R = \frac{M}{m} R = \|x\|. \end{aligned}$$

Hence $\|A_\lambda x\| \le \|x\|$, $x \in \partial\Omega_2 \cap P$. From Lemma 2.3, we have that

$$i(A_\lambda, \Omega_2 \cap P, P) = 1.$$

Thus $i(A_\lambda, \Omega_2 \backslash \bar{\Omega}_1, P) = 1$ and A_λ has a fixed point in $\Omega_2 \backslash \bar{\Omega}_1$, which is a positive T-periodic solution of (1.2) and

$$r < x(n) \le \frac{M}{m}R, \quad n \in \mathbb{Z}.$$

□

Theorem 3.2. *Assume that* (H1), (H2) *hold and there exist* R, r *such that* $R > r > 0$ *and*

$$m^2 \min_{x \in [R, \frac{M}{m}R]} f(x)/x > M^2 \max_{x \in [\frac{m}{M}r, r]} f(x)/x. \tag{3.3}$$

Then, for each λ *satisfying*

$$\frac{M}{m^2 B \min_{x \in [R, MR/m]} f(x)/x} \le \lambda < \frac{1}{MB \max_{x \in [m/rM, r]} f(x)/x}, \tag{3.4}$$

equation (1.2) *has a positive* T*-periodic solution* x *satisfying* $r < x \le \frac{M}{m}R$.

Proof. By (3.3), the set $\{\lambda : \lambda \text{ satisfies } (3.4)\}$ is nonempty. It follows from (3.4) that

$$\frac{f(x)}{x} < \frac{1}{\lambda MB}, \quad \forall x \in [\frac{m}{M}r, r] \quad \text{and} \quad \frac{f(x)}{x} \geq \frac{M}{\lambda m^2 B}, \quad x \in [R, \frac{M}{m}R].$$

The rest of the proof is similar to the proof of Theorem 3.1 and is omitted. □

Next we turn our attention to (1.3); i.e.,

$$x(n+1) = [1 - \frac{a(n, x(n))}{x(n)}]x(n) + \lambda b(n) f(x(n - \tau(n))), \quad n \in \mathbb{Z}, \tag{3.5}$$

where $\lambda, a(n), b(n), f(x(n - \tau(n)))$ satisfy the same assumptions stated for (1.2) except that

$$0 < \prod_{k=0}^{T-1}(1 - a_2(k)) \leq \prod_{k=0}^{T-1}(1 - a_1(k)) < 1,$$

for all $n \in \mathbb{Z}$. In view of (1.3) we have

$$x(n) = \lambda \sum_{s=n}^{n+T-1} K_x(n, s) b(s) f(x(s - \tau(s))), \tag{3.6}$$

where

$$K_x(n, s) = \frac{\prod_{k=s+1}^{n+T-1}(1 - \frac{a(k, x(k))}{x(k)})}{1 - \prod_{k=0}^{T-1}(1 - \frac{a(k, x(k))}{x(k)})}, \quad s \in [n, n + T - 1]. \tag{3.7}$$

Note that since $0 \leq a_1(n) \leq a(n, x(n)) \leq a_2(n) < 1$ for all $n \in \mathbb{Z}$, we have

$$\bar{m} := \frac{\rho_2}{1 - \rho_2} \leq K_x(n, s) \leq \frac{1}{1 - \rho_1} := \bar{M}, \quad n \leq s \leq n + T - 1,$$

here

$$\rho_i = \prod_{k=0}^{T-1}(1 - a_i(k)), \quad i = 1, 2 \quad \text{and} \quad 0 < \frac{\rho_2(1 - \rho_1)}{1 - \rho_2} < 1.$$

Similarly, we can get the following theorems.

Theorem 3.3. *Assume that* (H1), (H2) *hold and* $0 \leq a_1(n) \leq a_2(n) < 1$ *for* $n \in \mathbb{Z}$. *Moreover, there exist* R, r *such that* $R > r > 0$ *and*

$$\bar{m}^2 \min_{x \in [\frac{\bar{m}}{\bar{M}}r, r]} \frac{f(x)}{x} > \bar{M}^2 \max_{x \in [R, \frac{\bar{M}}{\bar{m}}R]} \frac{f(x)}{x}.$$

Then, for each λ *satisfying*

$$\frac{\bar{M}}{\bar{m}^2 B \min_{x \in [\frac{\bar{m}}{\bar{M}}r, r]} \frac{f(x)}{x}} < \lambda \leq \frac{1}{\bar{M} B \max_{x \in [R, \frac{\bar{M}}{\bar{m}}R]} \frac{f(x)}{x}},$$

equation (1.3) *has a positive* T*-periodic solution* x *satisfying* $r < x \leq \frac{\bar{M}}{\bar{m}}R$.

Theorem 3.4. *Assume that* (H1)-(H2) *hold and* $0 \leq a_1(n) \leq a_2(n) < 1$ *for* $n \in \mathbb{Z}$. *In addition, there exist* R, r *such that* $R > r > 0$ *and*

$$\bar{m}^2 \min_{x \in [R, \frac{\bar{M}}{\bar{m}}R]} \frac{f(x)}{x} > \bar{M}^2 \max_{x \in [\frac{\bar{m}}{\bar{M}}r, r]} \frac{f(x)}{x}.$$

Then, for each λ satisfying

$$\frac{\bar{M}}{\bar{m}^2 B \min_{x\in[R,\frac{\bar{M}}{\bar{m}}R]} \frac{f(x)}{x}} \le \lambda < \frac{1}{\bar{M} B \max_{x\in[\frac{\bar{m}}{\bar{M}}r,r]} \frac{f(x)}{x}},$$

equation (1.3) *has a positive T-periodic solution x satisfying $r < x \le \frac{\bar{M}}{\bar{m}}R$.*

4. Multiplicity of positive periodic solutions for (1.2) and (1.3)

To illustrate applications of Theorems 3.1-3.4, we will provide four corollaries in this section. For convenience, we introduce the notation

$$i_0 = \text{number of zeros in the set } \{f_0, f_\infty\},$$
$$i_\infty = \text{number of infinities in the set } \{f_0, f_\infty\}.$$

It is clear that $i_0, i_\infty = 0, 1$ or 2. Then we shall show that (1.2) has i_0 or i_∞ positive T-periodic solution(s) for sufficiently large or small λ, respectively.

Corollary 4.1. *Assume that* (H1), (H2) *hold and $c \in (0,\infty)$ is a fixed constant, then*

- (i) *If $i_0 = 1$ or 2, then* (1.2) *has i_0 positive T-periodic solution(s) for $\lambda > \frac{M}{m^2 B \min_{x\in[mc/M,c]} f(x)/x}$.*
- (ii) *If $i_\infty = 1$ or 2, then* (1.2) *has i_∞ positive T-periodic solution(s) for $0 < \lambda < \frac{1}{MB \max_{x\in[c,Mc/m]} f(x)/x}$.*

Proof. (i) If $f_0 = 0$, then there exists small enough r_1 such that $c > r_1 > 0$ and

$$m^2 \min_{x\in[mc/M,c]} \frac{f(x)}{x} \ge M^2 \max_{x\in[\frac{m^2}{M^2}r_1,\frac{m}{M}r_1]} \frac{f(x)}{x} \to 0 \quad (\text{as } r_1 \to 0).$$

By applying Theorem 3.2 with $R = \frac{m}{M}c$ and $r = \frac{m}{M}r_1$, Equation (1.2) has a positive T-periodic solution x satisfying

$$\frac{m}{M}r_1 < x \le c.$$

If $f_\infty = 0$, then there exists large enough R_1 such that $R_1 > c > 0$ and

$$m^2 \min_{x\in[mc/M,c]} \frac{f(x)}{x} \ge M^2 \max_{x\in[\frac{M}{m}R_1,\frac{M^2}{m^2}R_1]} \frac{f(x)}{x} \to 0 \quad (\text{as } R_1 \to \infty).$$

Thus, by applying Theorem 3.1 with $R = \frac{M}{m}R_1$ and $r = c$, there exists a positive T-solution x of Eq.(1.2) satisfying

$$c < x \le \frac{M^2}{m^2}R.$$

(ii) If $f_0 = \infty$, then there exists small enough r_2 such that $c > r_2 > 0$ and

$$M^2 \max_{x\in[c,\frac{M}{m}c]} \frac{f(x)}{x} \le m^2 \min_{x\in[\frac{m^2}{M^2}r_2,\frac{m}{M}r_2]} \frac{f(x)}{x} \to \infty \quad (\text{as } r_2 \to 0).$$

Thus, by applying Theorem 3.1 with $R = c$ and $r = \frac{m}{M}r_2$, Equation (1.2) has a positive T-periodic solution x satisfying

$$\frac{m}{M}r_2 < x \le \frac{M}{m}c.$$

If $f_\infty = \infty$, then there exists large enough $R_2 > c > 0$ such that

$$M^2 \max_{x\in[c,\frac{M}{m}c]} \frac{f(x)}{x} \le m^2 \min_{x\in[\frac{M}{m}R_2,\frac{M^2}{m^2}R_2]} \frac{f(x)}{x} \to \infty \quad (\text{as } R_2 \to \infty).$$

Thus, by applying Theorem 3.2 with $R = \frac{M}{m}R_2$ and $r = \frac{M}{m}c$, there exists a positive T-solution x of (1.2) satisfying

$$\frac{M}{m}c < x \le \frac{M^2}{m^2}R_2.$$

□

Corollary 4.2. *Assume that* (H1), (H2) *hold and* $i_0 = i_\infty = 0$*, then*

(1) *If* $m^2 f_0 > M^2 f_\infty$*, Equation* (1.2) *has a positive* T*-periodic solution for*

$$\frac{M}{m^2 B f_0} < \lambda < \frac{1}{MBf_\infty}.$$

(2) *If* $m^2 f_\infty > M^2 f_0$*, Equation* (1.2) *has a positive* T*-periodic solution for*

$$\frac{M}{m^2 B f_\infty} < \lambda < \frac{1}{MBf_0}.$$

Proof. (1) Since $m^2 f_0 > M^2 f_\infty$, inequality (3.1) is satisfied by taking r small enough and R large enough. According to Theorem 3.1, Equation (1.2) has a positive T-periodic solution for

$$\frac{M}{m^2 B(f_0+\epsilon)} < \lambda < \frac{1}{MB(f_\infty-\epsilon)},$$

where $\epsilon > 0$ is sufficiently small.

(2) Since $m^2 f_\infty > M^2 f_0$, inequality (3.3) is satisfied by taking r small enough and R large enough. As a consequence of Theorem 3.2, Equation (1.2) has a positive T-periodic solution for

$$\frac{M}{m^2 B(f_\infty+\epsilon)} < \lambda < \frac{1}{MB(f_0-\epsilon)},$$

where $\epsilon > 0$ is sufficiently small. □

Remark 4.3. Corollary 4.1 improves the results in Ma [10, Theorem 4.1]. Since assertion (b) in [10, Theorem 4.1] fails to the case $\lim_{x\to 0^+} f(x) = +\infty$, which is due to the definition of $M(r) = \max\{f(x) : 0 \le x \le r\}$. However, Corollary 4.1 is valid to the case $\lim_{x\to 0^+} f(x) = +\infty$ and provides more desirable intervals of λ.

If $a(n,x)$ of (1.2) is replaced with $a(n)g(x(n))x(n)$ of (1.4), then Corollary 4.2 is exactly the same as [10, Theorem 4.3].

The following results are direct consequences of Theorems 3.3 and 3.4.

Corollary 4.4. *Assume that* (H1), (H2) *hold and* $c \in (0,\infty)$ *is a fixed constant, then*

(i) *If* $i_0 = 1$ *or* 2*, then* (1.3) *has* i_0 *positive* T*-periodic solutions for*

$$\lambda > \frac{\bar{M}}{\bar{m}^2 B \min_{x\in[\frac{\bar{m}}{\bar{M}}c,c]} f(x)/x}.$$

(ii) *If* $i_\infty = 1$ *or* 2*, then* (1.3) *has* i_∞ *positive* T*-periodic solutions for*

$$0 < \lambda < \frac{1}{\bar{M}B\max_{x\in[c,\frac{\bar{M}}{\bar{m}}c]} f(x)/x}.$$

Corollary 4.5. *Assume that* (H1), (H2) *hold and* $i_0 = i_\infty = 0$*, then*

(1) *If* $\bar{m}^2 f_0 > \bar{M}^2 f_\infty$*, Equation* (1.3) *has a positive* T*-periodic solution for*

$$\frac{\bar{M}}{\bar{m}^2 B f_0} < \lambda < \frac{1}{\bar{M}Bf_\infty}.$$

(2) *If* $\bar{m}^2 f_\infty > \bar{M}^2 f_0$*, Equation* (1.3) *has a positive* T*-periodic solution for*

$$\frac{\bar{M}}{\bar{m}^2 B f_\infty} < \lambda < \frac{1}{\bar{M}Bf_0}.$$

Remark 4.6. Corollary 4.4 improves the results in [10, Theorem 4.4]. Since assertion (b) in [10, Theorem 4.4] fails to the case $\lim_{x\to 0^+} f(x) = +\infty$, which is due to the definition of $M(r) = \max\{f(x) : 0 \le x \le r\}$. However, Corollary 4.4 is valid to the case $\lim_{x\to 0^+} f(x) = +\infty$ and provides more desirable intervals of λ.

If $a(n,x)$ of (1.3) is replaced with $a(n)g(x(n))x(n)$ of (1.5), then Corollary 4.5 is exactly the same as [10, Theorem 4.6].

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