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EXISTENCE OF POSITIVE SOLUTIONS FOR SELF-ADJOINT BOUNDARY-VALUE PROBLEMS WITH INTEGRAL BOUNDARY CONDITIONS AT RESONANCE

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Abstract. In this article, we study the self-adjoint second-order boundary-value problem with integral boundary conditions,

$$(p(t)x'(t))' + f(t, x(t)) = 0, \quad t \in (0, 1),$$

$$p(0)x'(0) = p(1)x'(1), \quad x(1) = \int_0^1 x(s)g(s)ds,$$

which involves an integral boundary condition. We prove the existence of positive solutions using a new tool: the Leggett-Williams norm-type theorem for coincidences.

1. Introduction

This paper concerns the existence of positive solutions to the following boundary value problem at resonance:

$$(p(t)x'(t))' + f(t, x(t)) = 0, \quad t \in (0, 1), \tag{1.1}$$

$$p(0)x'(0) = p(1)x'(1), \quad x(1) = \int_0^1 x(s)g(s)ds, \tag{1.2}$$

where $g \in L^1[0, 1]$ with $g(t) \geq 0$ on $[0, 1]$, $\int_0^1 g(s)ds = 1$, $p \in C[0, 1] \cap C^1(0, 1)$, $p(t) > 0$ on $[0, 1]$.

Recently much attention has been paid to the study of certain nonlocal boundary value problems (BVPs). The methodology for dealing with such problems varies. For example, Kosmatov [7] applied a coincidence degree theorem due to Mawhin and obtained the existence of at least one solution of the BVP at resonance

$$u''(t) = f(t, u(t), u'(t)), \ t \in (0, 1),$$

$$u'(0) = u'(\eta), \quad \sum_{i=1}^{n} \alpha_i u(\eta_i) = u(1),$$

under the assumptions $\sum_{i=1}^{n} \alpha_i = 1$ and $\sum_{i=1}^{n} \alpha_i \eta_i = 1$.

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Han [5] studied the three-point BVP at resonance

$$x''(t) = f(t, x(t)), \quad t \in (0,1),$$
$$x'(0) = 0, \quad x(\eta) = x(1).$$

The author rewrote the original BVP as an equivalent problem, and then used the Krasnolsel'skii-Gue fixed point theorem.

Although the existing literature on solutions of BVPs is quite wide, to the best of our knowledge, only a few papers deal with the existence of positive solutions to multi-point BVPs at resonance. In particular, there has been no work done for the BVP (1.1)-(1.2). Moreover, Our main approach is different from the ones existing and our main ingredient is the Leggett-Williams norm-type theorem for coincidences obtained by O'Regan and Zima [9].

2. Related Lemmas

For the convenience of the reader, we review some standard facts on Fredholm operators and cones in Banach spaces. Let X, Y be real Banach spaces. Consider a linear mapping $L : \operatorname{dom} L \subset X \to Y$ and a nonlinear operator $N : X \to Y$. Assume that

- (A1) L is a Fredholm operator of index zero; that is, $\operatorname{Im} L$ is closed and $\dim \ker L = \operatorname{codim} \operatorname{Im} L < \infty$.

This assumption implies that there exist continuous projections $P : X \to X$ and $Q : Y \to Y$ such that $\operatorname{Im} P = \ker L$ and $\ker Q = \operatorname{Im} L$. Moreover, since $\dim \operatorname{Im}Q = \operatorname{codim} \operatorname{Im} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \ker L$. Denote by L_p the restriction of L to $\ker P \cap \operatorname{dom} L$. Clearly, L_p is an isomorphism from $\ker P \cap \operatorname{dom} L$ to $\operatorname{Im} L$, we denote its inverse by $K_p : \operatorname{Im} L \to \ker P \cap \operatorname{dom} L$. It is known (see [8]) that the coincidence equation $Lx = Nx$ is equivalent to

$$x = (P + JQN)x + K_P(I - Q)Nx.$$

Let C be a cone in X such that

- (i) $\mu x \in C$ for all $x \in C$ and $\mu \geq 0$,
- (ii) $x, -x \in C$ implies $x = \theta$.

It is well known that C induces a partial order in X by

$$x \preceq y \quad \text{if and only if} \quad y - x \in C.$$

The following property is valid for every cone in a Banach space X.

Lemma 2.1 ([10]). *Let C be a cone in X. Then for every $u \in C \setminus \{0\}$ there exists a positive number $\sigma(u)$ such that*

$$\|x + u\| \geq \sigma(u)\|u\| \quad \textit{for all } x \in C.$$

Let $\gamma : X \to C$ be a retraction; that is, a continuous mapping such that $\gamma(x) = x$ for all $x \in C$. Set

$$\Psi := P + JQN + K_p(I - Q)N \quad \text{and} \quad \Psi_\gamma := \Psi \circ \gamma.$$

We use the following result due to O'Regan and Zima, with the following assumptions:

- (A2) $QN : X \to Y$ is continuous and bounded and $K_p(I - Q)N : X \to X$ be compact on every bounded subset of X,
- (A3) $Lx \neq \lambda Nx$ for all $x \in C \cap \partial\Omega_2 \cap ImL$ and $\lambda \in (0, 1)$,

(A4) γ maps subsets of $\overline{\Omega}_2$ into bounded subsets of C,
(A5) $\deg\{[I-(P+JQN)\gamma]|_{\ker L}, \ker L\cap\Omega_2, 0\}\neq 0$,
(A6) there exists $u_0\in C\setminus\{0\}$ such that $\|x\|\leq\sigma(u_0)\|\Psi x\|$ for $x\in C(u_0)\cap\partial\Omega_1$, where $C(u_0)=\{x\in C:\mu u_0\preceq x\ for\ some\ \mu>0\}$ and $\sigma(u_0)$ such that $\|x+u_0\|\geq\sigma(u_0)\|x\|$ for every $x\in C$,
(A7) $(P+JQN)\gamma(\partial\Omega_2)\subset C$,
(A8) $\Psi_\gamma(\overline{\Omega}_2\setminus\Omega_1)\subset C$.

Theorem 2.2 ([9]). *Let C be a cone in X and let Ω_1, Ω_2 be open bounded subsets of X with $\overline{\Omega}_1\subset\Omega_2$ and $C\cap(\overline{\Omega}_2\setminus\Omega_1)\neq\emptyset$. Assume that* (A1)–(A8) *hold. Then the equation $Lx=Nx$ has a solution in the set $C\cap(\overline{\Omega}_2\setminus\Omega_1)$.*

For simplicity of notation, we set

$$\begin{gathered}\omega:=\int_0^1\Big(\int_s^1\frac{1}{p(\tau)}d\tau\Big)g(s)ds,\\ l(s):=\int_s^1\Big(\int_\tau^1\frac{1}{p(r)}dr\Big)g(\tau)d\tau+\int_s^1\frac{1}{p(\tau)}d\tau\int_0^s g(\tau)d\tau,\end{gathered}\tag{2.1}$$

and

$$G(t,s)=\begin{cases}\frac{1}{\omega}\big[\int_0^s(\int_s^1\frac{1}{p(r)}dr-\int_\tau^1\frac{r}{p(r)}dr)g(\tau)d\tau+\int_s^1\int_\tau^1\frac{1-r}{p(r)}dr\,g(\tau)d\tau\big]\\ \times\big[\int_0^1\frac{\tau}{p(\tau)}d\tau-\int_t^1\frac{1}{p(\tau)}d\tau\big]+1+\int_0^1\frac{\tau^2}{p(\tau)}d\tau+\int_t^1\frac{1-\tau}{p(\tau)}d\tau-\int_s^1\frac{\tau}{p(\tau)}d\tau,\\ \quad\text{if } 0\leq s<t\leq 1,\\ \frac{1}{\omega}\big[\int_0^s(\int_s^1\frac{1}{p(r)}dr-\int_\tau^1\frac{r}{p(r)}dr)g(\tau)d\tau+\int_s^1\int_\tau^1\frac{1-r}{p(r)}dr\,g(\tau)d\tau\big]\\ \times\big[\int_0^1\frac{\tau}{p(\tau)}d\tau-\int_t^1\frac{1}{p(\tau)}d\tau\big]+1+\int_0^1\frac{\tau^2}{p(\tau)}d\tau+\int_s^1\frac{1-\tau}{p(\tau)}d\tau-\int_t^1\frac{\tau}{p(\tau)}d\tau,\\ \quad\text{if } 0\leq t\leq s\leq 1.\end{cases}$$

Note that $G(t,s)\geq 0$ for $t,s\in[0,1]$, and set

$$\kappa:=\min\big\{1,\ \frac{1}{\max_{t,s\in[0,1]}G(t,s)}\big\}.\tag{2.2}$$

3. Main result

To prove the existence result, we present here a definition.

Definition 3.1. We say that the function $f:[0,1]\times\mathbb{R}\to\mathbb{R}$ satisfies the L^1-Carathéodory conditions, if

(i) for each $u\in\mathbb{R}$, the mapping $t\mapsto f(t,u)$ is Lebesgue measurable on $[0,1]$,
(ii) for a.e. $t\in[0,1]$, the mapping $u\mapsto f(t,u)$ is continuous on $\mathbb{R}$,
(iii) for each $r>0$, there exists $\alpha_r\in L^1[0,1]$ satisfying $\alpha_r(t)>0$ on $[0,1]$ such that
$$|u|\leq r \text{ implies } |f(t,u)|\leq\alpha_r(t).$$

Now, we state our result on the existence of positive solutions for (1.1)-(1.2). under the following assumptions:

(H1) $f:[0,1]\times\mathbb{R}\to\mathbb{R}$ satisfies the L^1-Carathéodory conditions,
(H2) there exist positive constants b_1,b_2,b_3,c_1,c_2,B with
$$B>\frac{c_2}{c_1}+3\Big(\frac{b_2c_2}{b_1c_1}+\frac{b_3}{b_1}\Big)\int_0^1\frac{1+s}{p(s)}ds,\tag{3.1}$$

such that

$$-\kappa x \le f(t,x), \quad f(t,x) \le -c_1 x + c_2, \quad f(t,x) \le -b_1|f(t,x)| + b_2 x + b_3$$

for $t \in [0,1]$, $x \in [0,B]$,

(H3) there exist $b \in (0,B)$, $t_0 \in [0,1]$, $\rho \in (0,1]$, $\delta \in (0,1)$ and $q \in L^1[0,1]$, $q(t) \ge 0$ on $[0,1]$, $h \in C([0,1] \times (0,b], \mathbb{R}^+)$ such that $f(t,x) \ge q(t)h(t,x)$ for $t \in [0,1]$ and $x \in (0,b]$. For each $t \in [0,1]$, $\frac{h(t,x)}{x^\rho}$ is non-increasing on $x \in (0,b]$ with

$$\int_0^1 G(t_0,s)q(s)\frac{h(s,b)}{b}ds \ge \frac{1-\delta}{\delta^\rho}. \tag{3.2}$$

Theorem 3.2. *Under assumptions* (H1)–(H3)*, The problem* (1.1)-(1.2) *has at least one positive solution on* $[0,1]$.

Proof. Consider the Banach spaces $X = C[0,1]$ with the supremum norm $\|x\| = \max_{t\in[0,1]}|x(t)|$ and $Y = L^1[0,1]$ with the usual integral norm $\|y\| = \int_0^1 |y(t)|dt$. Define $L : \operatorname{dom} L \subset X \to Y$ and $N : X \to Y$ with

$$\begin{gathered}\operatorname{dom} L = \big\{x \in X : p(0)x'(0) = p(1)x'(1),\ x(1) = \int_0^1 x(s)g(s)ds,\\ x, px' \in AC[0,1],\ (px')' \in L^1[0,1]\big\}\end{gathered}$$

with $Lx(t) = -(p(t)x'(t))'$ and $Nx(t) = f(t,x(t))$, $t \in [0,1]$. Then

$$\ker L = \{x \in \operatorname{dom} L : x(t) \equiv c \text{ on } [0,1]\},$$

$$\operatorname{Im} L = \{y \in Y : \int_0^1 y(s)ds = 0\}.$$

Next, we define the projections $P : X \to X$ by $(Px)(t) = \int_0^1 x(s)ds$ and $Q : Y \to Y$ by

$$(Qy)(t) = \int_0^1 y(s)ds.$$

Clearly, $\operatorname{Im} P = \ker L$ and $\ker Q = \operatorname{Im} L$. So $\dim\ker L = 1 = \dim \operatorname{Im} Q = \operatorname{codim} \operatorname{Im} L$. Notice that $\operatorname{Im} L$ is closed, L is a Fredholm operator of index zero; i.e. (A1) holds.

Note that the inverse $K_p : \operatorname{Im} L \to \operatorname{dom} L \cap \ker P$ of L_p is given by

$$(K_p y)(t) = \int_0^1 k(t,s)y(s)ds,$$

where

$$k(t,s) := \begin{cases} -\int_s^1 \frac{\tau}{p(\tau)}d\tau + \frac{1}{\omega}l(s)\big[\int_0^1 \frac{\tau}{p(\tau)}d\tau - \int_t^1 \frac{1}{p(\tau)}d\tau\big] \\ +\int_t^1 \frac{1}{p(\tau)}d\tau, & 0 \le s \le t \le 1, \\ -\int_s^1 \frac{\tau}{p(\tau)}d\tau + \frac{1}{\omega}l(s)\big[\int_0^1 \frac{\tau}{p(\tau)}d\tau - \int_t^1 \frac{1}{p(\tau)}d\tau\big] \\ +\int_s^1 \frac{1}{p(\tau)}d\tau, & 0 \le t < s \le 1, \end{cases} \tag{3.3}$$

It is easy to see that $|k(t,s)| \le 3\int_0^1 \frac{1+s}{p(s)}ds$. Since f satisfies the L^1-Carathéodory conditions, (A2) holds.

Consider the cone

$$C = \{x \in X : x(t) \ge 0 \text{ on } [0,1]\}.$$

Let

$$\Omega_1 = \{x \in X : \delta\|x\| < |x(t)| < b \text{ on } [0,1]\},$$
$$\Omega_2 = \{x \in X : \|x\| < B\}.$$

Clearly, Ω_1 and Ω_2 are bounded and open sets and

$$\overline{\Omega}_1 = \{x \in X : \delta\|x\| \le |x(t)| \le b \text{ on } [0,1]\} \subset \Omega_2$$

(see [9]). Moreover, $C \cap (\overline{\Omega}_2 \setminus \Omega_1) \neq \emptyset$. Let $J = I$ and $(\gamma x)(t) = |x(t)|$ for $x \in X$. Then γ is a retraction and maps subsets of $\overline{\Omega}_2$ into bounded subsets of C, which means that 4° holds.

To prove (A3), suppose that there exist $x_0 \in \partial\Omega_2 \cap C \cap \operatorname{dom} L$ and $\lambda_0 \in (0,1)$ such that $Lx_0 = \lambda_0 N x_0$, then $(p(t)x_0'(t))' + \lambda_0 f(t, x_0(t)) = 0$ for all $t \in [0,1]$. In view of (H2), we have

$$-\frac{1}{\lambda_0}(p(t)x_0'(t))' = f(t, x_0(t)) \le -\frac{1}{\lambda_0} b_1 |(p(t)x_0'(t))'| + b_2 x_0(t) + b_3.$$

Hence,

$$0 \le -b_1 \int_0^1 |(p(t)x_0'(t))'|dt + \lambda_0 b_2 \int_0^1 x_0(t)dt + \lambda_0 b_3,$$

which gives

$$\int_0^1 |(p(t)x_0'(t))'|dt \le \frac{b_2}{b_1}\int_0^1 x_0(t)dt + \frac{b_3}{b_1}. \tag{3.4}$$

Similarly, from (H2), we also obtain

$$\int_0^1 x_0(t)dt \le \frac{c_2}{c_1}. \tag{3.5}$$

On the other hand,

$$\begin{aligned} x_0(t) &= \int_0^1 x_0(t)dt + \int_0^1 k(t,s)(p(s)x_0'(s))'ds \\ &\le \int_0^1 x_0(t)dt + \int_0^1 |k(t,s)|\,|(p(s)x_0'(s))'|ds. \end{aligned} \tag{3.6}$$

From (3.4), (3.5) and (3.6), we have

$$B = \|x_0\| \le \frac{c_2}{c_1} + 3\Big(\frac{b_2c_2}{b_1c_1} + \frac{b_3}{b_1}\Big)\int_0^1 \frac{1+s}{p(s)}ds,$$

which contradicts (H2).

To prove (A5), consider $x \in \ker L \cap \overline{\Omega}_2$. Then $x(t) \equiv c$ on $[0,1]$. Let

$$H(c,\lambda) = c - \lambda|c| - \lambda\int_0^1 f(s,|c|)ds$$

for $c \in [-B, B]$ and $\lambda \in [0,1]$. It is easy to show that $0 = H(c,\lambda)$ implies $c \ge 0$. Suppose $0 = H(B,\lambda)$ for some $\lambda \in (0,1]$. Then, (H2) leads to

$$0 \le B(1-\lambda) = \lambda\int_0^1 f(s,B)ds \le \lambda(-c_1B + c_2) < 0$$

which is a contradiction. In addition, if $\lambda = 0$, then $B = 0$, which is impossible. Thus, $H(x,\lambda) \neq 0$ for $x \in \ker L \cap \partial\Omega_2$, $\lambda \in [0,1]$. As a result,

$$\deg\{H(\cdot,1), \ker L \cap \Omega_2, 0\} = \deg\{H(\cdot,0), \ker L \cap \Omega_2, 0\}.$$

However,

$$\deg\{H(\cdot,0),\ker L\cap\Omega_2,0\}=\deg\{I,\ker L\cap\Omega_2,0\}=1.$$

Then

$$\deg\{[I-(P+JQN)\gamma]_{\ker L},\ker L\cap\Omega_2,0\}=\deg\{H(\cdot,1),\ker L\cap\Omega_2,0\}\neq 0.$$

Next, we prove (A8). Let $x\in\overline{\Omega}_2\setminus\Omega_1$ and $t\in[0,1]$,

$$\begin{aligned}
(\Psi_\gamma x)(t)&=\int_0^1|x(s)|ds+\int_0^1 f(s,|x(s)|)ds\\
&\quad+\int_0^1 k(t,s)[f(s,|x(s)|)-\int_0^1 f(\tau,|x(\tau)|)d\tau]ds\\
&=\int_0^1|x(s)|ds+\int_0^1 G(t,s)f(s,|x(s)|)ds\\
&\geq\int_0^1(1-\kappa G(t,s))|x(s)|ds\geq 0.
\end{aligned}$$

Hence, $\Psi_\gamma(\overline{\Omega}_2\setminus\Omega_1)\subset C$; i.e. (A8) holds.

Since for $x\in\partial\Omega_2$,

$$\begin{aligned}
(P+JQN)\gamma x&=\int_0^1|x(s)|ds+\int_0^1 f(s,|x(s)|)ds\\
&\geq\int_0^1(1-\kappa)|x(s)|ds\geq 0.
\end{aligned}$$

Thus, $(P+JQN)\gamma x\subset C$ for $x\in\partial\Omega_2$, (A7) holds.

It remains to verify (A6). Let $u_0(t)\equiv 1$ on $[0,1]$. Then $u_0\in C\setminus\{0\}$, $C(u_0)=\{x\in C: x(t)>0 \text{ on } [0,1]\}$ and we can take $\sigma(u_0)=1$. Let $x\in C(u_0)\cap\partial\Omega_1$. Then $x(t)>0$ on $[0,1]$, $0<\|x\|\leq b$ and $x(t)\geq\delta\|x\|$ on $[0,1]$. For every $x\in C(u_0)\cap\partial\Omega_1$, by (H3), we have

$$\begin{aligned}
(\Psi x)(t_0)&=\int_0^1 x(s)ds+\int_0^1 G(t_0,s)f(s,x(s))ds\\
&\geq\delta\|x\|+\int_0^1 G(t_0,s)q(s)h(s,x(s))ds\\
&=\delta\|x\|+\int_0^1 G(t_0,s)q(s)\frac{h(s,x(s))}{x^\rho(s)}x^\rho(s)ds\\
&\geq\delta\|x\|+\delta^\rho\|x\|^\rho\int_0^1 G(t_0,s)q(s)\frac{h(s,b)}{b^\rho}ds\\
&=\delta\|x\|+\delta^\rho\|x\|\cdot\frac{b^{1-\rho}}{\|x\|^{1-\rho}}\int_0^1 G(t_0,s)q(s)\frac{h(s,b)}{b}ds\\
&\geq\delta\|x\|+\delta^\rho\|x\|\int_0^1 G(t_0,s)q(s)\frac{h(s,b)}{b}ds\geq\|x\|.
\end{aligned}$$

Thus, $\|x\|\leq\sigma(u_0)\|\Psi x\|$ for all $x\in C(u_0)\cap\partial\Omega_1$.

By Theorem 2.2, the BVP (1.1)-(1.2) has a positive solution x^* on $[0,1]$ with $b\leq\|x^*\|\leq B$. This completes the proof. □

Remark 3.3. Note that with the projection $P(x)=x(0)$, conditions (A7) and (A8) of Theorem 2.2 are no longer satisfied.

To illustrate how our main result can be used in practice, we present here an example.

Example. Consider the problem

$$\begin{gathered}(e^{54t}(1+t)x'(t))' + f(t,x(t)) = 0, \quad t \in (0,1), \\ x'(0) = 2e^{54}x'(1), \quad x(1) = \int_0^1 2sx(s)ds.\end{gathered} \tag{3.7}$$

Corresponding to (1.1)-(1.2), we have

$$p(t) = e^{54t}(1+t), \quad g(t) = 2t,$$

$$f(t,x) = \begin{cases} \sin(\pi x/2), & (t,x) \in [0,1] \times (-\infty, 3), \\ 2-x, & (t,x) \in [0,1] \times [3, +\infty). \end{cases}$$

When $\kappa = 1/2$, choose $c_1 = 1$, $c_2 = 3$, $b_1 = 1/2$, $b_2 = 3/2$, $b_3 = 9/2$, $B = 4$ and $b = 1/2$, $t_0 = 0$, $\rho = 1$, $\delta = 1/2$, $q(t) = 1-t$, $h(t,x) = \sin(\pi x/2)$. We can check that all the conditions of Theorem 3.2 are satisfied, then the BVP (3.7) has a positive solution on $[0,1]$.

References

[1] K. Deimling; *Nonlinear Functional Analysis.* New York, 1985.

[2] R. E. Gaines and J. Santanilla; *A coincidence theorem in convex sets with applications to periodic solutions of ordinary differential equations.* Rocky Mountain. J. Math., 12 (1982) 669-678.

[3] W. Ge; *Boundary value problems for ordinary nonlinear differential equations*, Science Press, Beijing, 2007.

[4] D. Guo and V. Lakshmikantham; *Nonlinear Problems in Abstract Cones.* New York, 1988.

[5] X. Han; *Positive solutions for a three-point boundary value problem at resonance*, J. Math. Anal. Appl., 336 (2007), 556-568.

[6] G. Infante and M. Zima; *Positive solutions of multi-point boundary value problems at resonance*, Nonlinear Analysis, 69 (2008), No. 8, 2458-2465.

[7] N. Kosmatov; *A multi-point boundary value problem with two critical conditions*, Nonlinear Anal., 65 (2006), 622-633.

[8] J. Mawhin; *Topological degree methods in nonlinear boundary value problems*, in NSFCBMS Regional Conference Series in Mathematics, American Mathematical Society, Providence, RI, 1979.

[9] D. O'Regan and M. Zima; *Leggett-Williams norm-type theorems for coincidences*, Arch. Math., 87 (2006), 233-244.

[10] W. V. Petryshyn; *On the solvability of $x \in Tx + \lambda Fx$ in quasinormal cones with T and F k-set contractive*, Nonlinear Anal., 5 (1981), 585-591.

[11] A. J. Yang; *An extension of Leggett-Williams norm-type theorem for coincidences and its applications* Topological Methods in Nonlinear Analysis, in press.

[12] A. Yang and W. Ge; *Positive solutions for boundary value problems of N-dimension nonlinear fractional differential system*, Boundary Value Problems, 2008, 437-453.

[13] A. Yang and H. Wang; *Positive solutions for higher-order nonlinear fractional differential equation with integral boundary condition*, E. J. Qualitative Theory of Diff. Equ., 1 (2011), 1-15.

Addendum posted on March 14, 2011

In response to comments from a reader, we want to make the following corrections:

Page 2, Line 9: Delete the last sentence in the introduction: "Moreover, ... by O'Regan and Zima [9]". Then insert the following paragraph:

Using the Legget-Williams norm-type theorem for coincidences, which is a tool introduced by O'Regan and Zima [9], Infante and Zima [6] studied the multi-point boundary-value problem

$$x''(t) = f(t, x(t)) = 0,$$
$$x'0) = 0, \quad x(1) = \sum_{i=1}^{m-2} \alpha_i x(\eta_i)\,.$$

Inspired by the work in [6, 9], we follow their steps, use the Legget-Williams norm-type theorem, and quote some of their results.

Page 6, Line -3: Replace $b \le \|x^*\| \le B$ by $\|x^*\| \le B$.

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