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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO A FIRST-ORDER NON-HOMOGENEOUS DELAY DIFFERENTIAL EQUATION

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Abstract. In this article, we study the asymptotic behavior of solutions to the delay differential equation

$$x'(t) = f(t, x(t), x(t - r(t)))\,.$$

It is shown that every solution tends to either ∞ or a constant as $t \to \infty$.

1. Introduction

Consider the delay differential equation

$$x'(t) = f(t, x(t), x(t - r(t))), \tag{1.1}$$

where $f \in C(\mathbb{R} \times \mathbb{R} \times \mathbb{R})$ and $r \in C(\mathbb{R})$. In this article, we assume the following: $f(t, u, v)$ is non-increasing in u; the delay may be unbounded from above, but it is bounded from below by a positive constant, $0 < \tau \le r(t)$; the function $\lambda(t) := t - r(t)$ is non-decreasing, and $\lim_{t\to\infty} \lambda(t) = \infty$. Let

$$\alpha(s) = \sup\{t : \lambda(t) = s\}. \tag{1.2}$$

Then $\alpha(\lambda(t)) \ge t$ and $\alpha(s) > s$, and when $\lambda(t)$ is strictly increasing, α is the inverse function of λ; i.e., $\alpha(\lambda(t)) = t$.

The initial condition for (1.1) is a continuous function ϕ such that

$$x(t) = \phi(t) \quad \text{for } t \in E_{t_0} := \{t_0\} \cup \{t - r(t) < t_0 : t \ge t_0\}.$$

Examples of equation (1.1) include the following:

$$x'(t) = -x^{1/3}(t) + x^{1/3}(t - r(t)), \tag{1.3}$$

$$x'(t) = p(t)[-x^{1/3}(t) + x^{1/3}(t - r(t))] + q(t), \tag{1.4}$$

where r, p and q are continuous functions. For $r(t)$ a positive constant, Bernfeld and Haddock [1] proposed the conjecture:

> Every solution of (1.3) tends to a constant as $t \to \infty$,

which was proved by Ding [5]. Chen [3] obtained the following result:

> If $p(t)$ is bounded and $q \in L^1[0, \infty)$, then every solution of (1.4) tends to a constant as $t \to \infty$.

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Subsequently, Chen [7] considered (1.1) and obtained the following result.

Theorem 1.1. *Assume that $f(t,u,v)$ is strictly decreasing in u, and*

$$f(t,u,v) \le p(t)G(u,v) + q_1(t)u + q_2(t)v + q_3(t),$$

where $G(u,v) \in C(\mathbb{R}\times\mathbb{R})$ and $p, q_i \in C(\mathbb{R})$ satisfying the following conditions: $G(u,v)$ is strictly decreasing in u, and strictly increasing in v; $G(u,u) \equiv 0$ for all $u \in \mathbb{R}$; $G(u_1,u_2) > 0$ for all $u_2 > u_1$; $q_i \in L^1[0,\infty)$ $(i = 1,2,3)$, p, q_1, q_2; and

$$\int_t^{\alpha(t)} p(s)ds \le M, \quad t \in \mathbb{R}\,. \tag{1.5}$$

Then every solution of (1.1) is bounded above, and tends to either a constant or to $-\infty$, as $t \to \infty$.

Recently, Yi [7] pointed out a mistake in [3, Proposition 4]. Unfortunately, the similar mistake appears in [2, Lemma 1] and [3, Lemma 2]. Moreover, we found that [3, condition (1.5)] should be strengthened to be

$$p(t) > 0, \quad \int_t^{\alpha(t)} p(s)ds \le M, \quad \forall t \in \mathbb{R}\,. \tag{1.6}$$

The main purpose of this paper is to show the convergence of the solutions of (1.1). Our approach is quite different from the one in [5, 7], and our conditions are weaker than those in [7]. Ofcourse, our proofs ovoid the mistakes in [2, 3].

2. Preliminary results

We start with a well known result in differential equations.

Lemma 2.1 (See [6])**.** *Let $x_0 \in \mathbb{R}$, $\beta > 0$, $h \in C([x_0, x_0+\beta]\times\mathbb{R}, \mathbb{R})$, and h be a non-increasing in the second variable. Then the initial value problem*

$$\begin{gathered} \frac{dy}{dx} = h(x,y) \\ y(x_0) = y_0 \end{gathered} \tag{2.1}$$

has a unique solution on the interval $[x_0, x_0+\beta]$.

Lemma 2.2. *Assume $\phi : E_{t_0} \to \mathbb{R}$ is a continuous function. Then the initial-value problem*

$$\begin{gathered} x'(t) = f(t, x(t), x(t-r(t))), \quad t \ge t_0 \\ x(t) = \phi(t), \quad t \in E_{t_0} \end{gathered} \tag{2.2}$$

has a unique solution on $[t_0, \infty)$.

Proof. We find the solution on intervals of length τ, where τ is the lower bound for the delay $r(t)$. For $t \in [t_0, t_0+\tau]$, let

$$h(t, x(t))) = f(t, x(t), x(t-r(t))) = f(t, x(t), \phi(t-r(t))).$$

Then by Lemma 2.1, there exists a unique solution $x(t)$ on $[t_0, t_0+\tau]$. Recursively, we can build a unique solution of (2.2) for any interval $[t_0, T]$. The proof is complete. $\square$

Using the same argument as in [5, Proposition 3], we can prove the following result.

Lemma 2.3. *Suppose that $G(u,u) \equiv 0$ for all $u \in \mathbb{R}$, $G(u,v)$ is non-increasing in u, and non-decreasing in v. Then the initial-value problem*

$$\frac{du}{dt} = G(u,k), \tag{2.3}$$

$$u(t_0) = u_0, \tag{2.4}$$

where k is constant in $\mathbb{R}$, has a unique solution $u = u(t,k)$ on $[t_0,+\infty)$, and the function $\phi(k) = u(t,k)$ is continuous in k.

Lemma 2.4. *Suppose that $G(u,u) \equiv 0$ for all $u \in \mathbb{R}$, $G(u,v)$ is non-increasing in u, and non-decreasing in v. Consider the initial-value problem*

$$\frac{du}{dt} = p(t)G(u,c+\varepsilon), \tag{2.5}$$

$$u(t_0) = u_0, \quad u_0 < c, \tag{2.6}$$

where c is a nonzero constant and ε is a parameter such that $0 \le \varepsilon \le |c|/2$. Let $u(t,t_0,\varepsilon)$ denote the solution to (2.5)-(2.6). Assume that

- (A1) *for each $\eta \neq 0$, and t_0 in $\mathbb{R}$, the initial-value problem $\frac{du}{dt} = G(u,\eta)$, $u(t_0) = \eta$ has a unique left-hand solution.*

Moreover, assume that there exists a postive constant M such that

$$p(t) > 0, \quad \int_t^{\alpha(t)} p(s)ds \le M \quad \text{for all } t \in \mathbb{R}\,, \tag{2.7}$$

where $\alpha(t)$ is defined by (1.2). Then there exists a positive constant μ, independent of t_0 and ε, such that

$$u(t,t_0,\varepsilon) \le c+\varepsilon-\mu, \quad \text{for } t_0 \le t \le \alpha(t_0)\,.$$

Proof. The change of variables

$$s = \int_{t_0}^t p(\xi)d\xi, \quad v(s) = u(t) \tag{2.8}$$

transform (2.5)–(2.6) into

$$\frac{dv}{ds} = G(v(s),c+\varepsilon), \quad s \ge 0, \tag{2.9}$$

$$v(0) = u_0, \quad u_0 < c. \tag{2.10}$$

From Lemma 2.3, there exists a unique solution $v(s,\varepsilon)$ defined on $[0,\infty)$. Since $c+\varepsilon$ is a continuous function in ε, it follows that

$$\psi(\varepsilon) = (c+\varepsilon) - \bar{u}(M,\varepsilon)$$

is also a continuous function in ε. Note that because $G(u,u) = 0$, the constant $c+\varepsilon$ is also solution to (2.9). From the uniqueness of the solutions to (2.9)–(2.10), and $u_0 < c < c+\varepsilon$, we obtain

$$v(s,\varepsilon) < c+\varepsilon \quad \text{for all } s \in [0,+\infty). \tag{2.11}$$

This implies $\psi(\varepsilon) > 0$. Let

$$\mu = \min_{0 \le \varepsilon \le |c|/2} \psi(\varepsilon) > 0. \tag{2.12}$$

It follows from (2.11), (2.9), $G(u,u)=0$, and $G(u,v)$ being non-increasing in u, that $\frac{\partial v}{\partial s}v(s,\varepsilon)\geq 0$, and

$$v(s,\varepsilon)\leq v(M,\varepsilon)\quad \text{for all } s\in[0,M],$$

which implies

$$v(s,\varepsilon)\leq c+\varepsilon-\psi(\varepsilon)\leq c+\varepsilon-\mu\quad \text{for all } s\in[0,M]. \tag{2.13}$$

By the relationship between the initial value problems (2.5)-(2.6) and (2.9)–(2.10), it follows from (2.7) that

$$u(t,t_0,\varepsilon)\equiv v(s(t),\varepsilon)\quad \text{for } t_0\leq t\leq \alpha(t_0). \tag{2.14}$$

Again from (2.7) and (1.2), we obtain

$$s(\alpha(t_0))\leq M. \tag{2.15}$$

By (2.13), (2.14) and (2.15), we have

$$u(t,t_0,\varepsilon)\leq c+\varepsilon-\mu\quad \text{for } t_0\leq t\leq \alpha(t_0).$$

Since M is independent of t_0, and μ is independent of t_0 and ε, the proof is complete. □

By a similar argument, we can prove the following result.

Lemma 2.5. *Suppose that $G(u,u)\equiv 0$ for all $u\in\mathbb{R}$, $G(u,v)$ is non-increasing in u, and non-decreasing in v. Consider the initial-value problem*

$$\frac{du}{dt}=p(t)G(u,c-\varepsilon), \tag{2.16}$$
$$u(t_0)=u_0,\quad u_0>c, \tag{2.17}$$

where c is a nonzero constant and ε is a parameter such that $0\leq\varepsilon\leq|c|/2$. Denote by $u=u(t,t_0,\varepsilon)$ be the solution of the initial value problem. If (A1) and (2.7) hold, then there exists a positive constant ν independent of t_0 and ε such that

$$u(t,t_0,\varepsilon)\geq(c-\varepsilon)+\nu\quad \textit{for } t_0\leq t\leq\alpha(t_0).$$

Remark 2.6. If (A1) holds for all $\eta\in\mathbb{R}$ and $\varepsilon\in[0,1]$, using the method in the proof of Lemma 2.4, we can show that the conclusions in Lemmas 2.4 and 2.5 hold for any $c\in\mathbb{R}$.

3. Main results

Theorem 3.1. *Assume that $f(t,u,v)$ non-increasing in u, and*

$$f(t,u,v)\leq p(t)G(u,v)+q_1(t)u+q_2(t)v+q_3(t). \tag{3.1}$$

where $G(u,v)\in C(\mathbb{R}\times\mathbb{R})$ and $p,q_i\in C(\mathbb{R})$ satisfying the following conditions: $G(u,v)$ is non-increasing in u, and non-decreasing in v; $G(u,u)\equiv 0$ for all $u\in\mathbb{R}$; $q_i\in L^1[0,\infty)$ $(i=1,2,3)$; and p,q_1,q_2 are non-negative; and (A1) and (2.7) hold. Then every solution of (1.1) is bounded above. Furthermore, if $\limsup_{t\to\infty}x(t)\neq 0$, then $x(t)$ tends to either a constant or to $-\infty$ as $t\to\infty$.

Proof. We first prove that every solution of (1.1) is bounded above. Let

$$y_1(t) = \max\{\max_{t_0 - r(t_0) \le s \le t} x(s), 1\}, \quad S_1 = \{t \ge t_0 : y_1(t) = x(t)\}.$$

Let D^+ denote the upper right derivative. Then $D^+ y_1(t) = 0$ for $t \in [t_0, +\infty) \backslash S_1$, and $D^+ y_1(t) \le \max\{x'(t), 0\}$ a.e. on S_1. From (1.1) and (3.1),

$$\begin{aligned} x'(t) &\le p(t)G(x(t), x(t-r(t))) + q_1(t)x(t) + q_2(t)x(t-r(t)) + q_3(t) \\ &\le p(t)G(x(t), y_1(t)) + q_1(t)y_1(t) + q_2(t)y_1(t) + q_3(t) \\ &\le p(t)G(x(t), y_1(t)) + q_1(t)y_1(t) + q_2(t)y_1(t) + |q_3(t)| \quad \forall t \ge t_0. \end{aligned} \tag{3.2}$$

Since $G(u,u) \equiv 0$ for all $u \in \mathbb{R}$, and $D^+ y_1(t) \le \max\{x'(t), 0\}$ a.e. on $[t_0, +\infty)$, we obtain

$$D^+ y_1(t) \le q_1(t)y_1(t) + q_2(t)y_1(t) + |q_3(t)| \quad \text{a.e. on } [t_0, +\infty).$$

From $y_1(t) \ge 1$, we have

$$\frac{D^+ y_1(t)}{y_1(t)} \le q_1(t) + q_2(t) + |q_3(t)| \quad \text{a.e. on } [t_0, +\infty).$$

Again from the monotonicity of $y_1(t)$, we obtain that $y_1(t)$ is differentiable almost everywhere on $[t_0, \infty)$. Thus

$$\ln\big(\frac{y_1(t)}{y_1(t_0)}\big) \le \int_{t_0}^{+\infty} q_1(t)dt + \int_{t_0}^{+\infty} q_2(t)dt + \int_{t_0}^{+\infty} |q_3(t)|dt < +\infty \quad \forall t \ge t_0,$$

which implies $y_1(t)$ is bounded above; thus $x(t)$ is also bounded above. Set $A = \limsup_{t\to\infty} x(t) < \infty$. If $\limsup_{t\to\infty} x(t) = -\infty$, then $\lim_{t\to\infty} x(t) = -\infty$, which implies that Theorem 3.1 holds.

Next we assume that A is a nonzero real number and show that $\lim_{t\to\infty} x(t) = A$. By contradiction, assume that $\lim_{t\to\infty} x(t)$ does not exist. For each $\mu_1 \in [0, |A|/2]$, there exists let $t^* > t_0$ large enough such that

$$x(t) \le A + \mu_1, \quad x(t - r(t)) \le A + \mu_1 \quad \forall t \ge t^*, \tag{3.3}$$

$$\int_{t^*}^{+\infty} [(q_1(t) + q_2(t))A + \mu_1 + |q_3(t)|]dt \le \mu_1. \tag{3.4}$$

For $t \ge t^*$, let

$$n_1(t) = x(t) - \int_{t^*}^{t} [(q_1(s) + q_2(s))A + \mu_1 + |q_3(s)|]ds\,. \tag{3.5}$$

Obviously, $n_1(t)$ is bounded above, and $\lim_{t\to\infty} n_1(t)$ does not exist. Let $B = \limsup_{t\to\infty} n_1(t)$ and $b = \limsup_{t\to\infty} n_1(t)$; Thus $b < B \le A$. For $b < H < B$, there exists a sequence $\{t_m\}_{m=1}^{\infty}$ satisfying $n_1(t_m) = H$, $t_m > t^*$ and $t_m \to \infty$ as $m \to \infty$. It follows from (3.1) and (3.3) that

$$x'(t) \le p(t)G(x(t), A + \mu_1) + (q_1(t) + q_2(t))A + \mu_1 + |q_3(t)| \quad \text{for all } t \ge t^*.$$

From (3.5), we obtain $x(t) \ge n_1(t)$ and

$$n_1'(t) \le p(t)G(n_1(t), A + \mu_1) \quad \text{for all } t \ge t^*. \tag{3.6}$$

For each m, we consider the initial-value problem

$$\begin{gathered} u'(t) = p(t)G(u(t), A + \mu_1) \\ u(t_m) = H, \quad H < A\,. \end{gathered} \tag{3.7}$$

By Lemma 2.4, this problem has a unique solution $u = u(t)$ on $[t_m, +\infty)$, and there exists a $\mu > 0$ independent of t_m and of μ_1, such that

$$u(t) \le A + \mu_1 - \mu \quad \text{for } t_m \le t \le \alpha(t_m).$$

Then, by the comparison theorem and (3.6), we obtain

$$n_1(t) \le u(t) \le A + \mu_1 - \mu \quad \text{for } t_m \le t \le \alpha(t_m),$$

thus $x(t) \le A + 2\mu_1 - \mu$ for $t_m \le t \le \alpha(t_m)$. Choosing $\mu_1 \in (0, \mu/4]$, we have

$$x(t) \le A - \frac{\mu}{2} \quad \text{for } t_m \le t \le \alpha(t_m),\ m = 1, 2, \dots. \tag{3.8}$$

On the other hand, define

$$y_2(t) = \max_{\lambda(t) \le s \le t} x(s), \quad S_2 = \{t : t \in [t^*, \infty), y_2(t) = x(t)\}.$$

Then $D^+ y_2(t) \le 0$ for all $t \in [t^*, \infty) \backslash S_2$, and $D^+ y_2(t) \le \max\{x'(t), 0\}$ for all $t \in S_2$. Hence

$$D^+ y_2(t) \le (q_1(t) + q_2(t))(A + \mu_1) + |q_3(t)| \quad \forall t \ge t^*. \tag{3.9}$$

For $t \ge t^*$, denote

$$n_2(t) = y_2(t) - \int_{t^*}^{t} [(q_1(s) + q_2(s))(A + \mu_1) + |q_3(s)|] ds.$$

From (3.9), we obtain $D^+ n_2(t) \le 0$ for all $t \ge t^*$; therefore, $n_2(t)$ is non-increasing. Since $\limsup_{t\to\infty} x(t) > -\infty$,

$$\lim_{t\to\infty} y_2(t) = \lim_{t\to\infty} n_2(t) + \lim_{t\to\infty} \int_{t^*}^{t} [(q_1(s) + q_2(s))(A + \mu_1) + |q_3(s)|] ds$$

exists as real number. From the definition of y_2 and the the fact that $\{s : \lambda(t) \le s \le t, t \ge t_0\} \supset [t_0, \infty)$, it follows that

$$\lim_{t\to\infty} y_2(t) = \lim_{t\to\infty} \max_{\lambda(t) \le s \le t} x(s) = \limsup_{t\to\infty} x(t) = A. \tag{3.10}$$

Since $\lambda(t) \to +\infty$ as $t \to +\infty$, for each t_m there exists t'_m such that $t_m = \lambda(t'_m)$. Then $\alpha(t_m) = \alpha(\lambda(t'_m)) \ge t'_m$ and $t'_m \ge t_m \ge t^*$. By (3.8),

$$y_2(t'_m) \le A - \frac{\mu}{2} \quad \text{for } m = 1, 2, \dots. \tag{3.11}$$

However, (3.10) implies $\lim_{m\to+\infty} y_2(t'_m) = A$ which contradicts (3.11). Hence $\lim_{t\to\infty} x(t)$ exists, and $\lim_{t\to\infty} x(t) = A$. This completes the proof. $\square$

In a similar fashion, by using Lemma 2.5, we can show the following result.

Theorem 3.2. *Assume $f(t, u, v)$ non-increasing in u, and*

$$f(t, u, v) \ge p(t)G(u, v) + q_1(t)u + q_2(t)v + q_3(t), \tag{3.12}$$

where $G(u, v) \in C(\mathbb{R} \times \mathbb{R})$ and $p, q_i \in C(\mathbb{R})$ satisfying the following conditions: $G(u, v)$ non-increasing in u, and non-decreasing in v; $G(u, u) \equiv 0$ for all $u \in \mathbb{R}$; $q_i \in L^1[0, \infty)$ $(i = 1, 2, 3)$, p, q_1, q_2 are nonnegative; and (A1) *and* (2.7) *hold. Then every solution of* (1.1) *is bounded below. Furthermore, if $\limsup_{t\to\infty} x(t) \ne 0$, then $x(t)$ tends to either a constant or to ∞ as $t \to \infty$.*

Theorem 3.3. *Consider the differential equation*

$$x'(t) = p(t)G(x(t), x(t-r(t))) + q_1(t)x(t) + q_2(t)x(t-r(t)) + q_3(t), \tag{3.13}$$

where $G(u,v) \in C(\mathbb{R}\times\mathbb{R})$ and $p, q_i \in C(\mathbb{R})$ satisfying the following conditions: $G(u,v)$ is non-increasing in u, and non-decreasing in v; $G(u,u) \equiv 0$ for all $u \in \mathbb{R}$; $q_i \in L^1[0,\infty)$ $(i = 1, 2, 3)$, p, q_1, q_2 are nonnegative; and (A1) *and* (2.7) *hold. Then every solution of* (3.13) *tends to a constant as $t \to \infty$.*

The proof of the above theorem follows immediately from Theorems 3.1 and 3.2.

Remark 3.4. Let $G(u,v) = -u^\theta + v^\theta$, where θ is the ratio of two odd positive integers. Then $G(u,v)$ is strictly decreasing in u, and is strictly increasing in v. Moreover, $G(u,\eta)$ is continuously differentiable when $u \neq 0$. Applying Cauchy's uniqueness and existence theorem, we conclude that assumption (A1) holds. Therefore, Theorem 3.3 confirms the Bernfeld-Haddock conjecture.

From Remark 2.6, and using a similar argument as in the proof of Theorem 3.1, we can also show the following result, under the assumption

(A1') For each η and t_0 in $\mathbb{R}$, the initial-value problem $\frac{du}{dt} = G(u,\eta)$, $u(t_0) = \eta$ has a unique left-hand solution.

Theorem 3.5. *Assume* (A1'). *Under the hypotheses of Theorem 3.1, every solution of* (1.1) *is bounded above, and tends to either a constant or to $-\infty$, as $t \to \infty$.*

Theorem 3.6. *Assume* (A1'). *Under the hypotheses of Theorem 3.2, every solution of* (1.1) *is bounded below, and tends to either a constant or to $+\infty$, as $t \to \infty$.*

The proofs of the two theorems above are similar to the proof of Theorem 3.1: Replace $\mu_1 \in [0, |A|/2]$ with $\mu_1 \in [0,1]$, and then use Remark 2.6.

Remark 3.7. Note that the results in [2, 3] can be obtained only by assuming condition (A1'), and the strengthened condition (2.7). Since the function $G(u,v)$ in this article satisfies weaker conditions than those in [2, 3], their results there are special cases in this article. When $r(t)$ is constant and $p(t)$ is a bounded and positive function, (2.7) holds naturally. Hence, our results include those in [5, 7], and naturally extend the Bernfeld-Haddock conjecture.

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