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EXISTENCE OF GLOBAL SOLUTIONS FOR SYSTEMS OF SECOND-ORDER FUNCTIONAL-DIFFERENTIAL EQUATIONS WITH p-LAPLACIAN

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Abstract. We find sufficient conditions for the existence of global solutions for the systems of functional-differential equations

$$\big(A(t)\Phi_p(y')\big)' + B(t)g(y', y'_t) + R(t)f(y, y_t) = e(t),$$

where $\Phi_p(u) = (|u_1|^{p-1}u_1, \dots, |u_n|^{p-1}u_n)^T$ which is the multidimensional p-Laplacian.

1. Introduction

There are many papers concerning various problems for ordinary differential equations with p-Laplacian. From the recently published papers and books see e.g. [14, 15, 24, 25, 26]. The problems treated in this paper are close to those studied in [1]-[6], [8]-[26]. The recently published paper [10] contains some results on the existence of positive solutions of a boundary value problem for a p-Laplacian functional- differential equations. This paper motivated us to study the problem of the existence of global solutions for such type of equations. This problem for functional-differential equations of the first order on the Banach space has been recently studied in the paper [20]. A survey of papers on this problems concerning systems of ordinary differential equations and also scalar differential equations with p-Laplacian and some remarks on results close to the results proved in [21] can be found in the introduction of this paper.

In this paper, we are concerned with the initial value problem

$$\big(A(t)\Phi_p(y')\big)' + B(t)g(y', y'_t) + R(t)f(y, y_t) = e(t), \quad t \ge 0, \tag{1.1}$$

$$y(t) = \varphi_0(t), y'(t) = \varphi_1(t), \quad -r \le t \le 0, \tag{1.2}$$

where $n \in \{1, 2, \dots\}$, $\Phi_p(u) = (|u_1|^{p-1}u_1, \dots, |u_n|^{p-1}u_n)^T$, $u \in \mathbb{R}^n$, $y_t \in C^1 := C^1(\langle -r, 0\rangle, \mathbb{R}^n)$, $y_t(\Theta) = y(t+\Theta)$, $y'_t \in C = C(\langle -r, 0\rangle, \mathbb{R}^n)$, $y'_t(\Theta) = y'(t+\Theta)$, $A(t)$, $B(t)$, $R(t)$ are continuous, matrix-valued functions on $\mathbb{R}_+ := \langle 0, \infty)$, $A(t)$ is regular for all $t \in \mathbb{R}_+$, $e : \mathbb{R}_+ \to \mathbb{R}^n$, $\varphi_0 : \langle -r, 0\rangle \to \mathbb{R}^n$, $\varphi_1 : \langle -r, 0\rangle \to \mathbb{R}^n$ and $f : \mathbb{R}^n \times C^1 \to R^n, g : \mathbb{R}^n \times C \to \mathbb{R}^n$ are continuous mappings.

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The aim of the paper is to study the problem of the existence of global solutions of the equation (1.1) in the sense of the following definition.

Definition 1.1. A solution $y(t)$, $t \in \langle -r, T)$ of the initial value problem (1.1), (1.2) is called non-extendable to the right if either $T < \infty$ and $\lim_{t\to T^-}[\|y(t)\|+\|y'(t)\|] = \infty$, or $T = \infty$, i. e. $y(t)$ is defined on $\langle -r, \infty)$. In the second case the solution $y(t)$ is called global.

We shall use in the sequel the norm $\|z\| = \max_{0\le i\le n} |z_i|$ of $z = (z_1, z_2, \dots, z_n) \in R^n$. The main results of this paper are formulated in the following theorems.

Theorem 1.2. *Let $m > p, m \ge 1, A(t)$, $B(t)$, $R(t)$ be continuous matrix-valued functions on $\langle 0, \infty), A(t)$ be regular for all $t \in \mathbb{R}_+, e : \mathbb{R}_+ \to \mathbb{R}^n$, $f, g : \mathbb{R}^n \to \mathbb{R}^n$ be continuous mappings and $\varphi_0 \in C^1$, $\varphi_1 \in C$, $\varphi_0(0) = y_0$, $\varphi_1(0) = y_1$. Let*
$$\int_0^\infty \|R(s)\| s^{m-1} \mathrm{d}s < \infty \tag{1.3}$$
and there exist constants $K_1, K_2 > 0$ such that
$$\|g(u, v)\| \le K_1(\|u\|^m + \|v\|_C^m), \quad \|f(u, v)\| \le K_2(\|u\|^m + \|v\|_C^m)\,, \tag{1.4}$$
for all $(u, v) \in \mathbb{R}^n \times C$. Let $A_\infty = \sup_{0\le t<\infty} \|A(t)^{-1}\|$, $R_\infty = \int_0^\infty \|R(s)\|\,\mathrm{d}s$,
$$B_\infty := \sup_{0\le t<\infty} \int_0^t \|B(\tau)\|\mathrm{d}\tau < \infty, \quad E_\infty := \sup_{0\le t<\infty} \int_0^t \|e(s)\|\mathrm{d}s < \infty$$
and
$$\frac{m-p}{p} c^{\frac{m-p}{p}} \sup_{0\le t<\infty} \int_0^t F(s)\mathrm{d}s < 1, \tag{1.5}$$
where
$$\begin{aligned} c &:= A_\infty\{\|A(0)\Phi_p(y_1)\| + 2^{m-1}K_1\|\varphi_1\|_C^m B_\infty \\ &\quad + 2^{m-1}K_2\big(\|y_0\|^m + \big(\|\varphi_0\|_C + \|y_0\|\big)^m\big)R_\infty\}, \\ F(t) &= 2^m K_2 A_\infty \int_t^\infty \|R(s)\| s^{m-1}\mathrm{d}s + (2^{m-1}+1)K_1A_\infty\|B(t)\|\,. \end{aligned}$$
Then any nonextendable to the right solution $y(t)$ of the initial value problem (1.1), (1.2) *is global.*

Due to the continuous Jensen's inequality, Theorem 1.2 is valid for $m \ge 1$ only. A similar result is stated in the following theorem in case $m < 1$ under stronger assumptions.

Theorem 1.3. *Let $m > p > 0$, $0 < m < 1$, $A(t)$, $B(t)$, $R(t)$ be continuous matrix-valued functions on $\mathbb{R}_+$, A regular for all $t \in \mathbb{R}_+$, $e : \mathbb{R}_+ \to \mathbb{R}^n$, $f, g : \mathbb{R}^n \to \mathbb{R}^n$ be continuous mappings and $\varphi_0 \in C^1$, $\varphi_1 \in C$, $\varphi_0(0) = y_0$, $\varphi_1(0) = y_1$. Let constants K_1, $K_2 > 0$ exist such that*
$$\|g(u, v)\| \le K_1(\|u\|^m + \|v\|_C^m), \quad \|f(u, v)\| \le K_2(\|u\|^m + \|v\|_C^m)$$
for $(u, v) \in \mathbb{R}^n \times C$. Let
$$\frac{m-p}{p} C_1^{\frac{m-p}{p}} \sup_{0\le t<\infty} \int_0^t F_1(s)\,\mathrm{d}s < 1\,,$$

where B_∞ and E_∞ are given in Theorem 1.2.2 and

$$\begin{aligned} C_1 &= A_\infty\big\{\|A(0)\Phi_p(y_1)\| + 2^m K_1\|\varphi_1\|_C^m B_\infty \\ &\quad + 2^m K_2 R_\infty\big(\|y_0\|^m + (\|\varphi_0\| + \|y_0\|)^m\big)\big\}\,, \\ F_1(t) &= (2^m+1)A_\infty K_1\|B(t)\| + 2^{m+1}A_\infty K_2\|R(t)\|t^m\,. \end{aligned}$$

Then any nonextendable to the right solution $y(t)$ of the initial value problem (1.1), (1.2) *is global.*

The above theorem solves the problem in case $m \le p$.

Theorem 1.4. *Let $p > 0$, $0 < m \le p$, $A(t)$, $B(t)$, $R(t)$ be continuous matrix-valued functions on $\mathbb{R}_+$, A regular for all $t \in \mathbb{R}_+$, $e : \mathbb{R}_+ \to \mathbb{R}^n$, $f, g : \mathbb{R}^n \to \mathbb{R}^n$ be continuous mappings and $\varphi_0 \in C^1$, $\varphi_1 \in C$. Let constants K_1, K_2, K_3, K_4, K_5 and K_6 exist such that*

$$\|g(u,v)\| \le K_1\big(\|u\|^m + \|v\|_C^m\big)\,, \quad \|f(u,v)\| \le K_2\big(\|u\|^m + \|v\|_C^m\big)$$

for $\|u\| \ge 1$, $\|v\|_C \ge 1$,

$$\|g(u,v)\| \le K_3\|u\|^m\,, \quad \|f(u,v)\| \le K_4\|u\|^m \quad \text{for } \|u\| \ge 1,\ 0 \le \|v\|_C < 1$$

and

$$\|g(u,v)\| \le K_5\|v\|_C^m\,, \quad \|f(u,v)\| \le K_6\|v\|_C^m \quad \text{for } 0 \le \|u\| < 1,\ \|v\|_C \ge 1.$$

Then any nonextendable to the right solution $y(t)$ of the initial value problem (1.1), (1.2) *is global.*

A special case of the equation (1.1) with g, f independent of y_t' and y_t, respectively, i.e. the equation

$$A(t)\Phi_p(y')' + B(t)g(y') + R(t)f(y) = e(t), \quad t \ge 0, \tag{1.6}$$

and with the initial conditions

$$y(0) = y_0, \quad y'(0) = y_1 \tag{1.7}$$

has been studied in the paper [21]. A similar theorem to Theorem 1.2 on the existence of a global solution of the initial value problem (1.6), (1.7) is proved there. It is assumed there that there exist positive constants K_1, K_2 such that

$$\|g(u)\| \le K_1\|u\|^m, \quad \|f(u)\| \le K_2\|u\|^m, u \in \mathbb{R}^n, \tag{1.8}$$

where the constant c and the function $F(t)$ are defined in [21, Theorem 1.1] as follows:

$$c := n^{\frac{p}{2}} A_\infty\big\{\|A(0)\Phi_p(y_1)\| + 2^{m-1}K_2\|y_0\|R_\infty + E_\infty\big\}, \tag{1.9}$$

$$F(t) := K_1\|B(t)\| + 2^{m-1}K_2\int_t^\infty \|R(s)\|s^{m-1}\mathrm{d}s, \tag{1.10}$$

$\|w\|$ is the Euclidean norm of $w \in \mathbb{R}^n$. If the condition (1.8) and one of the assumptions 1., 2. of [21, Theorem 1.1] (with c, F defined by (1.9) and (1.10)) is satisfied, then a solution of the initial value problem (1.6), (1.7) is global.

We remark that in [21, Theorem 1.1] there is a misprint. There must be $A_\infty = \sup_{0\le t<\infty}\|A(t)^{-1}\|$ instead of $A_\infty = \sup_{0\le t<\infty}\|A(t)^{-1}\|^{-1}$.

Corollary 1.5. *Consider the differential equation*
$$y'' = t^{\alpha}|y|^m \operatorname{sgn} y \tag{1.11}$$
with $m > 1$*. Then* $\varepsilon > 0$ *exists such that a solution of the problem* (1.11)*,* $|y(0)| < \varepsilon$*,* $|y'(0)| < \varepsilon$ *is defined on* $\mathbb{R}_+$ *if and only if*
$$\alpha < -m - 1\,. \tag{1.12}$$

Corollary 1.5 shows that condition (1.3) cannot be weaken, the integral cannot be infinite.

2. Proofs of the main results

Proof of Theorem 1.2. Let $y : \langle -r, T) \to \mathbb{R}^n$ be a nonextendable to the right solution of the initial value problem (1.1), (1.2) with $0 < T < \infty$. If we denote $u(t) = y'(t)$ for $t \geq 0$, then $y(t) = y_0 + \int_0^t u(\tau)d\tau$ and we can write (1.1) as
$$\begin{aligned}\Phi_p(u(t)) = A(t)^{-1}\big\{A(0)\varphi(y_1) - \int_0^t B(s)g(u(s), y_s')\mathrm{d}s \\ - \int_0^t R(s)f(y_0 + \int_0^s u(\tau)\mathrm{d}\tau, y_s)\mathrm{d}s + \int_0^t e(s)\mathrm{d}s\big\}, t \geq 0\,.\end{aligned}$$
We need to estimate $\|y_s\|_C$ and $\|y_s'\|_C$. From the definition of the shift operators we have
$$\|y_s\|_C = \max_{-r\leq\Theta\leq 0} \|y(s+\Theta)\| = \max\{\rho_1(s), \rho_2(s)\} \leq \rho_1(s) + \rho_2(s)\,,$$
where
$$\rho_1(s) = \max_{-r\leq s+\Theta\leq 0} \|y(s+\Theta)\| \leq \|\phi_0\|_C,$$
$$\begin{aligned}\rho_2(s) = \max_{s+\Theta\geq 0} \|y(s+\Theta)\| \leq \max_{s+\Theta\geq 0}\{\|y_0\| + \int_0^{s+\Theta} \|u(\tau)\|d\tau\} \leq \|y_0\| \\ + \int_0^s \|u(\tau)\|\mathrm{d}\tau\end{aligned}$$
and this yields
$$\|y_s\|_C \leq \|\varphi_0\|_C + \|y_0\| + \int_0^s \|u(\tau)\|\mathrm{d}\tau\,. \tag{2.1}$$
We can estimate analogously $\|y_s'\|$:
$$\|y_s'\|_C = \max_{-r\leq\Theta\leq 0} \|y'(s+\Theta)\| = \max\{\sigma_1(s), \sigma_2(s)\} \leq \sigma_1(s) + \sigma_2(s)\,,$$
where
$$\begin{aligned}\sigma_1(s) = \max_{-r\leq s+\Theta\leq 0} \|y'(s+\Theta)\| = \max_{-r\leq s+\Theta\leq 0} \|\varphi_1(s+\Theta)\| \leq \|\varphi_1\|_C, \\ \sigma_2(s) = \max_{s+\Theta\geq 0} \|y'(s+\Theta)\| = \max_{s+\Theta\geq 0} \|u(s+\Theta)\| \leq \max_{0\leq\tau\leq s} \|u(\tau)\|.\end{aligned}$$
Thus we have
$$\|y_s'\|_C \leq \|\varphi_1\|_C + \max_{0\leq\tau\leq s} \|u(\tau)\|\,. \tag{2.2}$$

From (1.1), the inequalites (2.1), (2.2) and the assumptions of the theorem we obtain

$$\begin{aligned} \|\Phi_p(u(t)\| \le \|A(t)^{-1}\| \big\{ \|A(0)\Phi_p(y_1)\| &+ K_1 \int_0^t \|B(s)\|\, \|u(s)\|^m \mathrm{d}s \\ &+ K_1 \int_0^t \|B(s)\| \Big((\|\varphi_1\|_C + \max_{0\le\tau\le s} \|u(\tau)\| \big)^m \mathrm{d}s \\ &+ K_2 \int_0^t \|R(s)\| \|y_0 + \int_0^s u(\tau)\mathrm{d}\tau\|^m \mathrm{d}s + K_2 \int_0^t \|R(s)\| \big[\|\varphi_0\|_C \\ &+ \|y_0\| + \int_0^s \|u(\tau)\|\mathrm{d}\tau \big]^m \mathrm{d}s \Big) \big\}\,. \end{aligned} \tag{2.3}$$

Now applying the continuous and discrete versions of the Jensen's inequality (see [17, Theorem 2, Chapter VIII] and the Fubini theorem in a similar way as in the proof of [21, Theorem 1.2] we obtain the inequality

$$v(t)^p \le c + \int_0^t F_1(\tau) v(\tau)^m \mathrm{d}\tau + \int_0^t F_2(\tau) [\sup_{0\le s\le\tau} v(\tau)]^m \mathrm{d}\tau, \quad 0 \le t < T,$$

where c is given in the theorem and $v(t) = \|u(t)\|$. If we denote by $G(t)$ the right-hand side of this inequality then $v^p(s) \le G(t)$ for $s \le t$ and therefore we obtain the following inequality for $w(t) := \sup_{0\le\sigma\le t} v(\sigma)$:

$$w(t)^p \le c + \int_0^t F(\tau) w(\tau)^m \mathrm{d}\tau, \quad 0 \le t < T,$$

where $F = F_1 + F_2$ is the function from the theorem. From [21, Lemma] it follows that $M = \sup_{0\le t<T} \|u(t)\| < \infty$ and since $w(t) := \sup_{0\le\sigma\le t} \|u(\sigma)\|$ we obtain that for the solution $y(t) = y_0 + \int_0^t u(s)ds$ of the initial value problem (1), (2) we have $\lim_{t\to T^-} \|y(t)\| \le \lim_{t\to T^-} \big(\|y_0\| + t \sup_{0\le s<T} \|u(s)\| \big) < \infty$. Thus we have proved that $\lim_{t\to T^-} [\|y(t)\| + \|y'(t)\|] < \infty$, i. e. the solution $y(t)$ is global.

Proof of Theorem 1.3. Let $y : \langle -r, T) \to \mathbb{R}^n$ be a nonextendable to the right solution of the initial value problem (1.1), (1.2) with $0 < T < \infty$ and $u(t) = y'(t)$ for $t \ge 0$. Then (2.3) holds. Denote $w(t) = \max_{0\le s\le t} \|u(s)\|$ $0 \le t < T$. Then (2.3) and the inequality

$$(A_1 + \dots + A_l)^k \le l^k (A_1^k + \dots A_l^k) \tag{2.4}$$

for $A_1, A_1, \dots, A_l \ge 0$, $k > 0$ yield

$$\begin{aligned} \|\Phi_p(u(t))\| \le \|A(t)^{-1}\| \big\{ \|A(0)\Phi_p(y_1)\| &+ K_1 \int_0^t \|B(s)\| w^m(s)\, \mathrm{d}s \\ &+ 2^m K_1 \|\varphi_1\|_C^m B_\infty + 2^m K_1 \int_0^t \|B(s)\| w^m(s)\, \mathrm{de}s \\ &+ 2^m K_2 \|y_0\|^m R_\infty + 2^m K_2 \int_0^t \|R(s)\| s^m w^m(s)\, \mathrm{d}s \\ &+ 2^m K_2 (\|\varphi_0\|_C + \|y_0\|)^m R_\infty + 2^m K_2 \int_0^t \|R(s)\| s^m w^m(s)\, \mathrm{d}s \big\}\,. \end{aligned}$$

Hence,

$$w^p(t) \le C_1 + \int_0^t F_1(\tau) w^m(\tau)\, \mathrm{d}\tau\,,$$

where C_1 and F_1 are given in Theorem 1.3, and the rest of the proof is the same as in the end of the proof of Theorem 1.2.

Proof of Theorem 1.4. Let $y : \langle -r, T) \to \mathbb{R}^n$ be a nonextendable solution of the initial value problem (1.1), (1.2) with $0 < T < \infty$. If we denote $u(t) = y'(t)$ for $t \geq 0$ and $\varphi_0(0) = y_0$, $\varphi_1(0) = y_1$, then the estimations (2.1) and (2.2) are valid. Let $w(t) = \max\big(1, \max_{0\leq s\leq t} \|u(s)\|\big)$. Furthermore,

$$\begin{aligned} \|g(u,v)\| &\leq K_1\|u\|^m + K_1\|v\|_C^m + K_3\|u\|^m + K_5\|v\|_C^m \\ &\quad + \max_{\|u\|\leq 1, \|v\|_C\leq 1} \|g(u,v)\| = K_7(\|u\|^m + \|v\|_C^m + 1) \end{aligned} \tag{2.5}$$

on $u, v \in \mathbb{R}^n \times C$ with

$$K_7 = \max\big\{K_1 + K_3, K_1 + K_5, \max_{\|u\|\leq 1, \|v\|_C\leq 1} \|g(u,v)\|\big\}.$$

Similarly,

$$\|f(u,v)\| \leq K_8(\|u\|^m + \|v\|_C^m + 1) \tag{2.6}$$

on $u, v \in \mathbb{R}^n \times C$ with

$$K_8 = \max\big\{K_2 + K_4, K_2 + K_6, \max_{\|u\|\leq 1, \|v\|_C\leq 1} \|f(u,v)\|\big\}.$$

Then (2.1), (2.2), (2.5), (2.6), the equation (1.1) and the assumptions of the theorem yield

$$\begin{aligned} \|\Phi_p(u(t))\| \leq \|A(t)^{-1}\| \Big\{ &\|A(0)\Phi_p(y_1)\| + K_7 \int_0^t \|B(s)\| w^m(s)\,\mathrm{d}s \\ &+ K_7 \int_0^t \|B(s)\| \big(\|\varphi_1\|_C + w(s)\big)^m + K_7 \int_0^t \|B(s)\|\,\mathrm{d}s \\ &+ K_8 \int_0^t \|R(s)\| \big(\|y_0\| + s w(s)\big)^m \mathrm{d}s + K_8 \int_0^t \|R(s)\| \big[\|\varphi_0\|_C + \|y_0\| \\ &+ s w(s)\big]^m \mathrm{d}s + K_8 \int_0^t \|R(s)\|\,\mathrm{d}s \Big\}. \end{aligned} \tag{2.7}$$

From this, the inequalities (2.4) and $w(t) \geq 1$, we have

$$w^p(t) \leq 1 + H + \int_0^t F_2(s) w^m(s)\,\mathrm{d}s \leq H + 1 + \int_0^t F_2(s) w^p(s)\,\mathrm{d}s \tag{2.8}$$

for $t \in [0, T)$, where

$$\begin{aligned} H = \max_{0\leq t\leq T} \|A(t)^{-1}\| \big\{ &\|A(0)\Phi_p(y_1)\| + (2^m\|\varphi_1\|_C^m + 1) K_7 \int_0^T \|B(s)\|\,\mathrm{d}s \\ &+ K_8\big(2^m\|y_0\|^m + 2^m(\|\varphi_0\|_C + \|y_0\|)^m + 1\big) \int_0^T \|R(s)\|\,\mathrm{d}s \big\}, \\ F_2(t) = \max_{0\leq s\leq T} &\|A(t)^{-1}\| \big\{ (2^m + 1) K_7 \|B(t)\| + 2^{m+1} K_8 t^m \|R(t)\| \big\}. \end{aligned}$$

Hence, (2.8) and Gronwall's inequality yield $w(t)$ and $y'(t)$ are bounded on $\langle 0, T)$. As according to $y(t) = y_0 + \int_0^t u(\tau)\,\mathrm{d}\tau$, y is bounded on $\langle 0, T)$, too, y cannot be nonextendable. The contradiction proves the statement.

Proof of Corollary 1.5. The sufficiency of (1.12) follows from Theorem 1.2 and the necessity of (1.12) follows from [22, Theorem 17.3].

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