Electronic Journal of Differential Equations, Vol. 2008(2008), No. 158, pp. 1–14.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

POSITIVE PERIODIC SOLUTIONS FOR AN IMPULSIVE RATIO-DEPENDENT PREDATOR-PREY SYSTEM WITH DELAYS

YAN LIU, QUANYI WANG

ABSTRACT. In this paper, we study a periodic ratio-dependent predator-prey system of two species with impulse and multiple time delays. By means of analysis techniques and the continuation theorem of coincidence degree theory, we obtain sufficient conditions for the existence of positive periodic solutions of the system. Our results extend previous results obtained in [9].

1. INTRODUCTION

The existence of positive periodic solutions of predator-prey models has been extensively studied by many mathematicians and biologists in recent years. Some authors have already obtained many good conclusions, see [2, 6, 10, 11].

However in many cases, especially when predators have to search, share or compete for food, a more suitable general predator-prey model should be based on the ratio-dependent theory. This roughly states that the per capita predator growth rate should be a function of the ratio of prey to predator abundance, see [3, 5].

In addition, there are numerous examples of evolutionary systems which at certain instants in time are subjected to rapid changes (for example, those due to seasonal effects of weather, food supply, hunting or harvesting seasons, etc). Those short-time perturbations are often assumed to be in the form of impulses in the modelling process. Consequently, impulsive differential equations provide a natural description of such systems. Because equations of this kind are found in many fields such as chemotherapy, population dynamics, optimal control, ecology, biotechnology and physics, they have attracted the interest of many researchers, see [1, 8, 9] and the references cited therein.

2000 *Mathematics Subject Classification.* 34K45, 34K13, 92D25.
Key words and phrases. Positive periodic solution; ratio-dependent; time delay; predator-prey system; impulse; coincidence degree theory.

Submitted May 28, 2008. Published December 3, 2008.
Supported by grant Z0511026 from the Natural Science Foundation of Fujian Province of China.

For the above reasons, Liu and Li [9] considered the following ratio-dependent predator-prey system with only one time delay and impulsive effects

$$
\begin{gathered}
x' = x(t)(b_1(t) - a_1(t)x(t) - \frac{c(t)y(t)}{m_1(t)y(t)+x(t)}), \quad t \neq t_k, \\
y' = y(t)(-b_2(t) + \frac{a_2(t)x(t-\tau)}{m_2(t)y(t-\tau)+x(t-\tau)}), \quad t \neq t_k, \\
x(t_k^+) - x(t_k^-) = c_k x(t_k), \\
y(t_k^+) - y(t_k^-) = d_k y(t_k), \quad t = t_k, \\
(x(0+), y(0+)) = (x_0, y_0), \\
(x(t), y(t)) = (\varphi_1(t), \varphi_2(t)) > 0, \quad -\tau \leq t < 0,
\end{gathered} \tag{e1.1}
$$

where $x(t), y(t)$ represent the densities of prey and predator at time t, respectively; τ is a positive constant time delay, $b_1(t)$, $a_1(t)$, $m_1(t)$, $b_2(t)$, $c(t)$, $a_2(t)$, $m_2(t) > 0$ are continuous T-periodic functions, $Z_+ = \{1, 2, \dots\}$. The initial functions are $\varphi(t) = (\varphi_1(t), \varphi_2(t))$, where $0 < t_1 < t_2 < \dots < t_k < \dots$ and $\lim_{k\to\infty} t_k = +\infty$. Assume that c_k, d_k ($k \in Z_+$) are constants and there exists an integer $q > 0$ such that $c_{k+q} = c_k$, $d_{k+q} = d_k$, $t_{k+q} = t_k + T$, $0 < t_{k+1} - t_k < T$. Liu and Li [9] obtained the following result.

thmA([9]) *Assume that the following conditions hold:*

$$
\begin{gathered}
\overline{b_1}T + \ln(\prod_{k=1}^{q}(1+c_k)) > \overline{(\frac{c}{m_1})}T, \\
\overline{b_2}T - \ln(\prod_{k=1}^{q}(1+d_k)) > 0, \quad \overline{a_2} > \overline{b_2}, \quad T > \tau, \\
{a_2}^l(T-\tau) - \overline{b_2}T + \ln(\prod_{k=1}^{q}(1+d_k)) > 0, \\
c^l - {m_1}^u[\overline{b_1} + \frac{1}{T}\ln(\prod_{k=1}^{q}(1+c_k))] > 0\,.
\end{gathered}
$$

Then system (e1.1) has at least one positive T-periodic solution.

However, this theorem is not valid because the condition

$$
c^l - {m_1}^u[\overline{b_1} + \frac{1}{T}\ln(\prod_{k=1}^{q}(1+c_k))] > 0
$$

contradicts the condition

$$
\overline{b_1}T + \ln(\prod_{k=1}^{q}(1+c_k)) > \overline{(\frac{c}{m_1})}T
$$

because

$$
\frac{c^l}{{m_1}^u} \leq \overline{(\frac{c}{m_1})}.
$$

Thus, the existence of a solution to (e1.1) has not been proved. Moreover, there are also some mistakes in the course of the proof of thmA, such as the computations of $K_P(I-Q)N\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}$ (see [9, p. 719]) and QNX (see [9, p. 722]).

In the actual environment, scientific researches suggest that time delays often occur in the course of the interaction of species in many ecological systems. So, in the present paper, we study the following two-species ratio-dependent predator-prey system with multiple time delays and impulsive effects

$$\begin{gathered}
x'(t) = x(t)\big[b_1(t) - a_1(t)x(t-\tau_1(t)) - \frac{c(t)y(t-\sigma_1(t))}{m_1(t)y(t-\sigma_1(t)) + x(t-\tau_2(t))}\big], \quad t \neq t_k, \\
y'(t) = y(t)\big[-b_2(t) + \frac{a_2(t)x(t-\tau_3(t))}{m_2(t)y(t-\sigma_2(t)) + x(t-\tau_3(t))}\big], \quad t \neq t_k, \\
x(t_k^+) - x(t_k^-) = c_{1k}x(t_k), \\
y(t_k^+) - y(t_k^-) = c_{2k}y(t_k), \quad t = t_k, \quad k = 1,2,\dots, \\
(x(0+), y(0+)) = (x_0, y_0), \\
(x(t), y(t)) = (\varphi_1(t), \varphi_2(t)) > 0, \quad -\tau \leq t \leq 0,
\end{gathered} \tag{e1.2}$$

where $x(t), y(t)$ represent the densities of prey and predator at time t, respectively; $a_1(t), a_2(t), b_1(t), b_2(t), c(t), m_1(t)$ and $m_2(t)$ are all positive continuous ω-periodic functions; $\sigma_1(t), \sigma_2(t), \tau_1(t), \tau_2(t)$ and $\tau_3(t)$ are all nonnegative continuous ω-periodic functions, $\tau = \max_{0\leq t\leq\omega}\{\sigma_1(t), \sigma_2(t), \tau_1(t), \tau_2(t), \tau_3(t)\}$. For the study of (e1.2), we always assume that

- (H1) $\{c_{ik}\}$ is a real sequence and $1 + c_{ik} > 0$, $i = 1,2$, $k = 1,2,\dots$;
- (H2) There exists an integer $q > 0$ such that $c_{i(k+q)} = c_{ik}$, $i = 1,2$, $k = 1,2,\dots$;
- (H3) $0 < t_1 < t_2 < \dots < t_q < \omega$ are fixed impulsive points in a period and $t_{k+q} = t_k + \omega$, $k = 1,2,\dots$.

In what follows, we shall use the following notation

$$f^M = \max_{t\in[0,\omega]} f(t), \quad f^l = \min_{t\in[0,\omega]} f(t), \quad \overline{f} = \frac{1}{\omega}\int_0^\omega f(t)dt,$$

where $f(t)$ is a continuous ω-periodic function.

2. Preliminaries

In this section, we first introduce the continuation theorem of coincidence degree theory [7], which will be used in this paper.

Let X, Z be real Banach spaces; let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \to Z$ a continuous mapping. The mapping L is called a Fredholm mapping of index zero if $\dim\ker L = \operatorname{codim}\operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$, $\ker Q = \operatorname{Im} L$, $X = \ker L \oplus \ker P$ and $Z = \operatorname{Im} L \oplus \operatorname{Im} Q$, then the restriction L_P of L to $\operatorname{Dom} L \cap \ker P$ is one-to-one and onto $\operatorname{Im} L$, so that its (algebraic) inverse $K_P : \operatorname{Im} L \to \operatorname{Dom} L \cap \ker P$ is defined. Let Ω be an open bounded subset of X, the mapping N is called L-compact on $\overline{\Omega}$ if $QN : \overline{\Omega} \to Z$ and $K_P(I-Q)N : \overline{\Omega} \to X$ are compact. Since $\operatorname{Im} Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \operatorname{Im} Q \to \ker L$. The following results appears in [7].

lem2.1([7]) *Let $\Omega \subset X$ be an open bounded set. Let L be a Fredholm mapping of index zero and $N : X \to Z$ a continuous operator which is L-compact on $\overline{\Omega}$. Assume*

- (a) *for each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;*

(b) *for each* $x \in \partial\Omega \cap \ker L, QNx \neq 0$;
(c) $\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0$.

Then the operator equation $Lx = Nx$ *has at least one solution in* $\overline{\Omega} \cap \operatorname{Dom} L$.

To prove the main conclusion by means of the continuation theorem, we need to introduce some function spaces.

Let $J_1 \subset \mathbb{R}$, and $PC(J_1, \mathbb{R})$ be the set of functions $\psi : J_1 \to \mathbb{R}$ such that $\psi(t)$ is continuous for $t \in J_1$, $t \neq t_k$, and is continuous from the left for $t \in J_1$, and $\psi(t_k^+)$ exists for $k = 1, 2, \dots$. Also define

$$PC^1(J_1, \mathbb{R}) = \{\psi : J_1 \to \mathbb{R}, \psi' \in PC(J_1, \mathbb{R})\}.$$

Now we define

$$\begin{aligned} X = \big\{&u(t) = (u_1(t), u_2(t))^T : u_i(t) \in PC([0,\omega], \mathbb{R}) \text{ for } t \in [0,\omega], \\ &u_i(t+\omega) = u_i(t) \text{ for } t \in \mathbb{R}, i = 1,2\big\} \end{aligned}$$

and $Z = X \times \mathbb{R}^{2q}$, where

$$\mathbb{R}^{2q} = \underbrace{\mathbb{R}^2 \times \mathbb{R}^2 \times \dots \times \mathbb{R}^2}_{q}.$$

Denote

$$\begin{gathered} \|u\| = \max\big\{\sup_{t\in[0,\omega]} |u_1(t)|, \sup_{t\in[0,\omega]} |u_2(t)|\big\} \quad \text{for } u \in X \\ \|z\| = \|u\| + \sum_{k=1}^{q} \|r_k\| \quad \text{for } z = (u, r_1, r_2, \dots, r_q) \in Z, \end{gathered}$$

where

$$r_k = \begin{pmatrix} r_{1k} \\ r_{2k} \end{pmatrix} \in \mathbb{R}^2, \quad \|r_k\| = \max\{|r_{1k}|, |r_{2k}|\}, \quad k = 1, 2, \dots, q\,.$$

Then (X, $\|\cdot\|$) and (Z, $\|\cdot\|$) are both Banach spaces.

def2.1([1]) *The set* $F \subset PC([0,\omega], \mathbb{R})$ *is said to be quasi-equicontinuous in* $[0,\omega]$ *if for any* $\varepsilon > 0$ *there exists* $\delta > 0$ *such that if* $x \in F$, $k \in Z_+$, $t_1, t_2 \in (t_{k-1}, t_k) \cap [0,\omega]$, $|t_1 - t_2| < \delta$, *then* $|x(t_1) - x(t_2)| < \varepsilon$.

lem2.2([1]) *The set* $F \subset PC([0,\omega], \mathbb{R})$ *is relatively compact if and only if*

(1) F *is bounded; that is,* $\|\psi\| = \sup\{|\psi| : t \in [0,\omega]\} \leq M$ *for each* $\psi \in F$ *and some* $M > 0$;
(2) F *is quasi-equicontinuous in* $[0,\omega]$.

3. Existence of positive ω-periodic solutions

In this section, we demonstrate the existence of a positive ω-periodic solution of (e1.2).

thm3.1 *Assume that* (H1)–(H3) *hold, and further assume the following conditions:*

(1) $\overline{b_1}\omega + \sum_{k=1}^{q} \ln(1 + c_{1k}) > \overline{(\frac{c}{m_1})}\omega$
(2) $a_2^l\omega + \sum_{k=1}^{q} \ln(1 + c_{2k}) > \overline{b_2}\omega$
(3) $\overline{b_2}\omega > \sum_{k=1}^{q} \ln(1 + c_{2k})$

Then (e1.2) has at least one positive ω*-periodic solution.*

Proof. Let $x(t)=e^{u_1(t)}$, $y(t)=e^{u_2(t)}$, then system (e1.2) can be rewritten as

$$
\begin{gathered}
u_1'(t)=b_1(t)-a_1(t)e^{u_1(t-\tau_1(t))}-\frac{c(t)e^{u_2(t-\sigma_1(t))}}{m_1(t)e^{u_2(t-\sigma_1(t))}}+e^{u_1(t-\tau_2(t))},\quad t\neq t_k,\\
u_2'(t)=-b_2(t)+\frac{a_2(t)e^{u_1(t-\tau_3(t))}}{m_2(t)e^{u_2(t-\sigma_2(t))}+e^{u_1(t-\tau_3(t))}},\quad t\neq t_k,\\
\Delta u_1(t_k)=u_1(t_k^+)-u_1(t_k)=\ln(1+c_{1k}),\\
\Delta u_2(t_k)=u_2(t_k^+)-u_2(t_k)=\ln(1+c_{2k}),\quad k=1,2,\dots.
\end{gathered}
\tag{e3.1}
$$

For the sake of simplicity, we denote

$$
\begin{gathered}
f_1(t,u(t))=b_1(t)-a_1(t)e^{u_1(t-\tau_1(t))}-\frac{c(t)e^{u_2(t-\sigma_1(t))}}{m_1(t)e^{u_2(t-\sigma_1(t))}+e^{u_1(t-\tau_2(t))}},\\
f_2(t,u(t))=-b_2(t)+\frac{a_2(t)e^{u_1(t-\tau_3(t))}}{m_2(t)e^{u_2(t-\sigma_2(t))}+e^{u_1(t-\tau_3(t))}},\\
\Delta u(t_k)=u(t_k^+)-u(t_k)=\begin{pmatrix}\Delta u_1(t_k)\\ \Delta u_2(t_k)\end{pmatrix},\quad k=1,2,\dots,q,\\
u(t)=(u_1(t),u_2(t))^T,\quad C_{1k}=\ln(1+c_{1k}),\quad C_{2k}=\ln(1+c_{2k}).
\end{gathered}
$$

It is obvious that if system (e3.1) has an ω-periodic solution $u^*(t)=(u_1^*(t),u_2^*(t))^T$, then $(x^*(t),y^*(t))^T=(e^{u_1^*(t)},e^{u_2^*(t)})^T$ is a positive ω-periodic solution of system (e1.2). So, to complete the proof, it suffices to show that the system (e3.1) has one ω-periodic solution.

To apply lem2.1 for establishing the existence of ω-periodic solutions of system (e3.1), now let

$$
\operatorname{Dom}L=\{u(t)=(u_1(t),u_2(t))^T\in X:(u_1(t),u_2(t))^T\in PC^1([0,\omega],\mathbb{R})\},
$$

and take $L:\operatorname{Dom}L\subset X\to Z$ as follows:

$$
u\to(u',\triangle u(t_1),\dots,\triangle u(t_q)),
$$

and define $N:X\to Z$ by

$$
Nu=\left(\begin{pmatrix}f_1(t,u(t))\\ f_2(t,u(t))\end{pmatrix},\begin{pmatrix}C_{11}\\ C_{21}\end{pmatrix},\dots,\begin{pmatrix}C_{1q}\\ C_{2q}\end{pmatrix}\right)
$$

for $u=(u_1,u_2)^T\in X$. Evidently, we have

$$
\begin{gathered}
\ker L=\{u:u\in X,u=c\in\mathbb{R}^2\},\\
\operatorname{Im}L=\big\{z=(u,r_1,r_2,\dots,r_q)\in Z:\frac{1}{\omega}\big(\int_0^\omega u(t)dt+\sum_{k=1}^q r_k\big)=0\big\}.
\end{gathered}
$$

So, $\operatorname{Im}L$ is closed in Z, and $\dim\ker L=2=\operatorname{codim}\operatorname{Im}L$. Hence, L is a Fredholm mapping of index zero.

Set two projectors $P : X \to X$ and $Q : Z \to Z$ as follows:

$$Pu = \frac{1}{\omega}\int_0^\omega u(t)dt, \quad (\forall u = (u_1, u_2)^T \in X),$$

$$Qz = Q(u, r_1, \dots, r_q) = \Big(\frac{1}{\omega}\big(\int_0^\omega u(t)dt + \sum_{k=1}^q r_k\big), 0, 0, \dots, 0\Big),$$
$$(\forall z = (u, r_1, r_2, \dots, r_q) \in Z).$$

It is easy to see that P and Q are continuous projectors, such that

$$\operatorname{Im} P = \ker L, \quad \ker Q = \operatorname{Im} L,$$
$$X = \ker L \oplus \ker P, \quad Z = \operatorname{Im} L \oplus \operatorname{Im} Q.$$

Furthermore, through an easy computation, we find that the inverse K_P of L_P (the restriction of L to $\operatorname{Dom} L \cap \ker P$) has the form $K_P : \operatorname{Im} L \to \operatorname{Dom} L \cap \ker P$,

$$K_P(z(t)) = \int_0^t u(s)ds + \sum_{0<t_k<t} r_k - \frac{1}{\omega}\big[\int_0^\omega \int_0^t u(s)dsdt + \sum_{k=1}^q r_k(\omega - t_k)\big]$$

for $z = (u, r_1, r_2, \dots, r_q) \in Z$. Accordingly, $QN : X \to Z$ and $K_P(I - Q)N : X \to X$ read

$$QNu = \left(\begin{pmatrix} \frac{1}{\omega}\big(\int_0^\omega f_1(s, u(s))ds + \sum_{k=1}^q C_{1k}\big) \\ \frac{1}{\omega}\big(\int_0^\omega f_2(s, u(s))ds + \sum_{k=1}^q C_{2k}\big)\end{pmatrix}, 0, 0, \dots, 0\right),$$

and

$$\begin{aligned} K_P(I - Q)Nu = &\begin{pmatrix} \int_0^t f_1(s, u(s))ds + \sum_{0<t_k<t} C_{1k} \\ \int_0^t f_2(s, u(s))ds + \sum_{0<t_k<t} C_{2k}\end{pmatrix} \\ &- \frac{1}{\omega}\begin{pmatrix} \int_0^\omega \int_0^t f_1(s, u(s))dsdt + \sum_{k=1}^q C_{1k}(\omega - t_k) \\ \int_0^\omega \int_0^t f_2(s, u(s))dsdt + \sum_{k=1}^q C_{2k}(\omega - t_k)\end{pmatrix} \\ &- (\frac{t}{\omega} - \frac{1}{2})\begin{pmatrix} \int_0^\omega f_1(s, u(s))ds + \sum_{k=1}^q C_{1k} \\ \int_0^\omega f_2(s, u(s))ds + \sum_{k=1}^q C_{2k}\end{pmatrix}. \end{aligned}$$

Using the Lebesgue convergence theorem, it is easy to see that QN and $K_P(I-Q)N$ are continuous. Moreover, from the expressions of QNu and $K_P(I - Q)Nu$, it is easy to see that $QN(\overline{\Omega})$ and $K_P(I - Q)N(\overline{\Omega})$ are bounded for any open bounded set $\Omega \subset X$. Furthermore, we have that

$$\frac{d}{dt}(QNu) = (0, 0, \dots, 0), \quad t \neq t_k,\ k = 1, 2, \dots$$

and

$$\frac{d}{dt}(K_P(I - Q)Nu) = \begin{pmatrix} f_1(t, u(t)) - \frac{1}{\omega}(\int_0^\omega f_1(s, u(s))ds + \sum_{k=1}^q C_{1k}) \\ f_2(t, u(t)) - \frac{1}{\omega}(\int_0^\omega f_2(s, u(s))ds + \sum_{k=1}^q C_{2k})\end{pmatrix}$$

for $t \neq t_k$, $k = 1, 2, \dots$, and $u \in X$. It follows from these expressions that the sets $\{\frac{d}{dt}(QNu) : u \in \overline{\Omega}\}$ and $\{\frac{d}{dt}(K_P(I-Q)Nu) : u \in \overline{\Omega}\}$ are bounded. So we have that $QN(\overline{\Omega})$ and $K_P(I - Q)N(\overline{\Omega})$ are equi-continuous in $[0, \omega]$. It follows from lem2.2 that $QN(\overline{\Omega})$ and $K_P(I - Q)N(\overline{\Omega})$ are compact. Therefore N is L-compact on $\overline{\Omega}$.

Corresponding to the operator equation $Lu = \lambda Nu$ with $\lambda \in (0,1)$, we have

$$
\begin{gathered}
u_1'(t) = \lambda\big[b_1(t) - a_1(t)e^{u_1(t-\tau_1(t))} - \frac{c(t)e^{u_2(t-\sigma_1(t))}}{m_1(t)e^{u_2(t-\sigma_1(t))} + e^{u_1(t-\tau_2(t))}}\big], \quad t \neq t_k,\\
u_2'(t) = \lambda\big[- b_2(t) + \frac{a_2(t)e^{u_1(t-\tau_3(t))}}{m_2(t)e^{u_2(t-\sigma_2(t))} + e^{u_1(t-\tau_3(t))}}\big], \quad t \neq t_k,\\
\Delta u_1(t_k) = \lambda \ln(1 + c_{1k}),\\
\Delta u_2(t_k) = \lambda \ln(1 + c_{2k}), \quad k = 1, 2, \dots .
\end{gathered} \tag{e3.2}
$$

Suppose that $u(t) = (u_1(t), u_2(t))^T \in X$ is an ω-periodic solution of (e3.2) for a certain $\lambda \in (0,1)$. Integrating (e3.2) over the interval $[0, \omega]$, we obtain

$$
\begin{gathered}
\int_0^\omega \big[b_1(t) - a_1(t)e^{u_1(t-\tau_1(t))} - \frac{c(t)e^{u_2(t-\sigma_1(t))}}{m_1(t)e^{u_2(t-\sigma_1(t))} + e^{u_1(t-\tau_2(t))}}\big]dt = -\sum_{k=1}^{q} \ln(1+c_{1k}),\\
\int_0^\omega \big[- b_2(t) + \frac{a_2(t)e^{u_1(t-\tau_3(t))}}{m_2(t)e^{u_2(t-\sigma_2(t))} + e^{u_1(t-\tau_3(t))}}\big]dt = -\sum_{k=1}^{q} \ln(1 + c_{2k}),
\end{gathered}
$$

which yield

$$
\begin{aligned}
&\int_0^\omega \big[a_1(t)e^{u_1(t-\tau_1(t))} + \frac{c(t)e^{u_2(t-\sigma_1(t))}}{m_1(t)e^{u_2(t-\sigma_1(t))} + e^{u_1(t-\tau_2(t))}}\big]dt\\
&= \int_0^\omega b_1(t)dt + \sum_{k=1}^{q} \ln(1 + c_{1k}),
\end{aligned} \tag{e3.3}
$$

$$
\int_0^\omega \frac{a_2(t)e^{u_1(t-\tau_3(t))}}{m_2(t)e^{u_2(t-\sigma_2(t))} + e^{u_1(t-\tau_3(t))}}dt = \int_0^\omega b_2(t)dt - \sum_{k=1}^{q} \ln(1 + c_{2k}). \tag{e3.4}
$$

In view of (e3.2), (e3.3) and (e3.4), we have

$$
\int_0^\omega |u_1{}'(t)|dt \le 2\int_0^\omega b_1(t)dt + \sum_{k=1}^{q} \ln(1 + c_{1k}) = 2\overline{b_1}\omega + \sum_{k=1}^{q} \ln(1 + c_{1k}), \tag{e3.5}
$$

$$
\int_0^\omega |u_2{}'(t)|dt \le 2\int_0^\omega b_2(t)dt - \sum_{k=1}^{q} \ln(1 + c_{2k}) = 2\overline{b_2}\omega - \sum_{k=1}^{q} \ln(1 + c_{2k}). \tag{e3.6}
$$

Since $u(t) = (u_1(t), u_2(t))^T \in X$, there exist $\eta_i, \xi_i \in [0, \omega]$, $(i = 1, 2)$ such that

$$
\begin{gathered}
u_i(\eta_i^+) = \sup_{t\in[0,\omega]} u_i(t) \quad \text{or} \quad u_i(\eta_i^-) = \sup_{t\in[0,\omega]} u_i(t),\\
u_i(\xi_i^+) = \inf_{t\in[0,\omega]} u_i(t) \quad \text{or} \quad u_i(\xi_i^-) = \inf_{t\in[0,\omega]} u_i(t), \quad (i = 1, 2).
\end{gathered}
$$

Whichever they are, for the sake of simplicity, we can denote them as follows:

$$
u_i(\eta_i) = \sup_{t\in[0,\omega]} u_i(t), \quad u_i(\xi_i) = \inf_{t\in[0,\omega]} u_i(t), \quad (i = 1, 2).
$$

Furthermore, it follows from (e3.3) that

$$
\overline{b_1}\omega + \sum_{k=1}^{q} \ln(1 + c_{1k}) \ge \int_0^\omega a_1(t)e^{u_1(t-\tau_1(t))}dt \ge \overline{a_1}\omega e^{u_1(\xi_1)},
$$

$$
\begin{aligned}
&\overline{b_1}\omega+\sum_{k=1}^{q}\ln(1+c_{1k})\\
&=\int_0^\omega a_1(t)e^{u_1(t-\tau_1(t))}dt+\int_0^\omega \frac{c(t)e^{u_2(t-\sigma_1(t))}}{m_1(t)e^{u_2(t-\sigma_1(t))}+e^{u_1(t-\tau_2(t))}}dt\\
&\le \overline{a_1}\omega e^{u_1(\eta_1)}+\overline{(\frac{c}{m_1})}\omega.
\end{aligned}
$$

So, from the condition (1), we have

$$
u_1(\xi_1)\le \ln\frac{\overline{b_1}\omega+\sum_{k=1}^{q}\ln(1+c_{1k})}{\overline{a_1}\omega}=:H_1, \tag{e3.7}
$$

$$
u_1(\eta_1)\ge \ln\frac{\overline{b_1}\omega-\overline{(\frac{c}{m_1})}\omega+\sum_{k=1}^{q}\ln(1+c_{1k})}{\overline{a_1}\omega}=:H_2. \tag{e3.8}
$$

Hence, (e3.5)–(e3.8) yield

$$
\begin{aligned}
u_1(t)&\le u_1(\xi_1)+\int_0^\omega |u_1{}'(t)|dt+\sum_{k=1}^{q}|\ln(1+c_{1k})|\\
&\le H_1+2\overline{b_1}\omega+\sum_{k=1}^{q}\big[\ln(1+c_{1k})+|\ln(1+c_{1k})|\big]=:H_3,
\end{aligned} \tag{e3.9}
$$

$$
\begin{aligned}
u_1(t)&\ge u_1(\eta_1)-\int_0^\omega |u_1{}'(t)|dt-\sum_{k=1}^{q}|\ln(1+c_{1k})|\\
&\ge H_2-2\overline{b_1}\omega-\sum_{k=1}^{q}\big[\ln(1+c_{1k})+|\ln(1+c_{1k})|\big]=:H_4.
\end{aligned} \tag{e3.10}
$$

It follows from the two equations above that

$$
\sup_{t\in[0,\omega]}|u_1(t)|\le \max\{|H_3|,|H_4|\}=:K_1. \tag{e3.11}
$$

On the other hand, from (e3.4), we can easily get

$$
\overline{b_2}\omega-\sum_{k=1}^{q}\ln(1+c_{2k})\le \int_0^\omega \frac{a_2(t)e^{u_1(\eta_1)}}{m_2(t)e^{u_2(\xi_2)}+e^{u_1(\eta_1)}}dt\le \frac{a_2^Me^{u_1(\eta_1)}\omega}{m_2^le^{u_2(\xi_2)}+e^{u_1(\eta_1)}}, \tag{e3.12}
$$

and

$$
\overline{b_2}\omega-\sum_{k=1}^{q}\ln(1+c_{2k})\ge \int_0^\omega \frac{a_2(t)e^{u_1(\xi_1)}}{m_2(t)e^{u_2(\eta_2)}+e^{u_1(\xi_1)}}dt\ge \frac{a_2^le^{u_1(\xi_1)}\omega}{m_2^Me^{u_2(\eta_2)}+e^{u_1(\xi_1)}}. \tag{e3.13}
$$

Thus, from (e3.11)–(e3.13) and the conditions (2) and (3), one has

$$
u_2(\xi_2)\le \ln\frac{\big[a_2^M\omega-\bar b_2\omega+\sum_{k=1}^{q}\ln(1+c_{2k})\big]e^{K_1}}{m_2^l\big[\bar b_2\omega-\sum_{k=1}^{q}\ln(1+c_{2k})\big]}=H_5+K_1, \tag{e3.14}
$$

$$
u_2(\eta_2)\ge \ln\frac{\big[a_2^l\omega-\bar b_2\omega+\sum_{k=1}^{q}\ln(1+c_{2k})\big]e^{-K_1}}{m_2^M\big[\bar b_2\omega-\sum_{k=1}^{q}\ln(1+c_{2k})\big]}=H_6-K_1, \tag{e3.15}
$$

where

$$H_5 = \ln \frac{\left[a_2^M\omega - \bar{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k})\right]}{m_2^l\left[\bar{b}_2\omega - \sum_{k=1}^q \ln(1+c_{2k})\right]},$$
$$H_6 = \ln \frac{\left[a_2^l\omega - \bar{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k})\right]}{m_2^M\left[\bar{b}_2\omega - \sum_{k=1}^q \ln(1+c_{2k})\right]}.$$

Furthermore, from (e3.6), (e3.14) and (e3.15), it follows that

$$\begin{aligned} u_2(t) &\le u_2(\xi_2) + \int_0^\omega |{u_2}'(t)|dt + \sum_{k=1}^q |\ln(1+c_{2k})| \\ &\le H_5 + K_1 + 2\overline{b_2}\omega - \sum_{k=1}^q \ln(1+c_{2k}) + \sum_{k=1}^q |\ln(1+c_{2k})| =: H_7, \end{aligned} \tag{e3.16}$$

$$\begin{aligned} u_2(t) &\ge u_2(\eta_2) - \int_0^\omega |{u_2}'(t)|dt - \sum_{k=1}^q |\ln(1+c_{2k})| \\ &\ge H_6 - K_1 - 2\bar{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k}) - \sum_{k=1}^q |\ln(1+c_{2k})| =: H_8. \end{aligned} \tag{e3.17}$$

Therefore,

$$\sup_{t\in[0,\omega]} |u_2(t)| \le \max\{|H_7|, |H_8|\} =: K_2. \tag{e3.18}$$

Set $K = 1 + K_1 + K_2 + |H_1| + |H_2| + |H_5| + |H_6|$. Clearly, K is independent of λ ($\lambda \in (0,1)$). Then it follows from (e3.11) and (e3.18) that

$$\|u\| \le K. \tag{e3.19}$$

Suppose $u = (u_1, u_2)^T \in \mathbb{R}^2$. Then from the expression of QNu , we obtain

$$\begin{aligned} &QN\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ &= \left(\begin{pmatrix} \bar{b}_1 - \bar{a}_1 e^{u_1} - \frac{1}{\omega}\int_0^\omega \frac{c(t)e^{u_2}}{m_1(t)e^{u_2}+e^{u_1}}dt + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{1k}) \\ -\bar{b}_2 + \frac{1}{\omega}\int_0^\omega \frac{a_2(t)e^{u_1}}{m_2(t)e^{u_2}+e^{u_1}}dt + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{2k}) \end{pmatrix}, 0, \dots, 0 \right). \end{aligned} \tag{e3.20}$$

Consider the equation

$$\begin{aligned} \overline{b_1} - \overline{a_1}e^{u_1} - \frac{1}{\omega}\int_0^\omega \frac{c(t)e^{u_2}}{m_1(t)e^{u_2}+e^{u_1}}dt + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{1k}) = 0, \\ -\overline{b_2} + \frac{1}{\omega}\int_0^\omega \frac{a_2(t)e^{u_1}}{m_2(t)e^{u_2}+e^{u_1}}dt + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{2k}) = 0. \end{aligned} \tag{e3.21}$$

By analysis similar to the one for (e3.7), (e3.8), (e3.14) and (e3.15), it is not difficult to see that any solution $u^* = (u_1^*, u_2^*)^T \in \mathbb{R}^2$ of (e3.21) exists, it must satisfy:

$H_2 \le u_1^* \le H_1$,

$$\begin{aligned} u_2^* &\le \ln \frac{\big[a_2^M\omega - \overline{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k})\big]e^{u_1^*}}{m_2^l\big[\overline{b}_2\omega - \sum_{k=1}^q \ln(1+c_{2k})\big]} \\ &= \ln \frac{\big[a_2^M\omega - \overline{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k})\big]}{m_2^l\big[\overline{b}_2\omega - \sum_{k=1}^q \ln(1+c_{2k})\big]} + u_1^* \\ &= H_5 + H_1, \end{aligned}$$

$$\begin{aligned} u_2^* &\ge \ln \frac{\big[a_2^l\omega - \overline{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k})\big]e^{u_1^*}}{m_2^M\big[\overline{b}_2\omega - \sum_{k=1}^q \ln(1+c_{2k})\big]} \\ &= \ln \frac{\big[a_2^l\omega - \overline{b}_2\omega + \sum_{k=1}^q \ln(1+c_{2k})\big]}{m_2^M\big[\overline{b}_2\omega - \sum_{k=1}^q \ln(1+c_{2k})\big]} + u_1^* \\ &= H_6 + H_2, \end{aligned}$$

which yield

$$\|u^*\| \le K. \tag{e3.22}$$

Put $\Omega = \{u = (u_1, u_2)^T \in X : \|u\| < K_0\}$, where $K_0 = K + 1$. Then it follows from (e3.19) that condition (a) of lem2.1 is satisfied.

Furthermore, for each $u = (u_1, u_2)^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^2$, we know that $u = (u_1, u_2)^T$ is a constant vector in $\mathbb{R}^2$ with $\|u\| = K_0$, and then can directly get $QNu \ne 0$ by (e3.20)-(e3.22). This shows that condition (b) of lem2.1 is satisfied.

Finally, let us prove that the condition (c) of lem2.1 is satisfied. Define $\phi : \overline{\Omega} \cap \ker L \times [0,1] \to \mathbb{R}^2$ by

$$\begin{aligned} \phi(u_1,u_2,\eta) = &\begin{pmatrix} \overline{b}_1 - \overline{a_1}e^{u_1} + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{1k}) \\ -\overline{b_2} + \frac{1}{\omega}\int_0^\omega \frac{a_2(t)e^{u_1}}{m_2(t)e^{u_2}+e^{u_1}}dt + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{2k}) \end{pmatrix} \\ &+ \eta \begin{pmatrix} -\frac{1}{\omega}\int_0^\omega \frac{c(t)e^{u_2}}{m_1(t)e^{u_2}+e^{u_1}}dt \\ 0 \end{pmatrix}, \end{aligned}$$

where $u = (u_1, u_2)^T \in \overline{\Omega} \cap \ker L$, $\eta \in [0,1]$. First, we will prove that $\phi(u_1, u_2, \eta) \ne 0$ when $u = (u_1, u_2)^T \in \partial\Omega \cap \ker L, \eta \in [0,1]$. Assume the conclusion is not true, then there exists a constant vector $u = (u_1, u_2)^T$ with $\|u\| = K_0$, such that $\phi(u_1, u_2, \eta) = 0$ for a certain $\eta \in [0,1]$; i.e.,

$$\begin{gathered} \overline{b_1} - \overline{a_1}e^{u_1} + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{1k}) - \frac{\eta}{\omega}\int_0^\omega \frac{c(t)e^{u_2}}{m_1(t)e^{u_2}+e^{u_1}}dt = 0, \\ -\overline{b_2} + \frac{1}{\omega}\int_0^\omega \frac{a_2(t)e^{u_1}}{m_2(t)e^{u_2}+e^{u_1}}dt + \frac{1}{\omega}\sum_{k=1}^q \ln(1+c_{2k}) = 0. \end{gathered} \tag{e3.23}$$

By a similar discussion on the solutions of (e3.21), it is easy to see that any solution $u = (u_1, u_2)^T \in \mathbb{R}^2$ of (e3.23) must satisfy

$$\|u\| \le K < K_0, \tag{e3.24}$$

which contradicts $\|u\| = K_0$ ($\forall u \in \partial\Omega \cap \ker L$). This shows that $\phi(u_1, u_2, \eta) \ne 0$ when $u = (u_1, u_2)^T \in \partial\Omega \cap \mathbb{R}^2$ and $\eta \in [0,1]$. We next prove that the equation

$\phi(u_1, u_2, 0) = 0$ in $\Omega \cap \ker L$ has a unique solution $u^{**} = (u_1^{**}, u_2^{**})^T$. In fact, $\phi(u_1^{**}, u_2^{**}, 0) = 0$ means

$$\begin{gathered} \overline{b_1} - \overline{a_1} e^{u_1^{**}} + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{1k}) = 0, \\ -\overline{b_2} + \frac{1}{\omega} \int_0^\omega \frac{a_2(t) e^{u_1^{**}}}{m_2(t) e^{u_2^{**}} + e^{u_1^{**}}} dt + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}) = 0; \end{gathered} \tag{e3.25}$$

that is,

$$\begin{gathered} u_1^{**} = \ln \frac{\overline{b_1} + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{1k})}{\overline{a_1}}, \\ -\overline{b_2} + \frac{1}{\omega} \int_0^\omega \frac{a_2(t) e^{u_1^{**}}}{m_2(t) e^{u_2^{**}} + e^{u_1^{**}}} dt + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}) = 0. \end{gathered} \tag{e3.26}$$

For $u_2 \in \mathbb{R}$, let

$$g(u_2) = -\overline{b_2} + \frac{1}{\omega} \int_0^\omega \frac{a_2(t) e^{u_1^{**}}}{m_2(t) e^{u_2} + e^{u_1^{**}}} dt + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}), \tag{e3.27}$$

then $g(u_2) \in C^1(\mathbb{R}, \mathbb{R})$ and

$$g'(u_2) = -\frac{1}{\omega} \int_0^\omega \frac{a_2(t) m_2(t) e^{u_1^{**}} e^{u_2}}{\left[m_2(t) e^{u_2} + e^{u_1^{**}}\right]^2} dt < 0;$$

that is, $g(u_2)$ is strictly monotonous decreasing in $\mathbb{R}$. From the conditions (2) and (3), it is easy to obtain

$$g(+\infty) = -\overline{b_2} + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}) < 0,$$

$$g(-\infty) = \overline{a_2} - \overline{b_2} + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}) > 0.$$

Therefore, there exists a unique $u_2^{**} \in \mathbb{R}$ such that $g(u_2^{**}) = 0$; that is,

$$-\overline{b_2} + \frac{1}{\omega} \int_0^\omega \frac{a_2(t) e^{u_1^{**}}}{m_2(t) e^{u_2^{**}} + e^{u_1^{**}}} dt + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}) = 0. \tag{e3.28}$$

Then by integral mean value theorem, there exists a $t_0 \in [0, \omega]$ such that

$$-\overline{b_2} + \frac{a_2(t_0) e^{u_1^{**}}}{m_2(t_0) e^{u_2^{**}} + e^{u_1^{**}}} + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k}) = 0. \tag{e3.29}$$

It follows from (e3.29) and conditions (2), (3) that

$$u_2^{**} = \ln \frac{\left[a_2(t_0) - \overline{b_2} + \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k})\right]}{m_2(t_0) \left[\overline{b_2} - \frac{1}{\omega} \sum_{k=1}^{q} \ln(1 + c_{2k})\right]} + u_1^{**}. \tag{e3.30}$$

Thus, from (e3.24) and (e3.30), we obtain

$$H_2 \le u_1^{**} \le H_1, \quad H_2 + H_6 \le u_2^{**} \le H_1 + H_5,$$

which imply

$$\|u^{**}\| \le K < K_0;$$

i.e., the equation $\phi(u_1,u_2,0)=0$ in $\Omega\cap\ker L$ has a unique solution $u^{**}=(u_1^{**},u_2^{**})^T$.

Now define the isomorphism $J:\operatorname{Im}Q\to\ker L$ by

$$J\Big(\frac{1}{\omega}\Big(\int_0^\omega u(t)dt+\sum_{k=1}^q r_k\Big),0,0,\dots,0\Big)=\frac{1}{\omega}\Big(\int_0^\omega u(t)dt+\sum_{k=1}^q r_k\Big),$$

then $JQNu=\phi(u_1,u_2,1)$ for each $u=(u_1,u_2)^T\in\overline{\Omega}\cap\ker L$. Using the property of topological degree, from (e3.25) and (e3.29), we have

$$\begin{aligned}
&\deg\{JQN,\Omega\cap\ker L,0\}\\
&=\deg\{\phi(u_1,u_2,1),\Omega\cap\ker L,0\}\\
&=\deg\{\phi(u_1,u_2,0),\Omega\cap\ker L,0\}\\
&=\operatorname{sign}\begin{vmatrix}-\overline{a_1}e^{u_1^{**}} & 0\\ \frac{a_2(t_0)m_2(t_0)e^{u_1^{**}}e^{u_2^{**}}}{(m_2(t_0)e^{u_2^{**}}+e^{u_1^{**}})^2} & -\frac{a_2(t_0)m_2(t_0)e^{u_1^{**}}e^{u_2^{**}}}{(m_2(t_0)e^{u_2^{**}}+e^{u_1^{**}})^2}\end{vmatrix}\neq 0.
\end{aligned}$$

Thus, condition (c) of lem2.1 holds and by now we have proved that all the conditions of lem2.1 are satisfied. Hence, system (e3.1) has at least one ω-periodic solution. Accordingly, system (e1.2) has at least one positive ω–periodic solution. This completes the proof. □

If we set $\sigma_1(t)=\tau_1(t)=\tau_2(t)=0$, $\tau_3(t)=\sigma_2(t)=\tau$, $\omega=T$, then system (e1.2) is simplified to system (e1.1) which was studied by Liu and Li in [9].

Obviously, our result in this paper extends and improves greatly the result in [9]. Finally, let us consider the system without impulse

$$\begin{aligned}
x'(t)&=x(t)\big[b_1(t)-a_1(t)x(t-\tau_1(t))-\frac{c(t)y(t-\sigma_1(t))}{m_1(t)y(t-\sigma_1(t))+x(t-\tau_2(t))}\big],\\
y'(t)&=y(t)\big[-b_2(t)+\frac{a_2(t)x(t-\tau_3(t))}{m_2(t)y(t-\sigma_2(t))+x(t-\tau_3(t))}\big],
\end{aligned}\tag{e3.31}$$

where $a_1(t)$, $a_2(t)$, $b_1(t)$, $b_2(t)$, $c(t)$, $m_1(t)$ and $m_2(t)$ are all positive continuous ω-periodic functions; $\sigma_1(t)$, $\sigma_2(t)$, $\tau_1(t)$, $\tau_2(t)$ and $\tau_3(t)$ are all continuous ω-periodic functions. From 3 and its proof, we immediately get the following result.

thm3.2 *If system (e3.31) satisfies the conditions*

$$\overline{b_1}>\overline{(\frac{c}{m_1})},\quad a_2^l>\overline{b_2},$$

then (e3.31) has at least one positive ω-periodic solution.

4. An example

In this section, we give an example that illustrates the feasibility of our results. Consider the system

$$\begin{gathered}
x'(t)=x(t)\big[(\frac{1}{2}+|\cos t|)-x(t-|\sin t|)-\frac{(\frac{1}{2}|\sin t|+\frac{1}{4})y(t-|\cos t|)}{y(t-|\cos t|)+x(t-|\sin 2t|)}\big],\ t\neq t_k,\\
y'(t)=y(t)\big[-(\frac{3}{4}+\frac{1}{2}\sin t)+\frac{\frac{4}{\pi}x(t-|\sin 3t|)}{2y(t-|\cos 2t|)+x(t-|\sin 3t|)}\big],\quad t\neq t_k,\\
x(t_k^+)-x(t_k^-)=c_{1k}x(t_k),\\
y(t_k^+)-y(t_k^-)=c_{2k}y(t_k),\quad k=1,2,\dots,
\end{gathered}\tag{4.1}$$

where

$$t_1 = \frac{\pi}{2}, \quad t_2 = \frac{3\pi}{2}, \quad t_{k+2} = t_k + 2\pi, \quad c_{11} = 1, \quad c_{12} = -\frac{1}{2},$$
$$c_{21} = 1, \quad c_{22} = -\frac{3}{4}, \quad c_{i(k+2)} = c_{ik}, \quad i = 1, 2, \quad k = 1, 2, \dots.$$

Corresponding to (e1.2), we have

$$\omega = 2\pi, \quad a_1(t) = 1, \quad a_2(t) = \frac{4}{\pi}, \quad b_1(t) = \frac{1}{2} + |\cos t|, \quad b_2(t) = \frac{3}{4} + \frac{1}{2}\sin t,$$
$$c(t) = \frac{1}{2}|\sin t| + \frac{1}{4}, \quad m_1(t) = 1, \quad m_2(t) = 2, \quad \sigma_1(t) = |\cos t|,$$
$$\sigma_2(t) = |\cos 2t|, \quad \tau_1(t) = |\sin t|, \quad \tau_2(t) = |\sin 2t|, \quad \tau_3(t) = |\sin 3t|.$$

It is easy to obtain that,

$$\overline{b_1} = \frac{1}{2\pi}\int_0^{2\pi}(\frac{1}{2} + |\cos t|)dt = \frac{2}{\pi} + \frac{1}{2}, \quad \overline{b_2} = \frac{1}{2\pi}\int_0^{2\pi}(\frac{3}{4} + \frac{1}{2}\sin t)dt = \frac{3}{4},$$
$$\overline{(\frac{c}{m_1})} = \frac{1}{2\pi}\int_0^{2\pi}(\frac{1}{2}|\sin t| + \frac{1}{4})dt = \frac{1}{4} + \frac{1}{\pi},$$
$$\sum_{k=1}^{2}\ln(1 + c_{1k}) = \ln 2 - \ln 2 = 0, \quad \sum_{k=1}^{2}\ln(1 + c_{2k}) = \ln 2 - \ln 4 = -\ln 2,$$

and then

$$\overline{b_1}\omega + \sum_{k=1}^{2}\ln(1 + c_{1k}) = \pi + 4 > 2 + \frac{\pi}{2} = \overline{(\frac{c}{m_1})}\omega,$$
$$a_2^l\omega + \sum_{k=1}^{2}\ln(1 + c_{2k}) = 8 - \ln 2 > \frac{3\pi}{2} = \overline{b_2}\omega,$$
$$\overline{b_2}\omega = \frac{3\pi}{2} > -\ln 2 = \sum_{k=1}^{2}\ln(1 + c_{2k}).$$

Thus, all the conditions of thm3.1 are satisfied. Then system (e4.1) has at least one positive 2π-periodic solution.

Acknowledgements. The authors are grateful to the anonymous referees for their valuable suggestions.

References

[1] D. D. Bainov, P. S. Simeonov; *Impulsive Differential Equations: Periodic Solutions and Applications*. Longman Scientific and Technical, NewYork, 1993.

[2] Y. Chen; *Multiple periodic solutions of delayed predator-prey systems with type IV functional responses*, Nonlinear Anal. 5(2005), 45-53.

[3] S. H. Chen, F. Wang, T. Young; *Positive periodic solution of two-species ratio-dependent predator-prey system with time delay in two-patch environment*, Comput. Math. Appl. 150(2004), 737-748.

[4] J. M. Cushing; *Periodic time-dependent predator-prey system*, SIAM. J. Appl. Math. 32(1977), 82-95.

[5] Y. H. Fan, W. T. Li; *Permanence in delayed ratio-dependent predator-prey models with monotonic functional responses*, Nonlinear Anal. 8(2007), 424-434.

[6] M. Fan, K. Wang; *Global existence of positive periodic solutions for periodic predator-prey system with infinite delay*, J. Math. Anal. Appl. 262(2001), 1-11.

[7] R. E. Gains, J. L. Mawhin; *Coincidence degree and nonlinear differential equations*. Spring-Verlag, Berlin, 1977.

[8] M. Li, Y. R. Duan, W. P. Zhang, M. S. Wang; *The existence of positive periodic solutions of a class of Lotka-Volterra type impulsive systems with infinitely distributed delay*, Comput. Math. Appl. 49(2005), 1037-1044.

[9] X. S. Liu, G. Li, G. Luo; *Positive periodic solution for a two-species ratio-dependent predator-prey system with time delay and impulse*, J. Math. Anal. Appl. 325(2007), 715-723.

[10] L. L. Wang, W. T. Li; *Existence and global stability of positive periodic solutions of a predator-prey system with delays*, Applied Mathematics and Computation, 146(2003), 167-185.

[11] H. L. Wang, S. M. Zhang, B. D. Tian; *Permanence and existence of periodic solutions of a predator-prey patchy model with dispersal and time delay*, J. Math. Biol. 22(2007), 25-36.

Yan Liu

School of mathematical sciences, Huaqiao University, Quanzhou, Fujian 362021, China

E-mail address: `liuyanlxly@yahoo.com.cn`

Quanyi Wang

School of mathematical sciences, Huaqiao University, Quanzhou, Fujian 362021, China

E-mail address: `qywang@hqu.edu.cn`