Electronic Journal of Differential Equations, Vol. 2008(2008), No. 141, pp. 1–9.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

EXISTENCE OF GLOBAL SOLUTIONS FOR A SYSTEM OF REACTION-DIFFUSION EQUATIONS HAVING A TRIANGULAR MATRIX

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ABSTRACT. We consider the system of reaction-diffusion equations

$$u_t - a\Delta u = \beta - f(u,v) - \alpha u,$$
$$v_t - c\Delta u - d\Delta v = g(u,v) - \sigma v.$$

Our aim is to establish the existence of global classical solutions using the method used by Melkemi, Mokrane, and Youkana [19].

1. INTRODUCTION

In this manuscript, we consider a reaction-diffusion system that arises in the study of physical, chemistry, and various biological processes including population dynamics [3, 6, 7, 9, 14, 23, 24]. The system of equations is

$$\frac{\partial u}{\partial t} - a\Delta u = \beta - f(u,v) - \alpha u \quad (x,t) \in \Omega \times R_+ \tag{1.1}$$

$$\frac{\partial v}{\partial t} - c\Delta u - d\Delta v = g(u,v) - \sigma v \quad (x,t) \in \Omega \times R_+, \tag{1.2}$$

with the boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \quad \text{on } \partial\Omega \times R_+. \tag{1.3}$$

and the initial data

$$0 \le u(0,x) = u_0(x); \quad 0 \le v(0,x) = v_0(x) \quad \text{in } \Omega. \tag{1.4}$$

where Ω is a smooth open bounded domain in R^n, with boundary $\partial\Omega$ of class C^1 and η is the outer normal to $\partial\Omega$. The constants of diffusion a, c, d are such that $a > 0$, $d > 0$, $c > 0$ and $a > d$, $c^2 < 4ad$ which is the parabolic condition, and α, σ are positive constants, $\beta \ge 0$, and f, g are nonnegative functions of class $C(R_+ \times R_+)$, such that

- (H1) For all $\tau \ge 0$, $f(0,\tau) = 0$;
- (H2) For all $\xi \ge 0$ and all $\tau \ge 0$, $0 \le f(\xi,\tau) \le \varphi(\xi)(\mu + \tau)^r$;
- (H3) For all $\xi \ge 0$ and all $\tau \ge 0$, $g(\xi,\tau) \le \psi(\tau) f(\xi,\tau) + \phi(\tau)$,

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2000 *Mathematics Subject Classification.* 35K57, 35K45.
Key words and phrases. Reaction-diffusion systems; Lyapunov functional; global solution.

Submitted August 19, 2007. Published October 16, 2008.

where r, μ are positive constants such that $r \geq 1$, $\mu \geq 1$, φ, ψ and ϕ are nonnegative functions of class $C(R^+)$, such that

$$\lim_{\tau \to +\infty} \frac{\psi(\tau)}{\tau} = \lim_{\tau \to +\infty} \frac{\phi(\tau)}{\tau} = 0. \tag{1.5}$$

$$\phi(0) > \beta \frac{c}{a-d}. \tag{1.6}$$

In addition we suppose that

$$g(\xi, \frac{c}{a-d}\xi) + \frac{c}{a-d} f(\xi, \frac{c}{a-d}\xi) \geq \frac{c}{a-d}[(\sigma - \alpha)\xi + \beta], \quad \forall \xi \geq 0. \tag{1.7}$$

Melkemi et al [19] established the existence of global solutions, (eventually uniformly bounded in time) using a novel approach that involved the use of a Lyapunov function for system (1.1)–(1.4) when $c = 0$. Here, we apply the same method to the study of system (1.1)–(1.4) when $c > 0$; that is, for a model that involves a triangular matrix.

2. Existence of local and positive solutions

First we convert system (1.1)–(1.4) into an abstract first order system in $X = C(\overline{\Omega}) \times C(\overline{\Omega})$ of the form

$$U'(t) = AU(t) + F(U(t)), \quad t > 0, \tag{2.1}$$

$$U(0) = U_0 \in X, \tag{2.2}$$

where

$$AU(t) = (a\Delta u, c\Delta u + d\Delta v),$$
$$F(U(t)) = (\beta - f(u,v) - \alpha u, g(u,v) - \sigma v).$$

Since F is locally Lipschitz in U and X, for every initial data $U_0 \in X$, system (2.1)–(2.2) admits a unique strong local solution on $]0, T^*[$, where T^* is the eventual blowing-up time, (see Kirane [14], Friedman [8], Henry [12], Pazy [20]).

Multiplying (1.1) by $\frac{c}{a-d}$, and subtracting the resulting equation from (1.2) leads to the system

$$\frac{\partial u}{\partial t} - a\Delta u = \Lambda(u,z), \quad (x,t) \in \Omega \times R_+ \tag{2.3}$$

$$\frac{\partial z}{\partial t} - d\Delta z = \Upsilon(u,z), \quad (x,t) \in \Omega \times R_+. \tag{2.4}$$

where

$$\Lambda(u,z) = \beta - f(u,v) - \alpha u$$
$$\Upsilon(u,z) = g(u,v) + \frac{c}{a-d} f(u,v) + \frac{c}{a-d}(\alpha u - \beta) - \sigma v,$$
$$z = v - \frac{c}{a-d} u,$$

with the boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial z}{\partial \eta} = 0, \quad \text{on } \partial\Omega \times R_+ \tag{2.5}$$

and initial data

$$u(0,x) = u_0(x) \quad \text{in } \Omega, \tag{2.6}$$

$$z(0,x) = z_0(x) = v_0(x) - \frac{c}{a-d}u_0(x) \quad \text{in } \Omega. \tag{2.7}$$

If we assume (1.7) and (H1) then a simple application of a comparison theorem to system (2.3)-(2.4) implies (see [14]) that for positive initial data $u_0 \geq 0$ and $z_0 \geq o$ we have that

$$u(t,x) \geq 0, \quad v(t,x) \geq \frac{c}{a-d}u(t,x) \quad \forall (x,t) \in \Omega \times]0,T^*[.$$

3. Existence of global solutions

Before we establish the existence of a global solution, we introduce some notation. Here, we let

$$\|u\|_p^p = \frac{1}{|\Omega|}\int_\Omega |u(x)|^p dx \text{ and } \|u\|_\infty = \max_{x\in\Omega}|u(x)|.$$

denote the usual norms in spaces $L^p(\Omega)$, $L^\infty(\Omega)$ and $C(\overline{\Omega})$. Applying the comparison principle we get that

$$u(t,x) \leq \max(\|u_0\|_\infty, \frac{\beta}{\alpha}) = K. \tag{3.1}$$

To establish the uniform boundedness of v, it is sufficient to show the uniform boundedness of z. This task is carried out using a result found in Henry [12, pp. 35-62], from which it sufficient to derive a uniform estimate for $\|\Upsilon(u,z)\|_p$; that is, finding a constant C such that

$$\|\Upsilon(u,z)\|_p \leq C. \tag{3.2}$$

Here C is a nonnegative constant independent of t, for some $p > n/2$. The key is to establish the uniform boundedness of $\|v\|_p$ on $]0,T^*[$, taking into account assumptions (H2) and (H3). When $p \geq 2$, we put

$$\begin{gathered}\Gamma(p) = \frac{p\Gamma+1}{p-1}\,, \quad \Gamma = (a-d)^2[1+\frac{1}{4ad}]\,,\\ l = \frac{2\beta\rho}{\Gamma(p)\sigma}\,, \quad \omega = [\frac{S^2}{4adR^2} + \frac{p}{R^2} + \mu](p-1)\end{gathered} \tag{3.3}$$

where $\rho > 0$,

$$S = \frac{\rho}{\Gamma(p)l}, \quad R = \frac{(a-d)\rho}{\Gamma(p)(l+\rho K)}.$$

Using these definitions and notation we can state the following key proposition

Proposition 3.1. *Assume that $p \geq 2$ and let*

$$G_N(t) = N\int_\Omega u dx + \int_\Omega (v+\omega)^p \exp(-\frac{1}{\Gamma(p)}\ln(\Gamma(p)[l+\rho(K-u)]))dx, \tag{3.4}$$

where (u,v) is the solution of (1.1)–(1.4) *on $]0,T^*[$. Then under the assumptions (H3) and* (1.5) *there exist two positive constants N and s such that*

$$\frac{dG_N}{dt} \leq -(p-1)\sigma G_N + s. \tag{3.5}$$

The proof of the above proposition requires some lemmas.

Lemma 3.2. *If* (u,v) *be a solution of* (1.1) – (1.4) *then*

$$\int_\Omega f(u,v)dx \le \beta|\Omega| - \frac{d}{dt}\int_\Omega u(t,x)dx. \tag{3.6}$$

Proof. We integrate both sides of (1.1),

$$f(u,v) = \beta - \alpha u - \frac{d}{dt}u(t,x)$$

satisfied by u, which is positive and then we find (3.6). □

Lemma 3.3. *Assume that* $p \ge 2$, *then under the assumptions* (H1)–(H3), *and* (1.5) *there exists* N_1, *such that*

$$[p(g(\xi,\tau) - \phi(\tau))(\tau+\omega)^{p-1} - \theta f(\xi,\tau)(\tau+\omega)^p] \le N_1 f(\xi,\tau) \tag{3.7}$$

for all $0 \le \xi \le K$ *and* $\tau \ge 0$, $\theta > 0$.

Proof. from the assumption (H3) and (1.5), we conclude that there exists $\tau_0 > 0$, such that for all $0 \le \xi \le K, \tau \ge \tau_0$, one finds

$$[p\frac{\psi(\tau)}{\tau+\omega} - \theta](\tau+\omega)^p f(\xi,\tau) \le 0$$

now if τ is in the compact interval $[0,\tau_0]$, then the continuous function

$$\chi(\xi,\tau) = p\psi(\tau)(\tau+\omega)^{p-1} - \theta(\tau+\omega)^p$$

is bounded. □

Lemma 3.4. *For all* $\tau \ge 0$ *and* $\omega \ge 1$, *we have*

$$\frac{\beta\rho}{\Gamma(p)l}(\tau+\omega)^p - \sigma p(\tau+\omega)^{p-1}\tau + p\phi(\tau)(\tau+\omega)^{p-1} \le -(p-1)\sigma(\tau+\omega)^p + M_1 \tag{3.8}$$

where M_1 *is positive constant.*

Proof. Let us put

$$\begin{aligned}\Pi =& \frac{\beta\rho}{\Gamma(p)l}(\tau+\omega)^p - \sigma p(\tau+\omega)^{p-1}\tau \\ \le& [2\frac{\beta\rho}{\Gamma(p)l} - \sigma p]\tau(\tau+\omega)^{p-1} + [2\frac{\beta\rho}{\Gamma(p)l}\frac{\omega}{\tau+\omega} - \frac{\beta\rho}{\Gamma(p)l}](\tau+\omega)^p\end{aligned}$$

since $l = \frac{2\beta\rho}{\Gamma(p)\sigma}$, then

$$\Pi + p\phi(\tau)(\tau+\omega)^{p-1} \le -(p-1)\sigma(\tau+\omega)^p + [\frac{p(\sigma\omega+\phi(\tau))}{\sigma(\tau+\omega)} - \frac{1}{2}]\sigma(\tau+\omega)^p$$

using (1.5) and Lemma 3.3, we can establish Proposition 3.1. □

Proof of Proposition 3.1. Let

$$h(u) = -\frac{1}{\Gamma(p)}\ln(\Gamma(p)[l+\rho(K-u)])$$

so that

$$G_N(t) = N\int_\Omega u dx + L(t)$$

where

$$L(t) = \int_\Omega e^{h(u)}(v+\omega)^p dx.$$

Differentiating L with respect to t and using Greens formula, we obtain that

$$L'(t) = I + J$$

where

$$\begin{aligned} I = &- \int_\Omega [a(h''(u) + h'^2(u))(v+\omega)^p + pch'(u)(v+\omega)^{p-1}]e^{h(u)}\nabla u^2 dx \\ &- \int_\Omega [p(a+d)h'(u)(v+\omega)^{p-1} + p(p-1)c(v+\omega)^{p-2}]e^{h(u)}\nabla u \nabla v dx \\ &- \int_\Omega p(p-1)d(v+\omega)^{p-2}e^{h(u)}\nabla v^2 dx \end{aligned}$$

and

$$\begin{aligned} J = &\int_\Omega \beta h'(u)(v+\omega)^p e^{h(u)} dx \\ &+ \int_\Omega [pg(u,v)(v+\omega)^{p-1} - h'(u)f(u,v)(v+\omega)^p]e^{h(u)} dx \\ &- \int_\Omega \alpha h'(u)u(v+\omega)^p e^{h(u)} dx - \int_\Omega \sigma p(v+\omega)^{p-1} v e^{h(u)} dx. \end{aligned}$$

We see that I involves a quadratic form with respect to $\nabla u, \nabla v$.

$$\begin{aligned} D = &[a(h''(u) + h'^2(u))(v+\omega)^p + pch'(u)(v+\omega)^{p-1}]\nabla u^2 \\ &+ [p(a+d)h'(u)(v+\omega)^{p-1} + p(p-1)c(v+\omega)^{p-2}]\nabla u \nabla v \\ &+ p(p-1)d(v+\omega)^{p-2}\nabla v^2 \end{aligned}$$

which is nonnegative if

$$\begin{aligned} \delta = &[p(a+d)h'(u)(v+\omega)^{p-1} + p(p-1)(v+\omega)^{p-2}]^2 \\ &- 4p(p-1)d(v+\omega)^{p-2}[a(h''(u) + h'^2(u))(v+\omega)^p \\ &+ pch'(u)(v+\omega)^{p-1}] \le 0. \end{aligned}$$

Indeed,

$$\begin{aligned} \delta = &p^2(a+d)^2h'^2(u)(v+\omega)^{2p-2} + 2(a+d)cp^2(p-1)h'(u))(v+\omega)^{2p-3} \\ &+ c^2p^2(p-1)^2(v+\omega)^{2p-4} - 4p(p-1)ad(h''(u) + h'^2(u))(v+\omega)^{2p-2} \\ &- 4cdp^2(p-1)h'(u)(v+\omega)^{2p-3} \end{aligned}$$

and from (3.3) we have that $v + \omega \ge 1$. It follows that

$$\begin{aligned} \delta \le &[p^2(a+d)^2h'^2(u) - 4p(p-1)ad(h''(u) + h'^2(u))](v+\omega)^{2p-2} \\ &+ [2(a+d)cp^2(p-1)h'(u) + c^2p^2(p-1)^2 \\ &- 4cdp^2(p-1)h'(u)](v+\omega)^{2p-3}. \end{aligned}$$

Let us try

$$T = p^2(a+d)^2h'^2(u) - 4p(p-1)ad(h''(u) + h'^2(u))$$

the choice of $h(u)$ implies

$$\frac{\rho}{\Gamma(p)[l+\rho K]} \le h'(u) \le \frac{\rho}{\Gamma(p)l}, \tag{3.9}$$

and, consequently,

$$T = -\frac{\rho^2 4ad(a-d)^2p^2}{[\Gamma(p)(l+\rho(K-u))]^2} \le -4adp^2R^2 \le 0.$$

In addition

$$\begin{aligned}\delta \le &[2(a+d)cp^2(p-1)h'(u) + c^2p^2(p-1)^2 - 4cdp^2(p-1)h'(u)](v+\omega)^{2p-3} \\ &+ T(v+\omega)(v+\omega)^{2p-3} \\ \le &[(p-1)c^2 + 2(a-d)c\frac{\rho}{\Gamma(p)l} - \frac{4adR^2\omega}{(p-1)}]p^2(p-1)(v+\omega)^{2p-3} + Tv(v+\omega)^{2p-3}.\end{aligned}$$

If we replace w by its value (3.3) and use the parabolic condition $c^2 - 4ad < 0$, we deduce that

$$\begin{aligned}\delta \le &[p(c^2-4ad) - (c-S)^2 - 4adR^2\mu]p^2(p-1)(v+\omega)^{2p-3} \\ &+ Tv(v+\omega)^{2p-3} \le 0,\end{aligned}$$

that is, $I \le 0$.

We can control the second term by observing that

$$\begin{aligned}J \le &\int_\Omega [\beta h'(u)(v+\omega)^p - \sigma p(v+\omega)^{p-1}v + p\phi(v)(v+\omega)^{p-1}]e^{h(u)}dx \\ &+ \int_\Omega [p(g(u,v)-\phi(v))(v+\omega)^{p-1} - h'(u)f(u,v)(v+\omega)^p]e^{h(u)}dx.\end{aligned}$$

Using (3.9),

$$\begin{aligned}J \le &\int_\Omega [\frac{\beta\rho}{\Gamma(p)l}(v+\omega)^p - \sigma p(v+\omega)^{p-1}v + p\phi(v)(v+\omega)^{p-1}]e^{h(u)}dx \\ &+ \int_\Omega [p(g(u,v)-\phi(v))(v+\omega)^{p-1} - \frac{\rho}{\Gamma(p)[l+\rho k]}f(u,v)(v+\omega)^p]e^{h(u)}dx.\end{aligned}$$

Applying Lemmas 3.3 and 3.4, one finds that

$$J \le -(p-1)\sigma\int_\Omega (v+\omega)^p e^{h(u)}dx + M_1\int_\Omega e^{h(u)}dx + N_1\int_\Omega f(u,v)e^{h(u)}dx.$$

In addition we see that

$$h(u) \le -\frac{1}{\Gamma(p)}\ln\frac{2\beta\rho}{\sigma},$$

and, consequently,

$$J \le -(p-1)\sigma L + M_1|\Omega|e^{-\frac{1}{\Gamma(p)}\ln\frac{2\beta\rho}{\sigma}} + N_1 e^{-\frac{1}{\Gamma(p)}\ln\frac{2\beta\rho}{\sigma}}\int_\Omega f(u,v)dx.$$

Letting

$$M = M_1|\Omega|e^{-\frac{1}{\Gamma(p)}\ln\frac{2\beta\rho}{\sigma}}, \quad N = N_1 e^{-\frac{1}{\Gamma(p)}\ln\frac{2\beta\rho}{\sigma}}$$

and using Lemma 3.2 we conclude that

$$\begin{aligned}J \le &-(p-1)\sigma L + M + N[\beta|\Omega| - \frac{d}{dt}\int_\Omega u(t,x)dx] \\ \le &-(p-1)\sigma G_N + (p-1)\sigma N\int_\Omega udx + M + N\beta|\Omega| - N\frac{d}{dt}\int_\Omega u(t,x)dx \\ \le &-(p-1)\sigma G_N + |\Omega|N[(p-1)K\sigma + \beta] + M - N\frac{d}{dt}\int_\Omega u(t,x)dx.\end{aligned}$$

It follows that
$$\frac{dG_N}{dt} \le -(p-1)\sigma G_N + s$$
where $s = |\Omega| N[(p-1)K\sigma + \beta] + M$. □

We can now establish the main result of this manuscript.

Theorem 3.5. *Under the assumptions* (H1)-(H3) *and* (1.5)*, the solutions of* (1.1)–(1.4) *are global and uniformly bounded in* $[0,+\infty[$.

Proof. Multiplying (3.5) by $e^{(p-1)\sigma t}$ and integrating, implies the existence of a positive constant $C > 0$ independent of t such that
$$G_N(t) \le C.$$
Since
$$e^{h(u)} \ge e^{-\frac{1}{\Gamma(p)}\ln(\Gamma(p)[l+\rho k])},$$
for all $p \ge 2$, we have
$$\int_\Omega (v+\omega)^p dx \quad \le e^{\frac{1}{\Gamma(p)}\ln(\Gamma(p)[l+\rho k])} G_N(t) \le C e^{\frac{1}{\Gamma(p)}\ln \Gamma(p)[l+\rho k]} = C(p).$$
Consequently,
$$\int_\Omega (v+\mu)^p dx \le C(p), \quad \int_\Omega v^p dx \le C(p).$$
Now we chose $p > n/2$ and we search for a bound for $\|\Upsilon(u,v)\|_p$. We put
$$A_1 = \max_{0\le\tau\le\tau_0} \psi(\tau), \quad A_2 = \max_{0\le\xi\le K} \varphi(\xi), \quad A_3 = \max_{0\le\tau\le\tau_0} \phi(\tau)$$
where $\tau_0 = max(\tau_1,\tau_2)$ such that
$$\tau \ge \tau_1 \quad \Rightarrow \quad \psi(\tau) \le \tau, \quad \tau \ge \tau_2 \quad \Rightarrow \quad \phi(\tau) \le \tau.$$
Using (H1)–(H3) implies
$$g(u,v) \le \psi(v) f(u,v) + \phi(v) \le \psi(v)\varphi(u)(\mu+v)^r + \phi(v) \le A_2\psi(v)(\mu+v)^r + \phi(v).$$
Since $0 \le u \le K$, we have
$$\int_\Omega g(u,v)^p dx \le \int_\Omega [A_2\psi(v)(\mu+v)^r + \phi(v)]^p dx.$$
Now we use the inequality
$$(x+y)^q \le 2^{q-1}(x^q + y^q)$$
all $x, y \ge 0$ and $q \ge 1$, to obtain the following sequence of estimates
$$\begin{aligned}
&\int_\Omega g(u,v)^p dx \\
&\le \int_\Omega 2^{p-1}[A_2^p \psi(v)^p (\mu+v)^{rp} + \phi(v)^p] dx \\
&\le 2^{p-1}\Big[A_2^p\Big(\int_{v\le\tau_0} A_1^p(\mu+\tau_0)^{rp} dx + \int_{v\ge\tau_0} v^p(\mu+v)^{rp} dx\Big) + |\Omega| A_3^p + \int_{v\ge\tau_0} v^p dx\Big] \\
&\le 2^{p-1}\Big[\Big(A_2A_1(\mu+\tau_0)^r\Big)^p |\Omega| + A_2^p \int_{v\ge\tau_0} (\mu+v)^{(r+1)p} dx + |\Omega| A_3^p + \int_{v\ge\tau_0} v^p dx\Big] \\
&\le 2^{p-1}[(A_2A_1(\mu+\tau_0)^r)^p |\Omega| + A_2^p C((r+1)p) + |\Omega| A_3^p + C(p)] = E_g^p
\end{aligned}$$

and

$$\int_\Omega f(u,v)^p dx \leq A_2^p C(rp) = E_f^p.$$

We conclude that

$$\begin{aligned} \|\Upsilon(u,v)\|_p &\leq \|g(u,v)\|_p + \frac{c}{a-d}[\|f(u,v)\|_p + \alpha\|u\|_p + \beta|\Omega|] + \sigma\|v\|_p \\ &\leq E_g + \frac{c}{a-d}[E_f + (\alpha K + \beta)|\Omega|] + \sigma\sqrt[p]{C(p)}. \end{aligned}$$

We conclude that the unique solution of (1.1)–(1.4) is globally and uniformly bounded in $[0,+\infty[\times\Omega$. □

Acknowledgments. The author wants to thank Professors A. Youkana and M. Mechab for their kind advice, and the anonymous referee for his/her comments that improved the final version of this article.

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