Electronic Journal of Differential Equations, Vol. 2008(2008), No. 102, pp. 1–6.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu (login: ftp)

EXISTENCE OF POSITIVE SOLUTIONS FOR A SINGULAR p-LAPLACIAN DIRICHLET PROBLEM

WENSHU ZHOU

Abstract. By a argument based on regularization technique, upper and lower solutions method and Arzelá-Ascoli theorem, this paper shows sufficient conditions of the existence of positive solutions of a Dirichlet problem for singular p-Laplacian.

1. Introduction

This paper shows the existence of positive solutions for the singular p-Laplacian equation

$$\big(\phi_p(u')\big)' - \lambda\frac{|u'|^p}{u^m} + f(t,u') = 0, \quad 0 < t < 1, \tag{1.1}$$

subject to Dirichlet boundary conditions

$$u(1) = u(0) = 0, \tag{1.2}$$

where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, λ and m are positive constants, and f is a continuous function. We call $u \in C^1[0,1]$ is a solution if $u > 0$ in $(0,1)$, $|u'|^{p-2}u' \in C^1(0,1)$, and it satisfies (1.1)–(1.2).

Such equation arises in the studies of some degenerate parabolic equations and in Non-Newtonian fluids; see [2, 3, 4, 5, 13]. The interesting feature of (1.1) is the lower term both is singular at $u = 0$ and depends on the first derivative.

Recently, the one-dimensional singular p-Laplacian differential equations without dependence on the first derivative have been studied extensively, see [1, 7, 12] and references therein. When it depends on the first derivative, however, it has not received much attention, see [8, 9, 10, 11]. Recently, the authors [14], considered the equation

$$\big(\phi_p(u')\big)' - \lambda\frac{|u'|^p}{u} + g(t) = 0, \quad 0 < t < 1,$$

subject to (1.2), and proved, by the classical method of elliptic regularization, that the problem admits one positive solution if $p \ge 2, \lambda > 0$ and $g \in C[0,1]$ with $g > 0$ on $[0,1]$. In the present paper we extend the result and obtain the sufficient conditions of existence. Our argument is based on regularization technique, upper

---

2000 *Mathematics Subject Classification.* 34B18.
Key words and phrases. p-Laplacian; singularity; positive solution; regularization technique.

Submitted December 17, 2007. Published July 30, 2008.
Supported by grants 20076209 from Dalian Nationalities University, and 10626056 from Tianyuan Fund.

and lower solutions method and Arzelá-Ascoli theorem. In addition, an example is also given to illustrate our main result.

2. Main result

The following hypotheses will be adopted in this section:

- (H1) $1 \le m < p$.
- (H2) $f(t,r)$ is a positive, continuous function in $[0,+\infty) \times \mathbb{R}$, and there exist constants $\alpha > 0$, $\beta \in [0,1)$ such that $f(t,r) \le \alpha + \beta |r|^{p-1}$, for all $(t,r) \in [0,1] \times \mathbb{R}$.
- (H3) $\lambda > \inf_{r \ge 1} H(r)$, where $H(r) : \mathbb{R}^+ \to \mathbb{R}^+$ is defined by
$$H(r) = \alpha r^{m-p} + \beta r^{m-1}.$$

Remark 2.1. Let $m \in (1,p)$ and define
$$X_0 = \Big(\frac{\alpha(p-m)}{\beta(m-1)}\Big)^{1/(p-1)}; \quad X_* = \begin{cases} X_0, & X_0 \ge 1 \\ 1. & X_0 < 1 \end{cases}$$
Then $\inf_{s \ge 1} H(s) = H(X_*)$. Indeed, since $\lim_{s\to 0^+} H(s) = \lim_{s\to+\infty} H(s) = +\infty$, $H(s)$ must reach a minimum at some $s \in (0,\infty)$ satisfying $H'(s) = 0$. Solving it gives $s = X_0$ and hence, $\inf_{s>0} H(s) = H(X_0)$. Since $H'(s) \ge 0$ for all $s \ge X_0$, we see that $\inf_{s\ge 1} H(s) = H(X_0)$ if $X_0 \ge 1$, and $\inf_{s \ge 1} H(s) = H(1)$ if $X_0 < 1$.

The main result of this paper is stated as follows.

Theorem 2.2. *Under Assumptions* (H1)–(H3)*, problem* (1.1)–(1.2) *has at least one solution.*

Remark 2.3. If $m = 1$ and $f \equiv 1$ (taking $\alpha = 1, \beta = 0$), then $\inf_{s\ge 1} H(s) = 0$. Clearly, Theorem 2.2 is an extension of the corresponding result of [14].

Proof of Theorem 2.2. Let $\epsilon \in (0,1)$, and define $H_\epsilon(t,v,\xi) : (0,1) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by
$$H_\epsilon(t,v,\xi) = \lambda \frac{|\xi|^p}{[I_\epsilon(v)]^m} - f(t,\xi),$$
where $I_\epsilon(v) = v + \epsilon$ if $v \ge 0$, $I_\epsilon(v) = \epsilon$ if $v < 0$. By (H2) and using the inequality: $a^{p-1} \le a^p + 1$, for all $a \ge 0$, we have
$$|H_\epsilon(t,v,\xi)| \le \frac{\lambda}{\epsilon^m}|\xi|^p + \alpha + \beta|\xi|^{p-1} \le \big(\frac{\lambda}{\epsilon^m} + \alpha + \beta\big)\mathcal{H}(|\xi|)$$
for all $(t,v,\xi) \in (0,1) \times \mathbb{R} \times \mathbb{R}$, where $\mathcal{H}(s) = 1 + s^p$ for $s \ge 0$. Denote $\mathfrak{M} = \{u \in C^1(0,1); |u'|^{p-2}u' \in C^1(0,1)\}$, and define $\mathscr{L}_\epsilon : \mathfrak{M} \to C(0,1)$ by
$$(\mathscr{L}_\epsilon u)(t) = -\big(\phi_p(u')\big)' + H_\epsilon(t,u,u'), \quad 0 < t < 1.$$
Consider the problem:
$$\begin{gathered} (\mathscr{L}_\epsilon u)(t) = 0, \quad 0 < t < 1, \\ u(1) = u(0) = 0. \end{gathered} \tag{2.1}$$
We call $u \in \mathfrak{M}$ is an upper solution (lower solution) of problem (2.1) if $\mathscr{L}_\epsilon u \ge 0$ (≤ 0) in $(0,1)$ and $u(t) \ge (\le) 0$ for $t = 0,1$.

We will apply the upper and lower solutions method (see [8, Theorem 1 and Remark 2.4]) to show the existence of solutions of problem (2.1). Obviously,

$\int_0^{+\infty} \frac{s^{p-1}}{\mathcal{H}(s)} ds = +\infty$, thus the condition [8, Eq. (2.3)] is satisfied. Then it suffices to find a lower solution and an upper solution to obtain a solution.

Let $\inf_{s\geq 1} H(s) \equiv \delta$. Then it follows from the definition of infimum and $\lambda > \delta$ that for $\delta_0 = \frac{\lambda-\delta}{2} > 0$, there exists $S^* \geq 1$ such that $H(S^*) < \delta + \delta_0 < \lambda$.

Lemma 2.4. *There exists a constant $\epsilon_0 \in (0,1)$, such that for any $\epsilon \in (0,\epsilon_0)$, $U_\epsilon = S^*(t+\epsilon)$ is an upper solution of* (2.1).

Proof. Noticing $U_\epsilon \geq \epsilon$ in $(0,1)$ and $m \geq 1$, we have

$$\begin{aligned}
\mathscr{L}_\epsilon U_\epsilon &= -\big(|U_\epsilon'|^{p-2}U_\epsilon'\big)' + \lambda\frac{|U_\epsilon'|^p}{(U_\epsilon+\epsilon)^m} - f(t,U_\epsilon') \\
&= \frac{\lambda S^{*p-m}}{(t+\epsilon+\epsilon/S^*)^m} - f(t,S^*) \\
&\geq \frac{\lambda S^{*p-m}}{(1+\epsilon+\epsilon/S^*)^m} - \alpha - \beta S^{*p-1} \\
&= S^{*p-m}[\lambda - H(S^*)] + r_\epsilon, \quad 0<t<1,
\end{aligned}$$

where $r_\epsilon = \lambda S^{*p-m}[(1+\epsilon+\epsilon/S^*)^{-m} - 1]$. Clearly, $r_\epsilon \to 0$ $(\epsilon \to 0)$. Since $\lambda > H(S^*)$, there exists a constant $\epsilon_0 \in (0,1)$, such that for any $\epsilon \in (0,\epsilon_0)$ there holds $S^{*p-m}[\lambda - H(S^*)] + r_\epsilon \geq 0$. So that we obtain $\mathscr{L}_\epsilon U_\epsilon \geq 0$ in $(0,1)$ for all $\epsilon \in (0,\epsilon_0)$. The lemma follows. □

Lemma 2.5. *Let $W = C\Phi^\alpha$, where $\alpha = \frac{p}{p-m}$, $\Phi(t)$ is defined by*

$$\Phi(t) = \frac{p-1}{p}\Big[\big(\frac{1}{2}\big)^{p/(p-1)} - \Big|\frac{1}{2}-t\Big|^{p/(p-1)}\Big], \quad 0\leq t\leq 1,$$

and $C \in (0,1)$ such that $C\alpha < 1$ and $(C\alpha)^{p-1} + \lambda C^{p-m}\alpha^p \leq \min_{[0,1]\times[-1,1]} f(s,r)$. Then W is a lower solution of problem (2.1).

Proof. It is easy to check that Φ has the following properties:

(a) $\Phi > 0$ in $(0,1)$, $\Phi \in C^1[0,1]$.
(b) $(|\Phi'|^{p-2}\Phi')' = -1$ in $(0,1)$, $\Phi(1) = \Phi(0) = 0$.
(c) $\Phi(t) \leq t$, $|\Phi'(t)| \leq 1$, for all $t \in [0,1]$.

Using these properties of Φ, by some calculations, we have

$$\begin{aligned}
\mathscr{L}_\epsilon W &= -\big(|W'|^{p-2}W\big)' + \lambda\frac{|W'|^p}{(W+\epsilon)^m} - f(t,W') \\
&\leq -\big(|W'|^{p-2}W\big)' + \lambda\frac{|W'|^p}{W^m} - f(t,W') \\
&= -(C\alpha)^{p-1}\Phi^{(\alpha-1)(p-1)}\big(|\Phi'|^{p-2}\Phi'\big)' \\
&\quad - (C\alpha)^{p-1}(\alpha-1)(p-1)\Phi^{(\alpha-1)(p-1)-1}|\Phi'|^p \\
&\quad + \lambda C^{p-m}\alpha^p|\Phi'|^p - f(t, C\alpha\Phi^{\alpha-1}\Phi') \\
&\leq (C\alpha)^{p-1}\Phi^{(\alpha-1)(p-1)} + \lambda C^{p-m}\alpha^p|\Phi'|^p - \min_{[0,1]\times[-1,1]} f(s,r) \\
&\leq (C\alpha)^{p-1} + \lambda C^{p-m}\alpha^p - \min_{[0,1]\times[-1,1]} f(s,r) \leq 0, \quad 0<t<1.
\end{aligned}$$

Thus the lemma follows. □

According to [8, Theorem 1 and Remark 2.4], for fixed $\epsilon \in (0, \epsilon_0)$ problem (2.1) has a solution $u_\epsilon \in C^1[0,1]$ satisfying $|u_\epsilon'|^{p-2}u_\epsilon' \in C^1(0,1)$ and

$$U_\epsilon \geq u_\epsilon \geq W > 0, \quad t \in (0,1). \tag{2.2}$$

Hence u_ϵ satisfies

$$-\big(|u_\epsilon'|^{p-2}u_\epsilon'\big)' + \lambda\frac{|u_\epsilon'|^p}{(u_\epsilon+\epsilon)^m} - f(t,u_\epsilon') = 0, \quad t \in (0,1). \tag{2.3}$$

Lemma 2.6. *For all $\epsilon \in (0,\epsilon_0)$, we have*

$$|u_\epsilon'(t)| \leq [\alpha(1-\beta)^{-1}]^{1/(p-1)}, \quad \forall t \in [0,1]. \tag{2.4}$$

Proof. Noticing that $u_\epsilon(1) = u_\epsilon(0) = 0$ and $u_\epsilon \geq 0$ on $[0,1]$, we have

$$u_\epsilon'(0) \geq 0 \geq u_\epsilon'(1). \tag{2.5}$$

From (2.3), we obtain

$$\big(|u_\epsilon'|^{p-2}u_\epsilon'\big)' + \alpha + \beta|u_\epsilon'|^{p-1} \geq 0, \quad t \in (0,1). \tag{2.6}$$

Let $\chi = \phi_p(u_\epsilon')$. Then we obtain from (2.6), $\chi' + \alpha + \beta|\chi| \geq 0$, $t \in (0,1)$; i.e., $\big(\int_0^{\chi(t)} \frac{1}{\alpha+\beta|s|}ds + t\big)' \geq 0$, $t \in (0,1)$. This and (2.5) give $1 \geq \int_0^{\chi(t)} \frac{1}{\alpha+\beta|s|}ds + t \geq 0$, $t \in [0,1]$, hence $\big|\int_0^{\chi(t)} \frac{1}{\alpha+\beta|s|}ds\big| \leq 1$, $t \in [0,1]$. Using the inequality: $|\int_0^y \frac{1}{\alpha+\beta|s|}ds| \geq \frac{|y|}{\alpha+\beta|y|}$ $(y \in \mathbb{R})$, we deduce that $|\chi| \leq \alpha + \beta|\chi|$, $t \in [0,1]$; that is, $|\chi| \leq \alpha(1-\beta)^{-1}$ on $[0,1]$. The lemma is proved. □

Lemma 2.7. *For each $\delta \in (0,1/2)$, there exists a positive constant C_δ independent of ϵ, such that for all $\epsilon \in (0,\epsilon_0)$*

$$|u_\epsilon'(t_2) - u_\epsilon'(t_1)| \leq C_\delta|t_2-t_1|^\gamma, \quad \forall t_2,t_1 \in [\delta,1-\delta], \tag{2.7}$$

where $\gamma = 1/(p-1)$ if $p \geq 2$; $\gamma = 1$ if $1 < p < 2$.

Proof. By (2.2) and (2.4), it is easy to derive from (2.3) that for any $\delta \in (0,1/2)$ there exists a constant $C_\delta > 0$ independent of ϵ, such that for all $\epsilon \in (0,\epsilon_0)$

$$\big|(|u_\epsilon'|^{p-2}u_\epsilon')'\big| \leq C_\delta, \quad \delta \leq t \leq 1-\delta. \tag{2.8}$$

Recalling the inequality (see [6])

$$(|\eta|^{p-2}\eta - |\eta'|^{p-2}\eta')\cdot(\eta-\eta') \geq \begin{cases} C_1|\eta-\eta'|^p, & p \geq 2 \\ C_2(|\eta|+|\eta'|)^{p-2}|\eta-\eta'|^2, & 1<p<2 \end{cases}$$

for each $\eta \in \mathbb{R}$, where $C_i (i=1,2)$ are positive constants depending only on p, we derive, by (2.8), that if $p \geq 2$, then

$$\begin{aligned} |u_\epsilon'(t_2) - u_\epsilon'(t_1)|^p &\leq C_2^{-1}[u_\epsilon'(t_2) - u_\epsilon'(t_1)]\cdot[|u_\epsilon'(t_2)|^{p-2}u_\epsilon'(t_2) - |u_\epsilon'(t_1)|^{p-2}u_\epsilon'(t_1)] \\ &\leq C_\delta|u_\epsilon'(t_2) - u_\epsilon'(t_1)||t_2-t_1|, \quad \forall t_2,t_1 \in [\delta,1-\delta], \end{aligned}$$

hence

$$|u_\epsilon'(t_2) - u_\epsilon'(t_1)| \leq C_\delta|t_2-t_1|^{1/(p-1)}, \quad \forall t_2,t_1 \in [\delta,1-\delta],$$

and if $p \in (1,2)$, then

$$\begin{aligned} &|u_\epsilon'(t_2) - u_\epsilon'(t_1)|^2[|u_\epsilon'(t_2)| + |u_\epsilon'(t_1)|]^{p-2} \\ &\leq C_2^{-1}[u_\epsilon'(t_2) - u_\epsilon'(t_1)]\cdot[|u_\epsilon'(t_2)|^{p-2}u_\epsilon'(t_2) - |u_\epsilon'(t_1)|^{p-2}u_\epsilon'(t_1)] \\ &\leq C_\delta|u_\epsilon'(t_2) - u_\epsilon'(t_1)||t_2-t_1|, \quad \forall t_2,t_1 \in [\delta,1-\delta]. \end{aligned}$$

Then, (2.4) yields

$$|u_\epsilon'(t_2) - u_\epsilon'(t_1)| \le C_\delta |t_2 - t_1| [|u_\epsilon'(t_2)| + |u_\epsilon'(t_1)|]^{2-p} \le C_\delta |t_2 - t_1|$$

for all $t_2, t_1 \in [\delta, 1-\delta]$. This completes the proof. □

By (2.4) and (2.7) and using Arzelá-Ascoli theorem, there exist a subsequence of $\{u_\epsilon\}$, still denoted by $\{u_\epsilon\}$, and a function $u \in C^1(0,1) \cap C[0,1]$ such that, as $\epsilon \to 0$,

$$\begin{gathered} u_\epsilon \to u, \quad \text{uniformly in } C[0,1], \\ u_\epsilon \to u, \quad \text{uniformly in } C^1[\delta, 1-\delta], \end{gathered} \tag{2.9}$$

where $\delta \in (0, 1/2)$, and hence from $u_\epsilon(1) = u_\epsilon(0) = \epsilon$ and (2.2) we derive that $u(1) = u(0) = 0$, and $u(t) \ge C\Phi^{p/(p-m)}$, $t \in [0,1]$; therefore $u > 0$ in $(0,1)$.

We now show that u satisfies (1.1). Integrating (2.3) over (t_0, t) gives

$$|u_\epsilon'(t)|^{p-2} u_\epsilon'(t) = \int_{t_0}^t \Big(\lambda \frac{|u_\epsilon'|^p}{(u_\epsilon + \epsilon)^m} - f(s, u_\epsilon')\Big) ds + |u_\epsilon'(t_0)|^{p-2} u_\epsilon'(t_0),$$

and letting $\epsilon \to 0$ and using Lebesgue's dominated convergence theorem yield

$$|u'(t)|^{p-2} u'(t) = \int_{t_0}^t \Big(\lambda \frac{|u'|^p}{u^m} - f(s, u')\Big) ds + |u'(t_0)|^{p-2} u'(t_0). \tag{2.10}$$

This shows that $|u'(t)|^{p-2} u'(t) \in C^1(0,1)$ and (1.1) is satisfied.

It remains to show that $u \in C^1[0,1]$. Integrating (2.3) over $(0,1)$ and using (2.4) and (2.5), we derive that

$$\int_0^1 \frac{|u_\epsilon'|^p}{(u_\epsilon + \epsilon)^m} dt \le \frac{1}{\lambda} \min_{[0,1] \times [-Y,Y]} f(t,r), \quad Y := \big(\frac{\alpha}{1-\beta}\big)^{1/(p-1)},$$

and letting $\epsilon \to 0$ and using Fatou's lemma and (2.9), we obtain

$$\int_0^1 \frac{|u'|^p}{u^m} dt \le \frac{1}{\lambda} \min_{[0,1] \times [-Y,Y]} f(t,r).$$

So, $\frac{|u'|^p}{u^m} \in L^1[0,1]$. By (2.10), the function $\omega(t) = |u'(t)|^{p-2} u'(t) = \phi_p(u'(t))$ is absolutely continuous on $[0,1]$. Since $u'(t) = \phi_q(\omega(t)) (\frac{1}{p} + \frac{1}{q} = 1)$, $u' \in C[0,1]$. The proof of Theorem 2.2 is complete.

Example. Let $\lambda > 4/27$. Consider the problem

$$\begin{gathered} (|u'|^3 u')' - \lambda \frac{|u'|^5}{u^2} + \frac{(\frac{1}{2} + \frac{\sqrt{6}}{18} |u'|^2)^2}{2t + 2\cos t - 1} + \frac{\sin(\pi t)}{\sqrt{4 + |u'|^3}} = 0, \quad 0 < t < 1, \\ u(1) = u(0) = 0. \end{gathered} \tag{2.11}$$

Let $p = 5$, $m = 2$,

$$f(t,r) = \frac{(\frac{1}{2} + \frac{\sqrt{6}}{18} r^2)^2}{2t + 2\cos t - 1} + \frac{\sin(\pi t)}{\sqrt{4 + |r|^3}}.$$

Since $(2t + 2\cos t - 1)' = 2(1 - \sin t) \ge 0$, $1 + 2\cos 1 \ge 2t + 2\cos t - 1 \ge 1$ for all $t \in [0,1]$ and hence, noticing $0 \le \frac{\sin(\pi t)}{\sqrt{4+|r|^3}} \le \frac{1}{2}$ for $(t,r) \in [0,1] \times \mathbb{R}$, we obtain

$$\frac{1}{1 + 2\cos 1} \big(\frac{1}{2} + \frac{\sqrt{6}}{18} r^2\big)^2 \le f(t,r) \le \big(\frac{1}{2} + \frac{\sqrt{6}}{18} r^2\big)^2 + \frac{1}{2},$$

for $(t,r) \in [0,1] \times \mathbb{R}$. By the inequality $(a+b)^2 \le 2(a^2+b^2)$, we have

$$f(t,r) \le 1 + \frac{1}{27} r^4, \quad (t,r) \in [0,1] \times \mathbb{R}.$$

Let $\alpha = 1$, $\beta = \frac{1}{27}$. Then $X_0 = 3$, and therefore $X_* = 3$ and $\inf_{s \ge 1} H(s) = H(X_*) = H(X_0) = \frac{4}{27}$ (see Remark 2.1). Thus all assumptions of Theorem 2.2 are satisfied for any $\lambda > \frac{4}{27}$, so problem (2.11) has at least one solution.

Acknowledgments. The author wants to thank the anonymous referee for pointing out some errors on the original manuscript.

References

[1] R. P. Agarwal, H. Lü, D. O'Regan; Existence theorems for the one-dimensional singular p-Laplacian equation with sign changing nonlinearities, Appl. Math. Comput. 1(143)(2003), 15-38.

[2] C. Astarita, G. Marrucci; Principles of Non-Newtonian Fluid Mechanics, McGraw-Hill, 1974.

[3] G. I. Barenblatt, M. Bertsch, A. E. Chertock, V. M. Prostokishin; Self-similar intermediate asymptotic for a degenerate parabolic filtration-absorption equation, Proc. Nat. Acad. Sci. (USA), 18(2000), 9844-9848.

[4] M. Bertsch, R. Dal Passo, M. Ughi; Discontinuous viscosity solutions of a degenerate parabolic equation, Trans. Amer. Math. Soc. 320(1990), 779-798.

[5] M. Bertsch, M. Ughi; Positivity properties of viscosity solutions of a degenerate parabolic equation, Nonlinear Anal. TMA. 14(1990) 571-592.

[6] L. Damascelli; Comparison theorems for some quasilinear degenerate elliptic operations and applications to symmetry and monotonicity results, Ann. Ist. henri Poincaré, Analyse Nonlinèaire, 4(15)1998, 493-516.

[7] D. Jiang; Upper and lower solutions method and a singular boundary value problem, Zeitschrift fur Angewandte Mathematik und Mechanik, 7(82)(2002), 481 - 490.

[8] D. Jiang, W. Gao; Singular boundary value problems for the one-dimension p-Laplacian, J. Math. Anal. Appl. 270(2002), 561-581.

[9] H. Lu, D. O'Regan, R. P. Agarwal; Nonuniform nonresonance at the first eigenvalue of the one-dimensional singular p-Laplacian, Mem. Differential Equations Math. Phys. 34(2005), 97-114.

[10] D. O'Regan; Some general existence principles and results for $(\phi(y'))' = qf(t,y,y'), 0 < t < 1$, SIAM, J. Math. Anal. 24(1993), No.3, 648-668.

[11] Z. Wang, J. Zhang; Positive solutions for one-dimensional p-Laplacian boundary value problems with dependence on the first derivative, J. Math. Anal. Appl. 314(2006), 618-630.

[12] Q. Yao; Existence, multiplicity and infinite solvability of positive solutions for one-dimensional p-Laplacian, Acta Mathematica Sinica, 4(21)(2005), 691-698

[13] Z. Yao, W. Zhou; Nonuniqueness of solutions for a singular diffusion problem, J. Math. Anal. Appl. 325(2007), 183-204.

[14] Z. Yao, W. Zhou; Existence of positive solutions for the one-dimensional singular p-Laplacian, Nonlinear Anal. TMA. 68(2008), 2309-2318.

Wenshu Zhou
Department of Mathematics, Dalian Nationalities University, 116600, China
E-mail address: pdezhou@126.com, wolfzws@163.com