Oscillation of Second Order Nonlinear Impulsive Dynamic Equations on Time Scales *

Mugen Huang†, Weizhen Feng‡
School of Mathematical Sciences, South China Normal University,
Guangzhou 510631, P. R. China

Abstract: In this paper, oscillation of second order nonlinear impulsive dynamic equations on time scales is investigated by Riccati transformation techniques, some sufficient conditions of oscillation for all solutions are obtained. An example is given to show that the impulses play a dominant part in oscillations of dynamic equations on time scales.
Keywords: Oscillation; Impulsive dynamic equations; Time scales.

1. Introduction

This paper is concerned with the oscillations of second order nonlinear impulsive dynamic equations on time scales. Consider the following problem

$$
\begin{aligned}
&y^{\Delta\Delta}(t) + f(t, y^{\sigma}(t)) = 0, \quad t \in \mathbf{J_T} := [0, \infty) \cap \mathbf{T}, t \neq t_k, k = 1, 2, \cdots, \\
&y(t_k^+) = g_k(y(t_k)), y^{\Delta}(t_k^+) = h_k(y^{\Delta}(t_k)), \quad k = 1, 2, \cdots, \\
&y(t_0^+) = y_0, y^{\Delta}(t_0^+) = y_0^{\Delta},
\end{aligned}
\tag{1.1}
$$

where $\mathbf{T}$ is a unbounded-above time scale with $0 \in \mathbf{T}$, $t_k \in \mathbf{T}, 0 \leq t_0 < t_1 < t_2 < \cdots < t_k < \cdots, \lim_{k\to\infty} t_k = \infty$.

$$
y(t_k^+) = \lim_{h\to 0^+} y(t_k + h), \quad y^{\Delta}(t_k^+) = \lim_{h\to 0^+} y^{\Delta}(t_k + h), \tag{1.2}
$$

which represent right and left limits of $y(t)$ at $t = t_k$ in the sense of time scales, and in addition, if t_k is right scattered, then $y(t_k^+) = y(t_k), y^{\Delta}(t_k^+) = y^{\Delta}(t_k)$. We can defined $y(t_k^-), y^{\Delta}(t_k^-)$ similar to (1.2).

We always suppose that the following conditions hold

*Corresponding author.
†E-mail address: huangmugen@sohu.com.
‡E-mail address: wsy@scnu.edu.cn

(H_1) $f \in C_{rd}(\mathbf{T} \times \mathbf{R}, \mathbf{R}), xf(t,x) > 0 \quad (x \neq 0)$ and $\dfrac{f(t,x)}{\varphi(x)} \geq p(t) \quad (x \neq 0)$, where $p(t) \in C_{rd}(\mathbf{T}, \mathbf{R}_+)$ and $x\varphi(x) > 0 \quad (x \neq 0), \varphi^{'}(x) \geq 0$.

(H_2) $g_k, h_k \in C(\mathbf{R}, \mathbf{R})$ and there exist positive constants a_k, a_k^*, b_k, b_k^* such that

$$a_k^* \leq \frac{g_k(x)}{x} \leq a_k, \quad b_k^* \leq \frac{h_k(x)}{x} \leq b_k.$$

Throughout the remainder of the paper, we assume that, for each $k = 1, 2, \cdots$, the points of impulses t_k are right dense (rd for short). In order to define the solutions of the problem (1.1), we introduce the following space

$AC^i = \{y : \mathbf{J_T} \to \mathbf{R}$ is $i-$times $\Delta-$differentiable, whose $i-$th delta-derivative $y^{\Delta^{(i)}}$ is absolutely continuous $\}$.

$PC = \{y : \mathbf{J_T} \to \mathbf{R}$ is rd-continuous expect at the points $t_k, k = 1, 2, \cdots$ for which $y(t_k^-), y(t_k^+), y^{\Delta}(t_k^-)$ and $y^{\Delta}(t_k^+)$ exist with $y(t_k^-) = y(t_k), y^{\Delta}(t_k^-) = y^{\Delta}(t_k)\}$.

Definition 1. A function $y \in PC \bigcap AC^2(\mathbf{J_T} \backslash \{t_1, \cdots\}, \mathbf{R})$ is said to be a solution of (1.1), if it satisfies $y^{\Delta\Delta}(t) + f(t, y^{\sigma}(t)) = 0$ a.e. on $\mathbf{J_T} \backslash \{t_k\}, k = 1, 2, \cdots$, and for each $k = 1, 2, \cdots, y$ satisfies the impulsive condition $y(t_k^+) = g_k(y(t_k)), y^{\Delta}(t_k^+) = h_k(y^{\Delta}(t_k))$ and the initial condition $y(t_0^+) = y_0, y^{\Delta}(t_0^+) = y_0^{\Delta}$.

Definition 2. A solution y of (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise it is called nonoscillatory. Eq.(1.1) is called oscillatory if all solutions are oscillatory.

In recent years, the theory of dynamic equations on time scales which unify differential equations and difference equations, which provides powerful new tools for exploring connections between the traditionally separated fields, has been developing rapidly and has received much attention. We refer the reader to the book by Bohner and Peterson [4] and to the papers cited therein. The time scales calculus has a tremendous potential for applications in mathematical models of real processes, for instance, in biotechnology, chemical technology, economic, neural networks, physics, social sciences and so on, see the monographs of Aulbach and Hilger [2], Bohner and Perterson [4] and the references therein.

Very recently, impulsive dynamic equations on time scales have been investigate by Agarwal et al.[1], Belarbi et al.[5], Benchohra et al.[$6-9$] and so forth. In [9], M.Benchohra et al. considered the existence of extremal solutions for a class of second order impulsive dynamic equations on time scales, we can see that the existence of global solutions can be guaranted by some simple conditions. In [6], M.Benchohra et al. discuss the existence of oscillatory and nonoscillatory solutions for first order impulsive dynamic equations on time scales using lower and upper solutions method.

The oscillations of impulsive differential equations have been investigated by many authors and they gained many classical results. See Y. S. Chen and W. Z. Feng [10] and the papers cited therein. Using the method of Y. S. Chen and W. Z. Feng [10], the present paper is devoted to study the oscillations of a kind of very extensive second order impulsive nonlinear dynamic equations on time scales. An example is given to show that though a dynamic equations on time scales is nonoscillatory, it may become oscillatory if some impulses are added to it. That is, in some cases, impulses play a dominating part in oscillations of dynamic equations on time scales.

In the following, we always assume the solutions of (1.1) exist in $\mathbf{J_T}$. Our attention is restricted to those solution y of Eq.(1.1) which exist on half line $\mathbf{J_T}$ with $\sup\{|y(t)| : t \geq t_0\} \neq 0$ for any $t_0 \geq t_y$, where t_y is dependent on the solution y of (1.1). To the best of our knowledge, the question of the oscillations for second order nonlinear impulsive dynamic equations has not been yet considered. Hence, these results can be considered as a contribution to this field.

2. Main results

In this section, we give some new oscillation criteria for Eq.(1.1). In order to prove our main results, we need the following auxiliary result.

Lemma 1. Suppose that $(H_1) - (H_2)$ hold and $y(t) > 0, t \geq t_0^{'} \geq t_0$ is a nonoscillatory solution of (1.1). If

$$(H_3) \quad (t_1 - t_0) + \frac{b_1^*}{a_1}(t_2 - t_1) + \frac{b_1^* b_2^*}{a_1 a_2}(t_3 - t_2) + \cdots + \frac{b_1^* b_2^* \cdots b_n^*}{a_1 a_2 \cdots a_n}(t_{n+1} - t_n) + \cdots = \infty,$$

then $y^{\Delta}(t_k^+) \geq 0$ and $y^{\Delta}(t) \geq 0$ for $t \in (t_k, t_{k+1}]_{\mathbf{T}}$, where $t_k \geq t_0^{'}$.

Proof. At first, we prove that $y^{\Delta}(t_k) \geq 0$ for $t_k \geq t_0^{'}$, otherwise, there exists some j such that $t_j \geq t_0^{'}$ and $y^{\Delta}(t_j) < 0$, hence

$$y^{\Delta}(t_j^+) = h_j\left(y^{\Delta}(t_j)\right) \leq b_j^* y^{\Delta}(t_j) < 0.$$

Let $y^{\Delta}(t_j^+) = -\alpha \ \ (\alpha > 0)$. From (1.1), for $t \in (t_{j+i-1}, t_{j+i}]_{\mathbf{T}}, i = 1, 2, \cdots$, we obtain

$$y^{\Delta\Delta}(t) = -f(t, y(t)) \leq -p(t)\varphi(y(t)) \leq 0, \tag{2.1}$$

i.e. $y^{\Delta}(t)$ is nonincreasing in $(t_{j+i-1}, t_{j+i}]_{\mathbf{T}}, i = 1, 2, \cdots$, then

$$\begin{aligned} &y^{\Delta}(t_{j+1}) \leq y^{\Delta}(t_j^+) = -\alpha < 0, \\ &y^{\Delta}(t_{j+2}) \leq y^{\Delta}(t_{j+1}^+) = h_{j+1}\left(y^{\Delta}(t_{j+1})\right) \leq b_{j+1}^* y^{\Delta}(t_{j+1}) \leq -b_{j+1}^*\alpha < 0. \end{aligned} \tag{2.2}$$

It is easy to show that for any positive integer $n \geq 2$

$$y^{\Delta}(t_{j+n}) \leq -b_{j+n-1}^* b_{j+n-2}^* \cdots b_{j+1}^* \alpha < 0. \tag{2.3}$$

Now, we claim that for any positive integer $n \geq 2$

$$\begin{aligned} y(t_{j+n}) \quad &\leq a_{j+n-1}a_{j+n-2}\cdots a_{j+1}\Bigg[y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2} - t_{j+1}) \\ &- \cdots - \frac{b_{j+n-1}^* b_{j+n-2}^* \cdots b_{j+1}^*}{a_{j+n-1}a_{j+n-2}\cdots a_{j+1}}\alpha(t_{j+n} - t_{j+n-1})\Bigg]. \end{aligned} \tag{2.4}$$

Since $y^{\Delta}(t)$ is nonincreasing in $(t_j, t_{j+1}]_{\mathbf{T}}$, hence

$$y^{\Delta}(t) \leq y^{\Delta}(t_j^+) \quad t \in (t_j, t_{j+1}]_{\mathbf{T}}. \tag{2.5}$$

Integrating (2.5) and using (2.2), we obtain

$$y(t_{j+1}) \leq y(t_j^+) + y^{\Delta}(t_j^+)(t_{j+1} - t_j) = y(t_j^+) - \alpha(t_{j+1} - t_j). \tag{2.6}$$

Similarly to (2.6) and using (H_2), (2.2) and (2.6), we get

$$\begin{aligned} y(t_{j+2}) \quad & \leq y(t_{j+1}^+) + y^{\Delta}(t_{j+1}^+)(t_{j+2} - t_{j+1}) \\ & = g_{j+1}(y(t_{j+1})) + h_{j+1}\left(y^{\Delta}(t_{j+1})\right)(t_{j+2} - t_{j+1}) \\ & \leq a_{j+1}y(t_{j+1}) + b_{j+1}^* y^{\Delta}(t_{j+1})(t_{j+2} - t_{j+1}) \\ & \leq a_{j+1}\left[y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2} - t_{j+1})\right]. \end{aligned}$$

Then (2.4) holds for $n = 2$. Now we suppose that (2.4) holds for $n = m$, i.e.

$$\begin{aligned} y(t_{j+m}) \quad & \leq a_{j+1}a_{j+2}\cdots a_{j+m-1}\left[y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2} - t_{j+1})\right. \\ & \left. - \cdots - \frac{b_{j+1}^* b_{j+2}^* \cdots b_{j+m-1}^*}{a_{j+1}a_{j+2}\cdots a_{j+m-1}}\alpha(t_{j+m} - t_{j+m-1})\right], \end{aligned} \tag{2.7}$$

we go to prove that (2.4) holds for $n = m+1$. Since $y^{\Delta}(t)$ is nonincreasing in $(t_{j+m}, t_{j+m+1}]_{\mathbf{T}}$, we have

$$y^{\Delta}(t) \leq y^{\Delta}(t_{j+m}^+) \quad t \in (t_{j+m}, t_{j+m+1}]_{\mathbf{T}}.$$

Integrating it and using (H_2), (2.2), (2.3) and (2.7), we obtain

$$\begin{aligned} y(t_{j+m+1}) \quad & \leq y(t_{j+m}^+) + y^{\Delta}(t_{j+m}^+)(t_{j+m+1} - t_{j+m}) \\ & \leq a_{j+m}y(t_{j+m}) + b_{j+m}^* y^{\Delta}(t_{j+m})(t_{j+m+1} - t_{j+m}) \\ & \leq a_{j+1}a_{j+2}\cdots a_{j+m}\left[y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2} - t_{j+1}) - \cdots\right. \\ & \left. - \frac{b_{j+1}^* b_{j+2}^* \cdots b_{j+m-1}^*}{a_{j+1}a_{j+2}\cdots a_{j+m-1}}\alpha(t_{j+m} - t_{j+m-1})\right] - b_{j+1}^* b_{j+2}^* \cdots b_{j+m}^* \alpha(t_{j+m+1} - t_{j+m}) \\ & = a_{j+1}a_{j+2}\cdots a_{j+m}\left[y(t_j^+) - \alpha(t_{j+1} - t_j) - \frac{b_{j+1}^*}{a_{j+1}}\alpha(t_{j+2} - t_{j+1}) - \cdots\right. \\ & \left. - \frac{b_{j+1}^* b_{j+2}^* \cdots b_{j+m-1}^*}{a_{j+1}a_{j+2}\cdots a_{j+m-1}}\alpha(t_{j+m} - t_{j+m-1}) - \frac{b_{j+1}^* b_{j+2}^* \cdots b_{j+m}^*}{a_{j+1}a_{j+2}\cdots a_{j+m}}\alpha(t_{j+m+1} - t_{j+m})\right]. \end{aligned}$$

Then (2.4) holds for $n = m+1$. By induction, (2.4) holds for any positive integer $n \geq 2$. (2.4) and (H_3) is contrary to $y(t) > 0$. Therefore, $y^{\Delta}(t_k) \geq 0 \;\; (t_k \geq t_0')$. From (H_2), we get for any $t_k \geq t_0', y^{\Delta}(t_k^+) \geq b_k^* y^{\Delta}(t_k) \geq 0$. Since $y^{\Delta}(t)$ is nonincreasing in $(t_k, t_{k+1}]_{\mathbf{T}}$, we know $y^{\Delta}(t) \geq y^{\Delta}(t_{k+1}) \geq 0, t \in (t_k, t_{k+1}]_{\mathbf{T}}$. The proof of Lemma 1 is complete.

Remark 1. In the case of $y(t)$ is eventually negative, under the hypothesis $(H_1)-(H_3)$, it can be proved similarly that $y^{\Delta}(t_k^+)\le 0$ and for $t\in(t_k,t_{k+1}]_{\mathbf{T}}, y^{\Delta}(t)\le 0$ for $t_k\ge T$.

Theorem 1. Suppose that $(H_1)-(H_3)$ hold and there exists a positive integer k_0 such that $a_k^*\ge 1$ for $k\ge k_0$. If

$$\int_{t_0}^{t_1}p(t)\Delta t+\frac{1}{b_1}\int_{t_1}^{t_2}p(t)\Delta t+\frac{1}{b_1b_2}\int_{t_2}^{t_3}p(t)\Delta t+\cdots+\frac{1}{b_1b_2\cdots b_n}\int_{t_n}^{t_{n+1}}p(t)\Delta t+\cdots=\infty, \quad (2.8)$$

then (1.1) is oscillatory.

Proof. Suppose to the contrary that Eq.(1.1) has a nonoscillatory solution $y(t)$, without loss of generality, we may assume that $y(t)$ is eventually positive solution of (1.1), i.e. $y(t)>0, t\ge t_0$ and $k_0=1$. From lemma 1, we have $y^{\Delta}(t)\ge 0,\quad t\in(t_k,t_{k+1}]_{\mathbf{T}}, k=1,2,\cdots$. Let

$$w(t)=\frac{y^{\Delta}(t)}{\varphi(y(t))}, \quad (2.9)$$

then $w(t_k^+)\ge 0, k=1,2,\cdots$ and $w(t)>0, t\ge t_0$. Using (H_1) and (1.1), we get when $t\ne t_k$

$$\begin{aligned} w^{\Delta}(t) &= -\frac{f(t,y^{\sigma}(t))}{\varphi(y^{\sigma}(t))}-\frac{y^{\Delta}(t)}{\varphi(y(t))\varphi(y^{\sigma}(t))}\int_0^1\varphi'\left(y(t)+h\mu(t)y^{\Delta}(t)\right)dhy^{\Delta}(t)\\ &\le -p(t)-\frac{\varphi(y(t))}{\varphi(y^{\sigma}(t))}\left(\frac{y^{\Delta}(t)}{\varphi(y(t))}\right)^2\int_0^1\varphi'\left(y(t)+h\mu(t)y^{\Delta}(t)\right)dh\\ &\le -p(t), \end{aligned} \quad (2.10)$$

since $\varphi'(y(t))\ge 0$ and $\varphi(y(t))>0$. From (H_2) and $a_k^*\ge 1$, we obtain

$$w(t_k^+)=\frac{y^{\Delta}(t_k^+)}{\varphi(y(t_k^+))}\le\frac{b_ky^{\Delta}(t_k)}{\varphi(a_k^*y(t_k))}\le\frac{b_ky^{\Delta}(t_k)}{\varphi(y(t_k))}=b_kw(t_k),\quad k=1,2,\cdots. \quad (2.11)$$

Integrating (2.10), we have

$$w(t_1)\le w(t_0^+)-\int_{t_0}^{t_1}p(t)\Delta t. \quad (2.12)$$

Using (2.11) and (2.12), we obtain

$$w(t_1^+)\le b_1w(t_1)\le b_1w(t_0^+)-b_1\int_{t_0}^{t_1}p(t)\Delta t. \quad (2.13)$$

Similarly, we get

$$w(t_2^+)\le b_2w(t_2)\le b_2\left[w(t_1^+)-\int_{t_1}^{t_2}p(t)\Delta t\right]\le b_1b_2w(t_0^+)-b_1b_2\int_{t_0}^{t_1}p(t)\Delta t-b_2\int_{t_1}^{t_2}p(t)\Delta t. \quad (2.14)$$

By induction, for any positive integer n, we have

$$\begin{aligned} w(t_n^+) &\le b_1b_2\cdots b_nw(t_0^+)-b_1b_2\cdots b_n\int_{t_0}^{t_1}p(t)\Delta t-b_2\cdots b_n\int_{t_1}^{t_2}p(t)\Delta t-\cdots\\ &\quad -b_{n-1}b_n\int_{t_{n-2}}^{t_{n-1}}p(t)\Delta t-b_n\int_{t_{n-1}}^{t_n}p(t)\Delta t\\ &= b_1b_2\cdots b_n\left[w(t_0^+)-\int_{t_0}^{t_1}p(t)\Delta t-\frac{1}{b_1}\int_{t_1}^{t_2}p(t)\Delta t-\cdots\right.\\ &\quad\left.-\frac{1}{b_1b_2\cdots b_{n-2}}\int_{t_{n-2}}^{t_{n-1}}p(t)\Delta t-\frac{1}{b_1b_2\cdots b_{n-1}}\int_{t_{n-1}}^{t_n}p(t)\Delta t\right]. \end{aligned} \quad (2.15)$$

Using (2.8) and $b_k > 0, k = 1, 2, \cdots$, we obtain $w(t_n^+) \to -\infty, n \to \infty$, which contradicts to $w(t_n^+) \geq 0$.

Theorem 2. Assume that $(H_1) - (H_3)$ hold and $\varphi(ab) \geq \varphi(a)\varphi(b)$ for any $ab > 0$. If

$$\begin{aligned} &\int_{t_0}^{t_1} p(t)\Delta t + \frac{\varphi(a_1^*)}{b_1}\int_{t_1}^{t_2} p(t)\Delta t + \frac{\varphi(a_1^*)\varphi(a_2^*)}{b_1 b_2}\int_{t_2}^{t_3} p(t)\Delta t \\ &+ \cdots + \frac{\varphi(a_1^*)\varphi(a_2^*)\cdots\varphi(a_n^*)}{b_1 b_2\cdots b_n}\int_{t_n}^{t_{n+1}} p(t)\Delta t + \cdots = \infty, \end{aligned} \tag{2.16}$$

then (1.1) is oscillatory.

Proof. As before, we may suppose $y(t) > 0, t \geq t_0$ be a nonoscillatory solution of (1.1), Lemma 1 yields $y^{\Delta}(t) \geq 0, t \geq t_0$, define $w(t)$ as in (2.9) and we get $w(t_k) \geq 0, t \geq t_0, w(t_k^+) \geq 0, k = 1, 2, \cdots$ and (2.10) holds for $t \neq t_k$ and

$$w(t_k^+) = \frac{y^{\Delta}(t_k^+)}{\varphi(y(t_k^+))} \leq \frac{b_k y^{\Delta}(t_k)}{\varphi(a_k^* y(t_k))} \leq \frac{b_k y^{\Delta}(t_k)}{\varphi(a_k^*)\varphi(y(t_k))} = \frac{b_k}{\varphi(a_k^*)} w(t_k). \tag{2.17}$$

Similarly to proof (2.15), by induction, we get for any positive integer n

$$\begin{aligned} w(t_n^+) \quad &\leq \frac{b_1 b_2\cdots b_n}{\varphi(a_1^*)\varphi(a_2^*)\cdots\varphi(a_n^*)}\left[w(t_0^+) - \int_{t_0}^{t_1} p(t)\Delta t - \frac{\varphi(a_1^*)}{b_1}\int_{t_1}^{t_2} p(t)\Delta t - \cdots\right. \\ &\left. - \frac{\varphi(a_1^*)\varphi(a_2^*)\cdots\varphi(a_{n-2}^*)}{b_1 b_2\cdots b_{n-2}}\int_{t_{n-2}}^{t_{n-1}} p(t)\Delta t - \frac{\varphi(a_1^*)\varphi(a_2^*)\cdots\varphi(a_{n-1}^*)}{b_1 b_2\cdots b_{n-1}}\int_{t_{n-1}}^{t_n} p(t)\Delta t\right]. \end{aligned}$$

Let $n \to \infty$ and use (2.16), we obtain the desired contradiction.

In the following , we will use the hypothesis

(H_4) $\int_{\pm\epsilon}^{\pm\infty} \frac{\Delta u}{\varphi(u)} < \infty$, for any $\epsilon > 0$,

where $\int_{\pm\epsilon}^{\pm\infty} \frac{\Delta u}{\varphi(u)} < \infty$ denotes $\int_{\epsilon}^{\infty} \frac{\Delta u}{\varphi(u)} < \infty$ and $\int_{-\epsilon}^{-\infty} \frac{\Delta u}{\varphi(u)} < \infty$.

Theorem 3. Assume that $(H_1) - (H_4)$ hold and there exists a positive integer k_0 such that $a_k^* \geq 1$ for $k \geq k_0$. If

$$\begin{aligned} &\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\left[\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots\right. \\ &\left. + \frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots\right]\Delta s = \infty, \end{aligned} \tag{2.18}$$

then (1.1) is oscillatory.

Proof. As before, we may assume $y(t) > 0, t \geq t_0$ be a nonoscillatory solution of (1.1) and $k_0 = 1$, Lemma 1 shows that $y^{\Delta}(t_k^+) \geq 0, k = 1, 2, \cdots$ and $y^{\Delta}(t) \geq 0, t \geq t_0$. Since $a_k^* \geq 1, k = 1, 2, \cdots$, we get

$$y(t_0^+) \leq y(t_1) \leq y(t_1^+) \leq y(t_2) \leq y(t_2^+) \leq \cdots, \tag{2.19}$$

its easy to see that $y(t)$ is nondecreasing in $[t_0, \infty)$, hence (1.1) yields

$$y^{\Delta\Delta}(t) = -f(t, y(t)) \leq -p(t)\varphi(y(t)), \quad t \neq t_k, \tag{2.20}$$

hence, $y^{\Delta}(t_1) - y^{\Delta}(t_0^+) \le -\int_{t_0}^{t_1} p(t)\varphi(y(t))\Delta t$. Using (H_2), we obtain

$$y^{\Delta}(t_0^+) \ge y^{\Delta}(t_1) + \int_{t_0}^{t_1} p(t)\varphi(y(t))\Delta t \ge \frac{y^{\Delta}(t_1^+)}{b_1} + \int_{t_0}^{t_1} p(t)\varphi(y(t))\Delta t.$$

Similarly,

$$y^{\Delta}(t_1^+) \ge \frac{y^{\Delta}(t_2^+)}{b_2} + \int_{t_1}^{t_2} p(t)\varphi(y(t))\Delta t.$$

Generally, for any positive integer n, we get

$$y^{\Delta}(t_n^+) \ge y^{\Delta}(t_{n+1}) + \int_{t_n}^{t_{n+1}} p(t)\varphi(y(t))\Delta t \ge \frac{y^{\Delta}(t_{n+1}^+)}{b_{n+1}} + \int_{t_n}^{t_{n+1}} p(t)\varphi(y(t))\Delta t. \tag{2.21}$$

By (2.20) and (2.21), noting that $y^{\Delta}(t_k^+) \ge 0, k = 1, 2, \cdots$, we have for $s \in (t_k, t_{k+1}]_{\mathbf{T}}$,

$$\begin{aligned}
y^{\Delta}(s) \quad & \ge \textstyle\int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + y^{\Delta}(t_{k+1}) \ge \int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \dfrac{y^{\Delta}(t_{k+1}^+)}{b_{k+1}} \\
& \ge \textstyle\int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \dfrac{1}{b_{k+1}} \left[\int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y(t))\Delta t + \dfrac{y^{\Delta}(t_{k+2}^+)}{b_{k+2}}\right] \\
& \ge \textstyle\int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \dfrac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y(t))\Delta t + \dfrac{1}{b_{k+1}b_{k+2}} \int_{t_{k+2}}^{t_{k+3}} p(t)\varphi(y(t))\Delta t \\
& + \frac{y^{\Delta}(t_{k+3}^+)}{b_{k+1}b_{k+2}b_{k+3}} \ge \cdots \\
& \ge \textstyle\int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \dfrac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y(t))\Delta t + \cdots \\
& + \frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \textstyle\int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y(t))\Delta t + \dfrac{y^{\Delta}(t_{k+n+1}^+)}{b_{k+1}b_{k+2}\cdots b_{k+n+1}}.
\end{aligned} \tag{2.22}$$

Noting that $b_k > 0$ and $y^{\Delta}(t_k^+) \ge 0, k = 1, 2, \cdots$, (2.22) yields

$$\begin{aligned}
y^{\Delta}(s) \quad & \ge \textstyle\int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \dfrac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y(t))\Delta t + \cdots \\
& + \frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \textstyle\int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y(t))\Delta t,
\end{aligned} \tag{2.23}$$

holds for any positive integer n, then

$$\begin{aligned}
y^{\Delta}(s) \quad & \ge \textstyle\int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \dfrac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y(t))\Delta t + \cdots \\
& + \frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \textstyle\int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y(t))\Delta t + \cdots.
\end{aligned} \tag{2.24}$$

Using (H_1) and (2.24), we obtain for $s \in (t_k, t_{k+1}]_{\mathbf{T}}$,

$$\begin{aligned}
\frac{y^{\Delta}(s)}{\varphi(y(s))} \quad & \ge \textstyle\int_s^{t_{k+1}} p(t)\dfrac{\varphi(y(t))}{\varphi(y(s))}\Delta t + \dfrac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\dfrac{\varphi(y(t))}{\varphi(y(s))}\Delta t + \cdots \\
& + \frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \textstyle\int_{t_{k+n}}^{t_{k+n+1}} p(t)\dfrac{\varphi(y(t))}{\varphi(y(s))}\Delta t + \cdots \\
& \ge \textstyle\int_s^{t_{k+1}} p(t)\Delta t + \dfrac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots + \dfrac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots.
\end{aligned}$$

Integrating it from t_k to t_{k+1} and using (2.3), we have

$$\int_{t_k}^{t_{k+1}}\left[\int_s^{t_{k+1}}p(t)\Delta t+\frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}}p(t)\Delta t+\cdots+\frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}}p(t)\Delta t+\cdots\right]\Delta s$$
$$\leq\int_{t_k}^{t_{k+1}}\frac{y^{\Delta}(s)}{\varphi(y(s))}\Delta s=\int_{y(t_k^+)}^{y(t_{k+1})}\frac{1}{\varphi(u)}\Delta u. \tag{2.25}$$

Using (2.19),and (H_4), (2.25) yields

$$\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\left[\int_s^{t_{k+1}}p(t)\Delta t+\frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}}p(t)\Delta t+\cdots+\frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}}p(t)\Delta t+\cdots\right]$$
$$\leq\sum_{k=0}^{\infty}\int_{y(t_k^+)}^{y(t_{k+1})}\frac{1}{\varphi(u)}\Delta u\leq\int_{y(t_0^+)}^{\infty}\varphi(u)\Delta u<\infty, \tag{2.26}$$

which contradicts to (2.18).

Theorem 4. Suppose that $(H_1)-(H_4)$ hold and there exists a positive integer k_0 such that $a_k^*\geq 1$ for $k\geq k_0$. Assume, furthermore, that $\varphi(ab)\geq\varphi(a)\varphi(b)$ for any $ab>0$ and

$$\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}}\left[\int_s^{t_{k+1}}p(t)\Delta t+\frac{\varphi(a_{k+1}^*)}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}}p(t)\Delta t+\frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)}{b_{k+1}b_{k+2}}\int_{t_{k+2}}^{t_{k+3}}p(t)\Delta t+\cdots\right.$$
$$\left.+\frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\cdots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\cdots b_{k+n}}\int_{t_{k+n}}^{t_{k+n+1}}p(t)\Delta t+\cdots\right]\Delta s=\infty. \tag{2.27}$$

Then (1.1) is oscillatory.

Proof. As before, we may assume that $y(t)>0,t\geq t_0$ be a nonoscillatory solution of (1.1) and $k_0=1$. According to the proof of Theorem 3, (2.19) and (2.24) hold. Furthermore, from (H_1) and Lemma 1, we obtain $\varphi(y)$ is nondecreasing and $y(t)$ is also nondecreasing in $(t_k,t_{k+1}]_{\mathbf{T}}, k=0,1,2,\cdots$. Therefore, $\varphi(y(t))$ is nondecreasing in $(t_k,t_{k+1}]_{\mathbf{T}}$. Hence,

$$\varphi\left(y(t_{k+1}^+)\right)\geq\varphi(a_{k+1}^*y(t_{k+1}))\geq\varphi(a_{k+1}^*)\varphi(y(t_{k+1})),$$

and

$$\varphi\left(y(t_{k+2}^+)\right)\geq\varphi(a_{k+2}^*y(t_{k+2}))\geq\varphi(a_{k+2}^*)\varphi(y(t_{k+1}^+))\geq\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\varphi(y(t_{k+1})).$$

By induction, it can be proved that for any positive integer n

$$\varphi\left(y(t_{k+n}^+)\right)\geq\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\cdots\varphi(a_{k+n}^*)\varphi(y(t_{k+1})). \tag{2.28}$$

From (2.24), (2.28) and using the fact that $\varphi(y(t))$ is nondecreasing, we obtain, for $s \in (t_k, t_{k+1}]_{\mathbf{T}}$

$$\begin{aligned} y^{\Delta}(s) \quad & \geq \int_s^{t_{k+1}} p(t)\varphi(y(t))\Delta t + \frac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\varphi(y(t))\Delta t + \cdots \\ & + \frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\varphi(y(t))\Delta t + \cdots \\ & \geq \varphi(y(s)) \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(y(t_{k+1}^+))}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots \\ & + \frac{\varphi(y(t_{k+n}^+))}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots \\ & \geq \varphi(y(s)) \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)\varphi(y(t_{k+1}))}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots \\ & + \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\cdots\varphi(a_{k+n}^*)\varphi(y(t_{k+1}))}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots. \end{aligned}$$

Hence,

$$\begin{aligned} \frac{y^{\Delta}(s)}{\varphi(y(s))} \quad & \geq \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)}{b_{k+1}} \frac{\varphi(y(t_{k+1}))}{\varphi(y(s))} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots \\ & + \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\cdots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\cdots b_{k+n}} \frac{\varphi(y(t_{k+1}))}{\varphi(y(s))} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots \\ & \geq \int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots \\ & + \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\cdots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots. \end{aligned}$$

Integrating the above inequality and using (2.19), (2.8), we obtain

$$\begin{aligned} & \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \left[\int_s^{t_{k+1}} p(t)\Delta t + \frac{\varphi(a_{k+1}^*)}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \cdots \right. \\ & \left. + \frac{\varphi(a_{k+1}^*)\varphi(a_{k+2}^*)\cdots\varphi(a_{k+n}^*)}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots \right] \\ & \leq \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \frac{y^{\Delta}(s)}{\varphi(y(s))} = \sum_{k=0}^{\infty} \int_{y(t_k^+)}^{y(t_{k+1})} \frac{1}{\varphi(u)} \Delta u \leq \int_{y(t_0^+)}^{\infty} \frac{1}{\varphi(u)} \Delta u < \infty, \end{aligned}$$

which contradicts to (2.27).

From Theorem 1-4, we have the following corollaries.

Corollary 1. Suppose that $(H_1) - (H_3)$ hold and there exists a positive integer k_0 such that $a_k^* \geq 1, b_k \leq 1$ for $k \geq k_0$. If $\int^{\infty} p(t)\Delta t = \infty$, then (1.1) is oscillatory.

Proof. Without loss of generality, let $k_0 = 1$. By $b_k \leq 1$, we get

$$\begin{aligned} & \int_{t_0}^{t_1} p(t)\Delta t + \frac{1}{b_1} \int_{t_1}^{t_2} p(t)\Delta t + \frac{1}{b_1 b_2} \int_{t_2}^{t_3} p(t)\Delta t + \cdots + \frac{1}{b_1 b_2 \cdots b_n} \int_{t_n}^{t_{n+1}} p(t)\Delta t \\ & \geq \int_{t_0}^{t_1} p(t)\Delta t + \int_{t_1}^{t_2} p(t)\Delta t + \int_{t_2}^{t_3} p(t)\Delta t + \cdots + \int_{t_n}^{t_{n+1}} p(t)\Delta t = \int_{t_0}^{t_{n+1}} p(t)\Delta t. \end{aligned} \tag{2.29}$$

Let $n \to \infty$, from $\int^{\infty} p(t)\Delta t = \infty$, (2.29) yields (2.8) holding. By Theorem 1, we conclude that (1.1) is oscillatory.

Corollary 2. Assume that $(H_1) - (H_4)$ hold and there exists a positive integer k_0 such that $a_k^* \geq 1, b_k \leq 1$ for $k \geq k_0$. If $\int^{\infty} \int_s^{\infty} p(t)\Delta t\Delta s = \infty$, then (1.1) is oscillatory.

Similar to the proof of Corollary 1 and using Theorem 3 can prove it, so we omit it.

Corollary 3. Suppose that $(H_1)-(H_3)$ hold and there exist a positive integer k_0 and a constant $\alpha>0$ such that

$$a_k^*\geq 1,\quad \frac{1}{b_k}\geq\left(\frac{t_{k+1}}{t_k}\right)^{\alpha},\quad for\quad k\geq k_0. \tag{2.30}$$

If

$$\int^{\infty} t^{\alpha}p(t)\Delta t=\infty. \tag{2.31}$$

Then (1.1) is oscillatory.

Proof. As before, let $k_0=1$, (2.30) yields

$$\begin{aligned}
&\textstyle\int_{t_0}^{t_1} p(t)\Delta t+\frac{1}{b_1}\int_{t_1}^{t_2} p(t)\Delta t+\frac{1}{b_1b_2}\int_{t_2}^{t_3} p(t)\Delta t\cdots+\frac{1}{b_1b_2\cdots b_n}\int_{t_n}^{t_{n+1}} p(t)\Delta t\\
&\geq \frac{1}{t_1^{\alpha}}\left[\textstyle\int_{t_1}^{t_2} t_2^{\alpha}p(t)\Delta t+\int_{t_2}^{t_3} t_3^{\alpha}p(t)\Delta t+\cdots+\int_{t_n}^{t_{n+1}} t_{n+1}^{\alpha}p(t)\Delta t\right]\\
&\geq \frac{1}{t_1^{\alpha}}\left[\textstyle\int_{t_1}^{t_2} t^{\alpha}p(t)\Delta t+\int_{t_2}^{t_3} t^{\alpha}p(t)\Delta t+\cdots+\int_{t_n}^{t_{n+1}} t^{\alpha}p(t)\Delta t\right]\\
&=\frac{1}{t_1^{\alpha}}\textstyle\int_{t_1}^{t_{n+1}} t^{\alpha}p(t)\Delta t.
\end{aligned} \tag{2.32}$$

Let $n\to\infty$ and use (2.31), (2.32) yields (2.8) holds, By Theorem 1, we obtain (1.1) is oscillatory.

Corollary 4. Assume that $(H_1)-(H_3)$ hold and $\varphi(ab)\geq\varphi(a)\varphi(b)$ for any $ab>0$. Suppose there exist a positive integer k_0 and a constant $\alpha>0$ such that

$$\frac{\varphi(a_k^*)}{b_k}\geq\left(\frac{t_{k+1}}{t_k}\right)^{\alpha},\quad for\quad k\geq k_0.$$

If $\int^{\infty} t^{\alpha}p(t)\Delta t=\infty$, then (1.1) is oscillatory.

Corollary 4 can be deduced from Theorem 2. Its proof is similar to that of Corollary 3. Here we omit it.

Corollary 5. Suppose that $(H_1)-(H_4)$ hold and there exist a positive integer k_0 and a constant $\alpha>0$ such that

$$a_k^*\geq 1,\quad \frac{1}{b_k}\geq t_{k+1}^{\alpha},\quad for\quad k\geq k_0. \tag{2.33}$$

If

$$\sum_{k=0}^{\infty}(t_{k+1}-t_k)\int_{t_{k+1}}^{\infty} t^{\alpha}p(t)\Delta t=\infty. \tag{2.34}$$

Then (1.1) is oscillatory.

Proof. As before, we assume $k_0=1, t_1\geq 1$. From (2.33), we get

$$\frac{1}{b_{k+1}}\geq t_{k+2}^{\alpha},\frac{1}{b_{k+1}b_{k+2}}\geq t_{k+2}^{\alpha}t_{k+3}^{\alpha},\cdots,\frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}}\geq t_{k+2}^{\alpha}t_{k+3}^{\alpha}\cdots t_{k+n+1}^{\alpha},\cdots.$$

Similar to the proof of Corollary 3, we have

$$\begin{aligned}
&\textstyle\int_{s}^{t_{k+1}} p(t)\Delta t+\frac{1}{b_{k+1}}\int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t+\frac{1}{b_{k+1}b_{k+2}}\int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t+\cdots\\
&+\frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}}\textstyle\int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t\\
&\geq \textstyle\int_{t_{k+1}}^{t_{k+n+1}} t^{\alpha}p(t)\Delta t.
\end{aligned}$$

Let $n \to \infty$, we have

$$\begin{aligned}
&\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \frac{1}{b_{k+1}b_{k+2}} \int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t + \cdots \\
&+\frac{1}{b_{k+1}b_{k+2}\cdots b_{k+n}} \int_{t_{k+n}}^{t_{k+n+1}} p(t)\Delta t + \cdots \\
&\geq \int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t.
\end{aligned} \tag{2.35}$$

Using (2.34) and (2.35), we get

$$\begin{aligned}
&\sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \left[\int_s^{t_{k+1}} p(t)\Delta t + \frac{1}{b_{k+1}} \int_{t_{k+1}}^{t_{k+2}} p(t)\Delta t + \frac{1}{b_{k+1}b_{k+2}} \int_{t_{k+2}}^{t_{k+3}} p(t)\Delta t + \cdots\right] \Delta s \\
&\geq \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t\Delta s = \sum_{k=0}^{\infty}(t_{k+1}-t_k) \int_{t_{k+1}}^{\infty} t^\alpha p(t)\Delta t = \infty.
\end{aligned}$$

By Theorem 3, we obtain that (1.1) is oscillatory.

Corollary 6. Suppose that $(H_1)-(H_4)$ hold and there exists a positive integer k_0 and a constant $\alpha > 0$ such that

$$a_k^* \geq 1, \quad \frac{\varphi(a_k^*)}{b_k} \geq t_{k+1}^\alpha, \quad for \quad k \geq k_0.$$

Suppose that $\varphi(ab) \geq \varphi(a)\varphi(b)$ for any $ab > 0$ and

$$\sum_{k=0}^{\infty}(t_{k+1}-t_k) \int_{t_{k+1}}^{\infty} p(t)\Delta t = \infty.$$

Then (1.1) is oscillatory.

The proof is similar to that of Corollary 5, so we omit it.

4. Example

Example 1. Consider the the following second order impulsive dynamic equation

$$\begin{aligned}
&y^{\Delta\Delta}(t) + \frac{1}{t\sigma^2(t)} y^\gamma(\sigma(t)) = 0, \quad t \geq 1, t \neq k, k = 1,2,\cdots, \\
&y(k^+) = \tfrac{k+1}{k} y(k), y^\Delta(k^+) = y^\Delta(k), \quad k = 1,2,\cdots, \\
&y(1) = y_0, y^\Delta(1) = y_0^\Delta.
\end{aligned} \tag{4.1}$$

where $\gamma \geq 3$ and $\mu(t) \leq Kt$, where K is a positive constant. Since $a_k = a_k^* = \frac{k+1}{k}, b_k = b_k^* = 1, p(t) = \frac{1}{t\sigma^2(t)}, t_k = k$ and $\varphi(y) = y^\gamma$. it is easy to see that $(H_1)-(H_3)$ hold. Let $k_0 = 1, \alpha = 3$, hence

$$\frac{\varphi(a_k^*)}{b_k} = \left(\frac{k+1}{k}\right)^\gamma = \left(\frac{t_{k+1}}{t_k}\right)^\gamma \geq \left(\frac{t_{k+1}}{t_k}\right)^3,$$

and

$$\int^\infty t^\alpha p(t)\Delta t = \int^\infty t^3 \frac{1}{t\sigma^2(t)}\Delta t = \int^\infty \left(\frac{t}{\sigma(t)}\right)^2 \Delta t.$$

Since $\mu(t) \leq Kt$, we get

$$\frac{t}{\sigma(t)} = \frac{t}{t+\mu(t)} \geq \frac{1}{1+K},$$

hence

$$\int^{\infty} \left(\frac{t}{\sigma(t)}\right)^2 \Delta t \geq \frac{1}{1+K} \int^{\infty} \Delta t = \infty.$$

By Corollary 3, we obtain that (4.1) is oscillatory. But by [3] we know that the dynamic equation $y^{\Delta\Delta}(t) + \frac{1}{t\sigma^2(t)} y^{\gamma}(\sigma(t)) = 0$ is nonoscillatory.

In the above example, it is noticing that the dynamic equation without impulses is nonoscillatory, but when some impulses are added to it, it all become oscillatory. Therefore, this example shows that impulses play an important part in oscillations of dynamic equations on time scales.

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