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PERIODIC SOLUTIONS FOR PLANAR SYSTEMS WITH TIME-VARYING DELAYS

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Abstract. This paper concerns delay differential systems that can be regarded as a model of two-neuron artificial neural network with delayed feedback. Some interesting results are obtained for the existence of a periodic solution for the system. Our approach is based on the continuation theorem of coincidence degree, and a-priori estimates of the periodic solutions.

1. Introduction

Neural networks are complex and large-scale nonlinear dynamics, while the dynamics of the delayed neural network are even richer and more complicated [10]. To obtain a deep and clear understanding of the dynamics of neural networks, there has been an increasing interest in the investigations of delayed neural network models with two neurons, see [1, 3, 4, 6, 7, 8, 9, 11]. Táboas [9] considered the system of delay differential equations

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t) + \alpha f_1(x_1(t-\tau), x_2(t-\tau)),\\ \dot{x}_2(t) &= -x_2(t) + \alpha f_2(x_1(t-\tau), x_2(t-\tau)),\end{aligned} \tag{1.1}$$

which arises as a model for a network of two saturating amplifiers (or neurons) with delayed outputs, where $\alpha > 0$ is a constant, f_1, f_2, are bounded C^3 functions on $\mathbb{R}^2$ satisfying

$$\frac{\partial f_1}{\partial x_2}(0,0) \neq 0 \quad \text{and} \quad \frac{\partial f_2}{\partial x_1}(0,0) \neq 0,$$

and the negative feedback conditions : $x_2 f_1(x_1, x_2) > 0$, $x_2 \neq 0$; $x_1 f_2(x_1, x_2) < 0$, $x_1 \neq 0$. Táboas showed that there is an $\alpha_0 > 0$ such that for $\alpha > \alpha_0$, there exists a non-constant periodic solution with period greater than 4. Further study on the global existence of periodic solutions to system (1.1) can be found in [1] and [6]. All together there is only one delay appearing in both equations. Ruan and Wei [8] investigated the existence of non-constant periodic solutions of the following planar system with two delays

$$\begin{aligned}\dot{x}_1(t) &= -a_0 x_1(t) + a_1 f_1(x_1(t-\tau_1), x_2(t-\tau_2)),\\ \dot{x}_2(t) &= -b_0 x_2(t) + b_1 f_2(x_1(t-\tau_1), x_2(t-\tau_2)),\end{aligned} \tag{1.2}$$

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where $a_0 > 0$, $b_0 > 0$, a_1 and b_1 are constants, the function f_1 and f_2 satisfy $f_j \in C^3(\mathbb{R}^2)$, $f_j(0,0) = 0$, $\frac{\partial f_j}{\partial x_j}(0,0) = 0$, $j = 1,2$; $x_2 f_1(x_1,x_2) \neq 0$ for $x_2 \neq 0$; $x_1 f_2(x_1,x_2) \neq 0$ for $x_1 \neq 0$; $\frac{\partial f_1}{\partial x_2}(0,0) \neq 0$, $\frac{\partial f_2}{\partial x_1}(0,0) \neq 0$.

Recently, Zhang and Wang [11] investigated the system

$$\begin{aligned}\dot{x}_1(t) &= -a_1 x_1(t) + b_1 f_1(x_1(t-\tau_1), x_2(t-\tau_2)),\\ \dot{x}_2(t) &= -a_2 x_2(t) + b_2 f_2(x_1(t-\tau_3), x_2(t-\tau_4)),\end{aligned} \tag{1.3}$$

where $a_1, a_2, b_1, b_2, \tau_1, \tau_2, \tau_3, \tau_4$ are constants. By means of the continuation theorem of the coincidence degree, they get some results about the periodic solutions to system (1.3).

However, delays considered in all above systems are constant. It is well known that the delays in artificial neural networks are usually time-varying, and sometimes vary violently with time due to the finite switching speed of amplifiers and faults in the electrical circuit. They slow down the transmission rate and tend to introduce some degree of instability in circuits. Therefore, fast response must be required in practical artificial neural-network designs. The technique to achieve fast response troubles many circuit designers. So, it is more important to investigate the dynamic behave of neural networks with time-varying delays. Keeping this in mind, in this paper, we consider the following planar system where coefficients and delays are all periodically varying in time:

$$\begin{aligned}\dot{x}_1(t) &= -a_1(t) x_1(t) + b_1(t) f_1(x_1(t-\tau_1(t)), x_2(t-\tau_2(t))),\\ \dot{x}_2(t) &= -a_2(t) x_2(t) + b_2(t) f_2(x_1(t-\tau_3(t)), x_2(t-\tau_4(t))),\end{aligned} \tag{1.4}$$

where $a_i \in C(\mathbb{R}, (0,\infty))$, $b_i \in C(\mathbb{R},\mathbb{R})$, $i = 1,2$, are periodic with a common period $\omega(> 0)$, $f_i \in C(\mathbb{R}^2, \mathbb{R})$, $i = 1,2$; $\tau_i \in C(\mathbb{R}, [0,\infty))$, $i = 1,2,3,4$, being ω-periodic.

For a continuous function g: $[0,\omega] \to \mathbb{R}$, we introduce the following notation:

$$\begin{gathered}\overline{g} = \frac{1}{\omega}\int_0^\omega g(t)\mathrm{d}t,\\ a(t) = \min\{a_1(t), a_2(t)\}, \quad b(t) = \max\{|b_1(t)|, |b_2(t)|\}.\end{gathered}$$

Obviously, system (1.4) is more general than system (1.3). To our best knowledge, the existence of ω-periodic solution of the system (1.4) has not been studied in pervious works. We shall employ the powerful method of coincidence degree to establish the existence of a periodic solution to (1.4). These conditions in our results are very simple and easy to be verified.

2. Existence of Periodic Solution

In this section, we use the coincidence degree theory to obtain the existence of an ω-periodic solution to (1.4). For the sake of convenience, we briefly summarize the theory as below.

Let X and Z be normed spaces, L: $\operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\operatorname{Ker} L = \operatorname{codim}\operatorname{Im} L < \infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero, there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$ and $\operatorname{Im} L = \ker Q = \operatorname{Im}(I - Q)$. It follows that $L\big|\operatorname{Dom} L \cap \ker P : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of this map by K_p. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N : \overline{\Omega} \to X$ is compact. Because $\operatorname{Im} Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \operatorname{Im} Q \to \ker L$.

Let $\Omega \subset \mathbb{R}^n$ be open and bounded, $f \in C^1(\Omega, \mathbb{R}^n) \cap C(\overline{\Omega}, \mathbb{R}^n)$ and $y \in \mathbb{R}^n \setminus f(\partial\Omega \cup S_f)$, i.e., y is a regular value of f. Here, $S_f = \{x \in \Omega : J_f(x) = 0\}$, the critical set of f, and J_f is the Jacobian of f at x. Then the degree $\deg\{f, \Omega, y\}$ is defined by

$$\deg\{f, \Omega, y\} = \sum_{x \in f^{-1}(y)} \operatorname{sgn} J_f(x)$$

with the agreement that the above sum is zero if $f^{-1}(y) = \emptyset$. For more details about degree theory, we refer to the book by Deimiling [2].

Now, with the above notation, we are ready to state the continuation theorem.

Lemma 2.1 (Continuation Theorem [5, P.40])**.** *Let L be a Fredholm mapping of index zero and let N be L-compact on $\overline{\Omega}$. Suppose*

(a) *For each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$*
(b) *$QNx \neq 0$ for each $x \in \partial\Omega \cap \ker L$ and*

$$\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0.$$

Then the equation $Lx = Nx$ has at least one solution lying in $\operatorname{Dom} L \cap \overline{\Omega}$.

The following is the main result of this section.

Theorem 2.2. *Suppose that $|f_i(x_1, x_2)| \leq \alpha_i|x_1| + \beta_i|x_2| + M_i$ and $\frac{D_1}{D} > 0$, $\frac{D_2}{D} > 0$, where $\alpha_i \geq 0$, $\beta_i \geq 0$ and $M_i > 0$ are constants for $i = 1, 2$, $D = a^2 - ab\alpha_1 - ab\beta_2 + b^2\alpha_1\beta_2 - b^2\alpha_2\beta_1$, $D_1 = abM_1 + b^2M_2\beta_1 - b^2M_1\beta_2$, $D_2 = abM_2 - b^2M_2\alpha_1 + b^2M_1\alpha_2$, $a = \min_{t\in[0,\omega]} a(t)$, $b = \max_{t\in[0,\omega]} b(t)$. Then system* (1.4) *has an ω-periodic solution.*

Proof. Take $X = \{u(t) = (x_1(t), x_2(t))^T \in C(\mathbb{R}, \mathbb{R}^2) : u(t) = u(t+\omega) \text{ for } t \in \mathbb{R}\}$ and denote

$$\|x_i\| = \max_{t\in[0,\omega]} |x_i(t)|, \quad i = 1, 2;$$
$$\|u\|_0 = \max_{i=1,2} \|x_i\|.$$

Equipped with the norm $\|.\|_0$, X is a Banach space. For any $u(t) \in X$, because of the periodicity, it is easy to check that

$$t \mapsto \begin{pmatrix} -a_1(t)x_1(t) + b_1(t)f_1(x_1(t-\tau_1(t)), x_2(t-\tau_2(t))), \\ -a_2(t)x_2(t) + b_2(t)f_2(x_1(t-\tau_3(t)), x_2(t-\tau_4(t))). \end{pmatrix} \in X.$$

Let

$$L : \operatorname{Dom} L = \{u \in X : u \in C^1(\mathbb{R}, \mathbb{R}^2)\} \ni u \mapsto u' \in X,$$

$$P : X \ni u \to \overline{u} \in X, \quad Q : X \ni x \mapsto \overline{x} \in X,$$

where for any $K = (k_1, k_2)^T \in \mathbb{R}^2$, we identify it as the constant function in X with the value vector K $= (k_1, k_2)^T$. Define $N : X \to X$ given by

$$(Nu)(t) = \begin{pmatrix} -a_1(t)x_1(t) + b_1(t)f_1(x_1(t-\tau_1(t)), x_2(t-\tau_2(t))), \\ -a_2(t)x_2(t) + b_2(t)f_2(x_1(t-\tau_3(t)), x_2(t-\tau_4(t))). \end{pmatrix} \in X.$$

Then system (1.4) can be reduced to the operator equation $Lu = Nu$. Note that N is continuous, since f_i are uniformly continuous on compact sets of $\mathbb{R}^2$. It is easy to see that

$$\begin{gathered}\ker L = \mathbb{R}^2,\\ \operatorname{Im} L = \{x \in X : \overline{x} = 0\},\ \text{which is closed in } X,\\ \dim\ker L = \operatorname{codim}\operatorname{Im} L = 2 < \infty,\end{gathered}$$

and P, Q are continuous projectors such that

$$\operatorname{Im} P = \ker L, \quad \ker Q = \operatorname{Im} L = \operatorname{Im}(I - Q).$$

It follows that L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p : \operatorname{Im} L \to \ker P \cap \operatorname{Dom} L$ is given by

$$(K_p(u))(t) = \begin{pmatrix} \int_0^t x_1(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega\int_0^s x_1(v)\mathrm{d}v\mathrm{d}s \\ \int_0^t x_2(s)\mathrm{d}s - \frac{1}{\omega}\int_0^\omega\int_0^s x_2(v)\mathrm{d}v\mathrm{d}s \end{pmatrix}.$$

Thus,

$$(QNu)(t) = \begin{pmatrix} \frac{1}{\omega}\int_0^\omega\{-a_1(s)x_1(s) + b_1(s)f_1(x_1(s-\tau_1(s))x_2(s-\tau_2(s)))\}\mathrm{d}s \\ \frac{1}{\omega}\int_0^\omega\{-a_2(s)x_2(s) + b_2(s)f_2(x_1(s-\tau_3(s))x_2(s-\tau_4(s)))\}\mathrm{d}s \end{pmatrix},$$

and

$$\begin{aligned}
&(K_p(I-Q)Nu)(t)\\
&= \begin{pmatrix} \int_0^t\{-a_1(s)x_1(s) + b_1(s)f_1(x_1(s-\tau_1(s)), x_2(s-\tau_2(s)))\}\mathrm{d}s \\ \int_0^t\{-a_2(s)x_2(s) + b_2(s)f_2(x_1(s-\tau_3(s)), x_2(s-\tau_4(s)))\}\mathrm{d}s \end{pmatrix}\\
&\quad - \begin{pmatrix} \frac{1}{\omega}\int_0^\omega\int_0^s\{-a_1(v)x_1(v) + b_1(v)f_1(x_1(v-\tau_1(v)), x_2(v-\tau_2(v)))\}\mathrm{d}v\mathrm{d}s \\ \frac{1}{\omega}\int_0^\omega\int_0^s\{-a_2(v)x_2(v) + b_2(v)f_2(x_1(v-\tau_3(v)), x_2(v-\tau_4(v)))\}\mathrm{d}v\mathrm{d}s \end{pmatrix}\\
&\quad + \begin{pmatrix} (\frac{1}{2} - \frac{t}{\omega})\int_0^\omega\{-a_1(s)x_1(s) + b_1(s)f_1(x_1(s-\tau_1(s)), x_2(s-\tau_2(s)))\}\mathrm{d}s \\ (\frac{1}{2} - \frac{t}{\omega})\int_0^\omega\{-a_2(s)x_2(s) + b_2(s)f_2(x_1(s-\tau_3(s)), x_2(s-\tau_4(s)))\}\mathrm{d}s \end{pmatrix}.
\end{aligned}$$

Clearly, QN and $K_p(I-Q)N$ are continuous. For any bounded open subset $\Omega \subset X$, $QN(\overline{\Omega})$ is obviously bounded. Moreover, applying the Arzela-Ascoli theorem, one can easily show that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact. Note that $K_p(I-Q)N$ is a compact operator and $QN(\overline{\Omega})$ is bounded, therefore, N is L-compact on $\overline{\Omega}$ for any bounded open subset $\Omega \subset X$. Since $\operatorname{Im} Q = \ker L$, we take the isomorphism J of $\operatorname{Im} Q$ onto $\ker L$ to be the identity mapping. Corresponding to equation $Lu = \lambda Nu, \lambda \in (0,1)$, we have

$$\begin{aligned}
\dot{x}_1(t) &= \lambda\{-a_1(t)x_1(t) + b_1(t)f_1(x_1(t-\tau_1(t)), x_2(t-\tau_2(t)))\},\\
\dot{x}_2(t) &= \lambda\{-a_2(t)x_2(t) + b_2(t)f_2(x_1(t-\tau_3(t)), x_2(t-\tau_4(t)))\}.
\end{aligned} \tag{2.1}$$

Now we reach the position to search for an appropriate open bounded subset Ω for the application of the Lemma 2.1. Assume that $u = u(t) \in X$ is a solution of system (2.1). Then, the components $x_i(t)(i = 1, 2)$ of $u(t)$ are continuously differentiable. Thus, there exists $t_i \in [0, \omega]$ such that $|x_i(t_i)| = \max_{t\in[0,\omega]}|x_i(t)|$. Hence, $\dot{x}_i(t_i) = 0$. This implies

$$a_i(t_i)x_i(t_i) = b_1(t_i)f_i(x_1(t_i - \tau_1(t_i)), x_2(t_i - \tau_2(t_i))), \tag{2.2}$$

for $i = 1, 2$. Since

$$|f_i(x_1, x_2)| \le \alpha_i|x_1| + \beta_i|x_2| + M_i \quad \text{for} \quad i = 1, 2,$$

we get

$$|x_i(t_i)| \le \frac{\alpha_i b|x_1(t_i-\tau_1(t_i))|}{a} + \frac{\beta_i b|x_2(t_i-\tau_2(t_i))|}{a} + \frac{bM_i}{a}, \tag{2.3}$$

for $i = 1,2$. From $k_1 = \frac{D_1}{D}$, $k_2 = \frac{D_2}{D}$, we find that

$$\begin{aligned} k_1 &= \frac{\alpha_1 b}{a}k_1 + \frac{\beta_1 b}{a}k_2 + \frac{bM_1}{a}, \\ k_2 &= \frac{\alpha_2 b}{a}k_1 + \frac{\beta_2 b}{a}k_2 + \frac{bM_2}{a}. \end{aligned} \tag{2.4}$$

Now, we choose a constant number $d > 1$ and take

$$\Omega = \{(x_1,x_2)^T \in \mathbb{R}^2; |x_i| < dk_i \text{ for } i = 1,2\},$$

where $k_1 = \frac{D_1}{D} > 0$, $k_2 = \frac{D_2}{D} > 0$. We will show that Ω satisfies all the requirements given in Lemma 2.1. In fact, we will prove that if $x_i(t_i - \tau_1(t_i)) \in \Omega$ then $x_i(t_i) \in \Omega$ for $i = 1,2$. Therefore, it means that $u = u(t)$ is uniformly bounded with respect to λ when the initial value function belongs to Ω. It follows from (2.3) that

$$\begin{aligned} |x_i(t_i)| &\le \frac{\alpha_i b|x_1(t_i-\tau_1(t_i))|}{a} + \frac{\beta_i b|x_2(t_i-\tau_2(t_i))|}{a} + \frac{bM_i}{a} \\ &< d\big(\frac{\alpha_i b}{a}k_1 + \frac{\beta_i b}{a}k_2 + \frac{bM_i}{a}\big). \end{aligned}$$

This, together with (2.4), implies $|x_i(t_i)| < dk_i$, for $i = 1,2$. Therefore,

$$\|x_i\| < dk_i \quad \text{for} \quad i = 1,2. \tag{2.5}$$

Clearly, dk_i, i=1,2, are independent of λ. It is easy to see that there are no $\lambda \in (0,1)$ and $u \in \partial\Omega$ such that $Lu = \lambda Nu$. If $u = (x_1,x_2)^T \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^2$, then u is a constant vector in $\mathbb{R}^2$ with $|x_i| = dk_i$ for $i = 1,2$. Note that $QNu = JQNu$, we have

$$QNu = \begin{pmatrix} -\overline{a_1}x_1 + \overline{b_1}f_1(x_1,x_2) \\ -\overline{a_2}x_2 + \overline{b_2}f_2(x_1,x_2) \end{pmatrix} \tag{2.6}$$

We claim that

$$|(QNu)_i| > 0 \quad \text{for } i = 1,2. \tag{2.7}$$

Contrarily, suppose that there exists some i such that $|(QNu)_i| = 0$, i.e., $\overline{a_i}x_1 = \overline{b_i}f_i(x_1,x_2)$. So, we have

$$\begin{aligned} dk_i &= |x_i| \\ &\le \frac{b}{a}|f_i(x_1,x_2)| \\ &\le \frac{\alpha_i b}{a}dk_1 + \frac{\beta_i b}{a}dk_2 + \frac{bM_i}{a} \\ &< \frac{\alpha_i b}{a}dk_1 + \frac{\beta_i b}{a}dk_2 + d\frac{bM_i}{a} \\ &= dk_i, \end{aligned} \tag{2.8}$$

this is a contradiction. Therefore, (2.7) holds, and hence,

$$QNu \ne 0, \quad \text{for any } u \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^2. \tag{2.9}$$

Consider the homotopy $F : (\overline{\Omega} \cap \ker L) \times [0,1] \to \overline{\Omega} \cap \ker L$, defined by

$$F(u,\mu) = -\mu\,\mathrm{diag}(\overline{a_1},\overline{a_2})u + (1-\mu)QNu, \tag{2.10}$$

for all $u \in \overline{\Omega} \cap \ker L = \overline{\Omega} \cap \mathbb{R}^2$ and $\mu \in [0,1]$.

When $u \in \partial\Omega \cap \ker L = \partial\Omega \cap \mathbb{R}^2$ and $\mu \in [0,1]$, $u = (x_1, x_2)^T$ is a constant vector in $\mathbb{R}^2$ with $|x_i| = dk_i$ for $i = 1, 2$. Thus

$$\begin{aligned}
\|F(u,\mu)\|_0 &= \max_{i=1,2} | -\mu\overline{a_i}x_i + (1-\mu)[-\overline{a_i}x_i + \overline{b_i}f_i(x_1,x_2)]| \\
&= \max_{i=1,2} | -\overline{a_i}x_i + (1-\mu)\overline{b_i}f_i(x_1,x_2)|.
\end{aligned}$$

We claim that

$$\|F(u,\mu)\|_0 > 0\,. \tag{2.11}$$

Contrarily, suppose that $\|F(u,\mu)\|_0 = 0$, then

$$\overline{a_i}x_i = (1-\mu)\overline{b_i}f_i(x_1,x_2) \quad \text{for } i = 1, 2.$$

Thus

$$\begin{aligned}
dk_i &= |x_i| \\
&= (1-\mu)\frac{\overline{b_i}}{\overline{a_i}}|f_i(x_1,x_2)| \\
&\le \frac{b}{a}|f_i(x_1,x_2)| \\
&\le \frac{\alpha_i b}{a}dk_1 + \frac{\beta_i b}{a}dk_2 + \frac{bM_i}{a} \\
&< \frac{\alpha_i b}{a}dk_1 + \frac{\beta_i b}{a}dk_2 + d\frac{bM_i}{a} \\
&= dk_i.
\end{aligned}$$

This is impossible. Thus, (2.11) holds. Therefore,

$$F(u,\mu) \ne 0 \quad \text{for } (u,\mu) \in (\partial\Omega \cap \ker L) \times [0,1].$$

From the property of invariance under a homotopy, it follows that

$$\begin{aligned}
\deg\{JQN, \Omega \cap \ker L, 0\} &= \deg\{F(\cdot \quad, 0), \Omega \cap \ker L, 0\} \\
&= \deg\{F(\cdot \quad, 1), \Omega \cap \ker L, 0\} \\
&= \operatorname{sgn}\begin{vmatrix} -\overline{a_1} & 0 \\ 0 & -\overline{a_2} \end{vmatrix} \\
&= \operatorname{sgn}\{\overline{a_1} \cdot \overline{a_2}\} \ne 0.
\end{aligned}$$

We have shown that Ω satisfies all the assumptions of Lemma 2.1. Hence, $Lu = Nu$ has at least one ω-periodic solution on $\operatorname{Dom} L \cap \overline{\Omega}$. This completes the proof. □

Corollary 2.3. *Suppose there exist positive constants M_i such that $|f_i(x_1,x_2)| \le M_i$ for $i = 1, 2$. Then system (1.4) has at least an ω-periodic solution.*

Proof. Since $|f_i(x_1,x_2)| \le M_i$ $(i = 1, 2,)$ implies that $\alpha_i, \beta_i = 0$, hence the conditions in Theorem 2.2 are all satisfied. □

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